Velocity and Acceleration Analysis

Algebraic Method

The velocity of the point $B = B_1$ on the link 1 is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \boldsymbol{\omega}_1 \times \mathbf{r}_{AB} = \boldsymbol{\omega}_1 \times \mathbf{r}_B,$$

where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$. The angular velocity of link 1 is

$$\boldsymbol{\omega}_1 = \omega_1 \mathbf{k} = \frac{\pi n}{30}\mathbf{k} = \frac{\pi(50)}{30}\mathbf{k} = 5.235\mathbf{k} \text{ rad/s}.$$

the position vector of point B is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = \mathbf{r}_B = x_B\boldsymbol{\imath} + y_B\boldsymbol{\jmath} + z_B\mathbf{k} = 0.121\boldsymbol{\imath} + 0.070\boldsymbol{\jmath} \text{ m}.$$

The velocity of point B_2 on the link 2 is $\mathbf{v}_{B_2} = \mathbf{v}_{B_1}$ because between the links 1 and 2 there is a rotational joint.

The velocity of $B_1 = B_2$ is

$$\mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & 5.235 \\ 0.121 & 0.070 & 0 \end{vmatrix} = -0.366\boldsymbol{\imath} + 0.634\boldsymbol{\jmath} \text{ m/s}.$$

The acceleration of the point $B = B_1$ on the link 1 is

$$\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \mathbf{a}_A + \boldsymbol{\alpha}_1 \times \mathbf{r}_B + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_B) = \boldsymbol{\alpha}_1 \times \mathbf{r}_B - \boldsymbol{\omega}_1^2 \mathbf{r}_B.$$

The angular acceleration of link 1 is $\boldsymbol{\alpha}_1 = \dot{\boldsymbol{\omega}}_1 = \mathbf{0}$. The acceleration of $B_1 = B_2$ is

$$\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = -3.323\boldsymbol{\imath} - 1.919\boldsymbol{\jmath} \text{ m/s}^2.$$

The velocity of the point B_3 on the link 3 is calculated in terms of the velocity of the point B_2 on the link 2

$$\mathbf{v}_{B_3} = \mathbf{v}_{B_2} + \mathbf{v}^{rel}_{B_3B_2} = \mathbf{v}_{B_2} + \mathbf{v}_{B_{32}}, \tag{1}$$

where $\mathbf{v}^{rel}_{B_3B_2} = \mathbf{v}_{B_{32}}$ is the relative acceleration of B_3 with respect to B_2 on link 3. This relative velocity is parallel to the sliding direction BC, $\mathbf{v}_{B_{32}} || BC$, or

$$\mathbf{v}_{B_{32}} = v_{B_{32}} \cos\phi_2 \boldsymbol{\imath} + v_{B_{32}} \sin\phi_2 \boldsymbol{\jmath}. \tag{2}$$

The points B_3 and C are on the link 3 and

$$\mathbf{v}_{B_3} = \mathbf{v}_C + \boldsymbol{\omega}_3 \times \mathbf{r}_{CB} = \boldsymbol{\omega}_3 \times (\mathbf{r}_B - \mathbf{r}_C), \tag{3}$$

where $\mathbf{v}_C \equiv \mathbf{0}$ and the angular velocity of link 3 is

$$\boldsymbol{\omega}_3 = \omega_3 \mathbf{k}.$$

Equations (1), (2), and (3) give

$$\begin{vmatrix} \imath & \jmath & \mathbf{k} \\ 0 & 0 & \omega_3 \\ x_B - x_C & y_B - y_C & 0 \end{vmatrix} = \mathbf{v}_{B_2} + v_{B_{32}} \cos\phi_2 \imath + v_{B_{32}} \sin\phi_2 \jmath. \tag{4}$$

Equation (4) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns ω_3 and $v_{B_{32}}$. It results

$$\omega_3 = \omega_2 = 5.448 \text{ rad/s} \quad \text{and} \quad v_{B_{32}} = 0.313 \ \text{ m/s}.$$

The acceleration of the point B_3 on the link 3 is calculated in terms of the acceleration of the point B_2 on the link 2

$$\mathbf{a}_{B_3} = \mathbf{a}_{B_2} + \mathbf{a}^{rel}_{B_3B_2} + \mathbf{a}^{cor}_{B_3B_2} = \mathbf{a}_{B_2} + \mathbf{a}_{B_{32}} + \mathbf{a}^{cor}_{B_{32}}, \tag{5}$$

where $\mathbf{a}^{rel}_{B_3B_2} = \mathbf{a}_{B_{32}}$ is the relative acceleration of B_3 with respect to B_2 on link 3. This relative acceleration is parallel to the sliding direction BC, $\mathbf{a}_{B_{32}} || BC$, or

$$\mathbf{a}_{B_{32}} = a_{B_{32}} \cos\phi_2 \imath + a_{B_{32}} \sin\phi_2 \jmath. \tag{6}$$

The Coriolis acceleration of B_3 realative to B_2 is

$$\mathbf{a}^{cor}_{B_{32}} = 2\ \boldsymbol{\omega}_3 \times \mathbf{v}_{B_{32}} = 2\ \boldsymbol{\omega}_2 \times \mathbf{v}_{B_{32}} = -0.280\imath + 3.400\jmath \ \text{ m/s}^2. \tag{7}$$

The points B_3 and C are on the link 3 and

$$\mathbf{a}_{B_3} = \mathbf{a}_C + \boldsymbol{\alpha}_3 \times \mathbf{r}_{CB} - \omega_3^2 \mathbf{r}_{CB}, \tag{8}$$

where $\mathbf{a}_C \equiv \mathbf{0}$ and the angular acceleration of link 3 is

$$\boldsymbol{\alpha}_3 = \boldsymbol{\alpha}_3 \mathbf{k}.$$

Equations (5), (6), and (8) give

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_3 \\ x_B - x_C & y_B - y_C & 0 \end{vmatrix} - \omega_3^2(\mathbf{r}_B - \mathbf{r}_C) =$$

$$\mathbf{a}_{B_2} + a_{B_{32}}(\cos\phi_2\boldsymbol{\imath} + \sin\phi_2\boldsymbol{\jmath}) + 2\ \boldsymbol{\omega}_3 \times \mathbf{v}_{B_{32}}. \tag{9}$$

Equation (9) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns α_3 and $a_{B_{32}}$. It results

$$\alpha_3 = \alpha_2 = 14.568 \text{ rad/s}^2 \quad \text{and} \quad a_{B_{32}} = -0.140 \ \ \text{m/s}^2.$$

The velocity and acceleration of D_3 are

$$\begin{aligned} \mathbf{v}_{D_3} &= \mathbf{v}_{D_4} = \mathbf{v}_C + \boldsymbol{\omega}_3 \times \mathbf{r}_{CD} = \boldsymbol{\omega}_3 \times (\mathbf{r}_D - \mathbf{r}_C) = 0.067\boldsymbol{\imath} - 0.814\boldsymbol{\jmath} \text{ m/s}, \\ \mathbf{a}_{D_3} &= \mathbf{a}_{D_4} = \mathbf{a}_C + \boldsymbol{\alpha}_3 \times \mathbf{r}_{CD} - \omega_3^2\mathbf{r}_{CD} = \boldsymbol{\alpha}_3 \times (\mathbf{r}_D - \mathbf{r}_C) - \omega_3^2(\mathbf{r}_D - \mathbf{r}_C) \\ &= 4.617\boldsymbol{\imath} - 1.811\boldsymbol{\jmath} \text{ m/s}^2. \end{aligned}$$

The velocity of the point D_5 on the link 5 is calculated in terms of the velocity of the point D_4 on the link 4

$$\mathbf{v}_{D_5} = \mathbf{v}_{D_4} + \mathbf{v}_{D_{54}}, \tag{10}$$

This relative velocity of D_5 with respect to D_4 is parallel to the sliding direction DE, $\mathbf{v}_{D_{54}}||DE$, or

$$\mathbf{v}_{D_{54}} = v_{D_{54}}\cos\phi_5\boldsymbol{\imath} + v_{D_{54}}\sin\phi_5\boldsymbol{\jmath}. \tag{11}$$

The points D_5 and E are on the link 5 and

$$\mathbf{v}_{D_5} = \mathbf{v}_E + \boldsymbol{\omega}_5 \times \mathbf{r}_{ED} = \boldsymbol{\omega}_5 \times (\mathbf{r}_D - \mathbf{r}_E), \tag{12}$$

where $\mathbf{v}_E \equiv \mathbf{0}$ and the angular velocity of link 5 is

$$\boldsymbol{\omega}_5 = \omega_5\mathbf{k}.$$

Equations (10), (11), and (12) give

$$\begin{vmatrix} \boldsymbol{\imath} & \boldsymbol{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_5 \\ x_D - x_E & y_D - y_E & 0 \end{vmatrix} = \mathbf{v}_{D_4} + v_{D_{54}}(\cos\phi_5\boldsymbol{\imath} + \sin\phi_5\boldsymbol{\jmath}). \tag{13}$$

Equation (13) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns ω_5 and $v_{D_{54}}$. It results

$$\omega_5 = \omega_4 = 0.917 \text{ rad/s} \quad \text{and} \quad v_{D_{54}} = -0.757 \quad \text{m/s}.$$

The acceleration of the point D_5 on the link 5 is calculated in terms of the acceleration of the point D_4 on the link 4

$$\mathbf{a}_{D_5} = \mathbf{a}_{D_4} + \mathbf{a}_{D_{54}} + \mathbf{a}^{cor}_{D_{54}}, \tag{14}$$

This relative acceleration $\mathbf{a}_{B_{32}}$ is parallel to the sliding direction DE, $\mathbf{a}_{D_{54}}||DE$, or

$$\mathbf{a}_{D_{54}} = a_{D_{54}} \cos\phi_5 \mathbf{\imath} + a_{D_{54}} \sin\phi_5 \mathbf{\jmath}. \tag{15}$$

The Coriolis acceleration of D_5 realative to D_4 is

$$\mathbf{a}^{cor}_{D_{54}} = 2\ \boldsymbol{\omega}_5 \times \mathbf{v}_{D_{54}} = -1.242\mathbf{\imath} - 0.623\mathbf{\jmath} \quad \text{m/s}^2. \tag{16}$$

The points D_5 and E are on the link 5 and

$$\mathbf{a}_{D_5} = \mathbf{a}_E + \boldsymbol{\alpha}_5 \times \mathbf{r}_{ED} - \omega_5^2 \mathbf{r}_{ED}, \tag{17}$$

where $\mathbf{a}_E \equiv \mathbf{0}$ and the angular acceleration of link 5 is

$$\boldsymbol{\alpha}_5 = \boldsymbol{\alpha}_5 \mathbf{k}.$$

Equations (14), (15), and (17) give

$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \alpha_5 \\ x_D - x_E & y_D - y_E & 0 \end{vmatrix} - \omega_5^2(\mathbf{r}_D - \mathbf{r}_E) = \\ \mathbf{a}_{D_4} + a_{D_{54}}(\cos\phi_5 \mathbf{\imath} + \sin\phi_5 \mathbf{\jmath}) + 2\ \boldsymbol{\omega}_5 \times \mathbf{v}_{D_{54}}. \tag{18}$$

Equation (18) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns α_5 and $a_{D_{54}}$. It results

$$\alpha_5 = \alpha_4 = -5.771 \text{ rad/s}^2 \quad \text{and} \quad a_{D_{54}} = 3.411 \text{ m/s}^2.$$

The *Mathematica*^{TM} program for the velocity and acceleration analysis using the algebraic method is given in Program 4-II.