Velocity and Acceleration Analysis

Algebraic Method

The velocity of the point $B = B_1$ on the link 1 is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_A + \omega_1 \times \mathbf{r}_{AB} = \omega_1 \times \mathbf{r}_B,$$

where $\mathbf{v}_A \equiv \mathbf{0}$ is the velocity of the origin $A \equiv O$. The angular velocity of link 1 is

$$\omega_1 = \omega_1 \mathbf{k} = \frac{\pi n}{30} \mathbf{k} = \frac{\pi (50)}{30} \mathbf{k} = 5.235 \mathbf{k} \text{ rad/s}.$$  

The position vector of point $B$ is

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = x_B \mathbf{i} + y_B \mathbf{j} + z_B \mathbf{k} = 0.121 \mathbf{i} + 0.070 \mathbf{j} \text{ m}.$$  

The velocity of point $B_2$ on the link 2 is $\mathbf{v}_{B_2} = \mathbf{v}_{B_1}$ because between the links 1 and 2 there is a rotational joint.

The velocity of $B_1 = B_2$ is

$$\mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \begin{vmatrix} 1 & j & k \\ 0 & 0 & \omega \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} 1 & j & k \\ 0 & 0 & 5.235 \\ 0.121 & 0.070 & 0 \end{vmatrix} = -0.366 \mathbf{i} + 0.634 \mathbf{j} \text{ m/s}.$$  

The acceleration of the point $B = B_1$ on the link 1 is

$$\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = \mathbf{a}_A + \mathbf{\alpha}_1 \times \mathbf{r}_B + \omega_1 \times (\omega_1 \times \mathbf{r}_B) = \mathbf{\alpha}_1 \times \mathbf{r}_B - \omega_1^2 \mathbf{r}_B.$$  

The angular acceleration of link 1 is $\mathbf{\alpha}_1 = \dot{\omega}_1 = \mathbf{0}$. The acceleration of $B_1 = B_2$ is

$$\mathbf{a}_{B_1} = \mathbf{a}_{B_2} = -3.323 \mathbf{i} - 1.919 \mathbf{j} \text{ m/s}^2.$$  

The velocity of the point $B_3$ on the link 3 is calculated in terms of the velocity of the point $B_2$ on the link 2

$$\mathbf{v}_{B_3} = \mathbf{v}_{B_2} + \mathbf{v}_{B_3}^{rel} = \mathbf{v}_{B_2} + \mathbf{v}_{B_{32}},$$  

where $\mathbf{v}_{B_{31}}^{rel} = \mathbf{v}_{B_{32}}$ is the relative acceleration of $B_3$ with respect to $B_2$ on link 3. This relative velocity is parallel to the sliding direction $BC$, $\mathbf{v}_{B_{32}} \parallel BC$, or

$$\mathbf{v}_{B_{32}} = v_{B_{32}} \cos \phi_2 \mathbf{i} + v_{B_{32}} \sin \phi_2 \mathbf{j}.$$  

(1)
The points $B_3$ and $C$ are on the link 3 and

$$\mathbf{v}_{B_3} = \mathbf{v}_C + \omega_3 \times \mathbf{r}_{CB} = \omega_3 \times (\mathbf{r}_B - \mathbf{r}_C),$$

(3)

where $\mathbf{v}_C \equiv \mathbf{0}$ and the angular velocity of link 3 is

$$\omega_3 = \omega_3 \mathbf{k}.$$

Equations (1), (2), and (3) give

$$\begin{vmatrix}
1 & \mathbf{j} & \mathbf{k} \\
0 & 0 & \omega_3 \\
x_B - x_C & y_B - y_C & 0
\end{vmatrix} = \mathbf{v}_{B_2} + \mathbf{v}_{B_{32}} \cos \phi_2 \mathbf{i} + \mathbf{v}_{B_{32}} \sin \phi_2 \mathbf{j}.$$  

(4)

Equation (4) represents a vectorial equations with two scalar components on $x$-axis and $y$-axis and with two unknowns $\omega_3$ and $\mathbf{v}_{B_{32}}$. It results

$$\omega_3 = \omega_2 = 5.448 \text{ rad/s} \quad \text{and} \quad \mathbf{v}_{B_{32}} = 0.313 \text{ m/s}.$$

The acceleration of the point $B_3$ on the link 3 is calculated in terms of the acceleration of the point $B_2$ on the link 2

$$\mathbf{a}_{B_3} = \mathbf{a}_{B_2} + \mathbf{a}_{B_{32}}^{rel} + \mathbf{a}_{B_{32}}^{cor} = \mathbf{a}_{B_2} + \mathbf{a}_{B_{32}} + \mathbf{a}_{B_{32}}^{cor},$$

(5)

where $\mathbf{a}_{B_{32}}^{rel} = \mathbf{a}_{B_{32}}$ is the relative acceleration of $B_3$ with respect to $B_2$ on link 3. This relative acceleration is parallel to the sliding direction $BC$, $\mathbf{a}_{B_{32}} \parallel BC$, or

$$\mathbf{a}_{B_{32}} = a_{B_{32}} \cos \phi_2 \mathbf{i} + a_{B_{32}} \sin \phi_2 \mathbf{j}.$$  

(6)

The Coriolis acceleration of $B_3$ relative to $B_2$ is

$$\mathbf{a}_{B_{32}}^{cor} = 2 \omega_3 \times \mathbf{v}_{B_{32}} = 2 \omega_2 \times \mathbf{v}_{B_{32}} = -0.280 \mathbf{i} + 3.400 \mathbf{j} \text{ m/s}^2.$$  

(7)

The points $B_3$ and $C$ are on the link 3 and

$$\mathbf{a}_{B_3} = \mathbf{a}_C + \alpha_3 \times \mathbf{r}_{CB} - \omega_3^2 \mathbf{r}_{CB},$$

(8)

where $\mathbf{a}_C \equiv \mathbf{0}$ and the angular acceleration of link 3 is

$$\alpha_3 = \alpha_3 \mathbf{k}.$$
Equations (5), (6), and (8) give
\[
\begin{vmatrix}
1 & \mathbf{J} & \mathbf{k} \\
0 & 0 & \alpha_3 \\
x_B - x_C & y_B - y_C & 0
\end{vmatrix}
- \omega_3^2 (r_B - r_C) =
\]
\[a_{B_2} + a_{B_32} (\cos \phi_2 \mathbf{i} + \sin \phi_2 \mathbf{j}) + 2 \omega_3 \times \mathbf{v}_{B_{32}}. \tag{9}\]
Equation (9) represents a vectorial equations with two scalar components on x-axis and y-axis and with two unknowns \(\alpha_3\) and \(a_{B_{32}}\). It results \(\alpha_3 = \alpha_2 = 14.568 \text{ rad/s}^2\) and \(a_{B_{32}} = -0.140 \text{ m/s}^2\).

The velocity and acceleration of \(D_3\) are
\[
\mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \mathbf{v}_{C} + \omega_3 \times r_{CD} = \omega_3 \times (r_D - r_C) = 0.067\mathbf{i} - 0.814\mathbf{j} \text{ m/s},
\]
\[
\mathbf{a}_{D_3} = \mathbf{a}_{D_4} = \mathbf{a}_{C} + \alpha_3 \times r_{CD} - \omega_3^2 r_{CD} = \alpha_3 \times (r_D - r_C) - \omega_3^2 (r_D - r_C)
\]
\[= 4.617\mathbf{i} - 1.811\mathbf{j} \text{ m/s}^2.\]

The velocity of the point \(D_5\) on the link 5 is calculated in terms of the velocity of the point \(D_4\) on the link 4
\[
\mathbf{v}_{D_5} = \mathbf{v}_{D_4} + \mathbf{v}_{D_{54}}, \tag{10}\]
This relative velocity of \(D_5\) with respect to \(D_4\) is parallel to the sliding direction \(DE, \mathbf{v}_{D_{54}}\parallel DE\), or
\[
\mathbf{v}_{D_{54}} = v_{D_{54}} \cos \phi_5 \mathbf{i} + v_{D_{54}} \sin \phi_5 \mathbf{j}. \tag{11}\]
The points \(D_5\) and \(E\) are on the link 5 and
\[
\mathbf{v}_{D_5} = \mathbf{v}_E + \omega_5 \times r_{ED} = \omega_5 \times (r_D - r_E), \tag{12}\]
where \(\mathbf{v}_E \equiv \mathbf{0}\) and the angular velocity of link 5 is \(\omega_5 = \omega_5 \mathbf{k}\).

Equations (10), (11), and (12) give
\[
\begin{vmatrix}
1 & \mathbf{J} & \mathbf{k} \\
0 & 0 & \omega_5 \\
x_D - x_E & y_D - y_E & 0
\end{vmatrix}
= \mathbf{v}_{D_4} + v_{D_{54}} (\cos \phi_5 \mathbf{i} + \sin \phi_5 \mathbf{j}). \tag{13}\]
Equation (13) represents a vectorial equations with two scalar components on $x$-axis and $y$-axis and with two unknowns $\omega_5$ and $v_{D54}$. It results

$\omega_5 = \omega_4 = 0.917 \text{ rad/s}$ and $v_{D54} = -0.757 \text{ m/s}$.

The acceleration of the point $D_5$ on the link 5 is calculated in terms of the acceleration of the point $D_4$ on the link 4

$$a_{D5} = a_{D4} + a_{D54} + a_{D54}^{cor},$$

(14)

This relative acceleration $a_{B32}$ is parallel to the sliding direction $DE$, $a_{D54} \parallel DE$, or

$$a_{D54} = a_{D54} \cos \phi_5 \mathbf{i} + a_{D54} \sin \phi_5 \mathbf{j}.$$  

(15)

The Coriolis acceleration of $D_5$ relative to $D_4$ is

$$a_{D54}^{cor} = 2 \omega_5 \times v_{D54} = -1.242 \mathbf{i} - 0.623 \mathbf{j} \text{ m/s}^2.$$  

(16)

The points $D_5$ and $E$ are on the link 5 and

$$a_{D5} = a_E + \alpha_5 \times r_{ED} - \omega_5^2 r_{ED},$$  

(17)

where $a_E \equiv 0$ and the angular acceleration of link 5 is

$$\alpha_5 = \alpha_5 \mathbf{k}.$$

Equations (14), (15), and (17) give

$$\begin{bmatrix} 1 & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \alpha_5 \\ x_D - x_E & y_D - y_E & 0 \end{bmatrix} \begin{bmatrix} -\omega_5^2 (r_D - r_E) \\ a_{D4} + a_{D54} (\cos \phi_5 \mathbf{i} + \sin \phi_5 \mathbf{j}) + 2 \omega_5 \times v_{D54} \end{bmatrix}.$$  

(18)

Equation (18) represents a vectorial equations with two scalar components on $x$-axis and $y$-axis and with two unknowns $\alpha_5$ and $a_{D54}$. It results

$$\alpha_5 = \alpha_4 = -5.771 \text{ rad/s}^2 \text{ and } a_{D54} = 3.411 \text{ m/s}^2.$$  

The Mathematica$^{TM}$ program for the velocity and acceleration analysis using the algebraic method is given in Program 4-II.