Final Exam

A schematic representation of a robot arm consisting of three links 1, 2, and 3 is shown in the figure. The mass centers of links 1, 2, and 3 are \( C_1, C_2, \) and \( C_3 \), respectively. The generalized coordinates (quantities associated with the instantaneous position of the system) are \( q_1(t), q_2(t) \) and \( q_3(t) \).

The central principal axes of link \( p, p = 1, 2, 3 \) are parallel to \( \hat{1}_p, \hat{2}_p, \hat{3}_p \) and the associated moments of inertia have the values \( I_{px}, I_{py}, I_{pz} \), respectively. The central inertia dyadic of link \( p \) is

\[
\bar{I}_p = (I_{px}\hat{1}_p)\hat{1}_p + (I_{py}\hat{2}_p)\hat{2}_p + (I_{pz}\hat{3}_p)\hat{3}_p.
\]

If the joint between link \( p \) and link \( p + 1 \) is a rotational joint consider a control vector moment

\[
T_{p,p+1} = T_{(p,p+1)x}1_{p+1} + T_{(p,p+1)y}2_{p+1} + T_{(p,p+1)z}3_{p+1},
\]

translational joint consider a control vector force

\[
F_{p,p+1} = F_{(p,p+1)x}1_{p+1} + F_{(p,p+1)y}2_{p+1} + F_{(p,p+1)z}3_{p+1}.
\]

Find and solve the equations of motion (eom) using Lagrange or/and Kane methods.

The examination should contain the following:

1. write-up and symbolical (MATLAB/Mathematica\textsuperscript{TM}) program for:
   - reference frame transformations \( R_{ij} \),
   - positions, \( r_{Ci} \), velocities, \( \omega_{ij}, v_{Ci} \), accelerations, \( \alpha_{ij}, a_{Ci} \), and
   - generalized (active) forces \( Q_i \). ...................................................(30 p)

2. write-up and symbolical (MATLAB/Mathematica\textsuperscript{TM}) program for:
   - Lagrange’s equations of motion or/and Kane’s dynamical equations (30 p)

3. numerical solutions for:
   - inverse dynamics, Lagrange or/and Kane methods ..............................(20 p)
   - direct dynamics, Lagrange or/and Kane methods ..............................(20 p)

Select suitable numerical values for the input numerical data.

Graduate students: use Lagrange and Kane methods.