

Final Exam

A schematic representation of a robot arm consisting of three links 1, 2, and 3 is shown in the figure. The mass centers of links 1, 2, and 3 are C_1 , C_2 , and C_3 , respectively. The generalized coordinates (quantities associated with the instantaneous position of the system) are $q_1(t)$, $q_2(t)$ and $q_3(t)$.

The central principal axes of link p , $p = 1, 2, 3$ are parallel to \mathbf{i}_p , \mathbf{j}_p , \mathbf{k}_p and the associated moments of inertia have the values I_{px} , I_{py} , I_{pz} , respectively. The central inertia dyadic of link p is

$$\bar{I}_p = (I_{px}\mathbf{i}_p)\mathbf{i}_p + (I_{py}\mathbf{j}_p)\mathbf{j}_p + (I_{pz}\mathbf{k}_p)\mathbf{k}_p.$$

If the joint between link p and link $p + 1$ is a *rotational joint* consider a control vector moment

$$\mathbf{T}_{p,p+1} = T_{(p,p+1)x}\mathbf{i}_{p+1} + T_{(p,p+1)y}\mathbf{j}_{p+1} + T_{(p,p+1)z}\mathbf{k}_{p+1},$$

translational joint consider a control vector force

$$\mathbf{F}_{p,p+1} = F_{(p,p+1)x}\mathbf{i}_{p+1} + F_{(p,p+1)y}\mathbf{j}_{p+1} + F_{(p,p+1)z}\mathbf{k}_{p+1}.$$

Find and solve the equations of motion (eom) using Lagrange or/and Kane methods.

The examination should contain the following:

1. write-up and symbolical (MATLAB/*Mathematica*TM) program for:
 - reference frame transformations R_{ij} ,
 - positions, \mathbf{r}_{C_i} , velocities, $\boldsymbol{\omega}_{ij}$, \mathbf{v}_{C_i} , accelerations, $\boldsymbol{\alpha}_{ij}$, \mathbf{a}_{C_i} , and
 - generalized (active) forces Q_i (30 p)
 2. write-up and symbolical (MATLAB/*Mathematica*TM) program for:
 - Lagrange's equations of motion or/and Kane's dynamical equations (30 p)
 3. numerical solutions for:
 - inverse dynamics, Lagrange or/and Kane methods (20 p)
 - direct dynamics, Lagrange or/and Kane methods (20 p)
- Select suitable numerical values for the input numerical data.

Graduate students: use Lagrange and Kane methods.