```
(* RT *)
Apply [Clear,Names["Global`*"]];
Off[General::spell];
Off[General::spell1];
"Method I: cartesian coordinates "
rC= {L Cos[theta[t]]/2, L Sin[theta[t]]/2, 0} ;
rA= {r[t] Cos[theta[t]], r[t] Sin[theta[t]], 0} ;
rP= {p Cos[theta[t]], p Sin[theta[t]], 0} ;
Print["rC = ",rC];
Print["rA = ",rA];
Print["rP = ",rP];
F21= {-f21 Sin[theta[t]], f21 Cos[theta[t]], 0} ;
Print["F21 = ", F21];
" F21 joint forces at Q ( f21 and p  unknowns ) "
G1= {0, -m g, 0};
G2= {0, -m g, 0};
alpha= {0, 0, theta''[t]};
IO=m L^2/3;
"IO alpha - rC x G1 - rP x F21 = 0 , (1)"
e1=IO alpha-Cross[rC,G1]-Cross[rP,F21];
e1z=Simplify[e1[[3]]]==0
" aA = d^2(rA)/dt^2 = "
aA=D[rA,{t,2}]
"m2 aA  = - F21 + G2 => "
e2= m aA  +  F21 - G2;
"(x):   m2 aAx  +  F21x = 0 (2)"
e2x=e2[[1]]==0
"(y):   m2 aAy  +  F21y - G2 = 0 (3)"
e2y=e2[[2]]==0
"IA alpha - (rP-rA) x (-F21) = 0 , (4)"
e2=IA alpha-Cross[(rP-rA),-F21];
e2z=Simplify[e2[[3]]]==0
"From Eq.(4) => "
solP=Solve[e2z,p];
ps=p/.solP[[1]];
Print["p = ",ps];
"From Eq.(2) => "
solf=Solve[e2x,f21];
f21s=f21/.solf[[1]];
Print["f21 = ",f21s];
"From Eqs. (1) and (3) => two ODE "
eq1=e1z/.solP[[1]]/.solf[[1]]
eq2=e2y/.solP[[1]]/.solf[[1]]

"numerical application"
rule={m→1.,L→1,IA→1.,g→10.};
equation1=Simplify[eq1/.rule]
equation2=Simplify[eq2/.rule]
sol=NDSolve[{equation1,equation2,r[0]==.1,theta[0]==.1,
        r'[0]==0.,theta'[0]==0.},{r,theta},{t,0.,1.}];
Plot[Evaluate[r[t]]/.sol,{t,0.,1.},PlotRange→All,
    AxesLabel→{"t[s]","r[m]"}];
Plot[Evaluate[theta[t]]/.sol,{t,0.,1.},PlotRange→All,
  AxesLabel→{"t[s]","theta[rad]"}]


 r5= (Evaluate[r[t]]/.sol/.t->.5)[[1]] ;
```

```
 t5= (Evaluate[theta[t]]/.sol/.t->.5)[[1]];
Print["r[.5] = ",r5, " m"];
Print["theta[.5] = ",t5, " rad"];
```

Method I: cartesian coordinates

$$rC = \left\{\frac{1}{2} L \cos[\theta[t]], \frac{1}{2} L \sin[\theta[t]], 0\right\}$$

$$rA = \{\cos[\theta[t]]\, r[t], r[t] \sin[\theta[t]], 0\}$$

$$rP = \{p \cos[\theta[t]], p \sin[\theta[t]], 0\}$$

$$F21 = \{-f21 \sin[\theta[t]], f21 \cos[\theta[t]], 0\}$$

F21 joint forces at Q (f21 and p unknowns)

IO alpha − rC x G1 − rP x F21 = 0 , (1)

$$\frac{1}{6}\left(-6\, f21\, p + 3\, g\, L\, m \cos[\theta[t]] + 2\, L^2\, m\, \theta''[t]\right) == 0$$

aA = d^2(rA)/dt^2 =

$$\{-2 \sin[\theta[t]]\, r'[t]\, \theta'[t] + \cos[\theta[t]]\, r''[t] + r[t]\left(-\cos[\theta[t]]\, \theta'[t]^2 - \sin[\theta[t]]\, \theta''[t]\right),\ 2 \cos[\theta[t]]\, r'[t]\, \theta'[t] + \sin[\theta[t]]\, r''[t] + r[t]\left(-\sin[\theta[t]]\, \theta'[t]^2 + \cos[\theta[t]]\, \theta''[t]\right), 0\}$$

m2 aA = − F21 + G2 =>

(x): m2 aAx + F21x = 0 (2)

$$-f21 \sin[\theta[t]] + m\left(-2 \sin[\theta[t]]\, r'[t]\, \theta'[t] + \cos[\theta[t]]\, r''[t] + r[t]\left(-\cos[\theta[t]]\, \theta'[t]^2 - \sin[\theta[t]]\, \theta''[t]\right)\right) == 0$$

(y): m2 aAy + F21y − G2 = 0 (3)

$$g\, m + f21 \cos[\theta[t]] + m\left(2 \cos[\theta[t]]\, r'[t]\, \theta'[t] + \sin[\theta[t]]\, r''[t] + r[t]\left(-\sin[\theta[t]]\, \theta'[t]^2 + \cos[\theta[t]]\, \theta''[t]\right)\right) == 0$$

IA alpha − (rP−rA) x (−F21) = 0 , (4)

$$f21\, p - f21\, r[t] + IA\, \theta''[t] == 0$$

From Eq.(4) =>

$$p = \frac{f21\, r[t] - IA\, \theta''[t]}{f21}$$

From Eq.(2) =>

$$f21 = m \csc[\theta[t]]\left(-2 \sin[\theta[t]]\, r'[t]\, \theta'[t] + \cos[\theta[t]]\, r''[t] + r[t]\left(-\cos[\theta[t]]\, \theta'[t]^2 - \sin[\theta[t]]\, \theta''[t]\right)\right)$$

From Eqs. (1) and (3) => two ODE

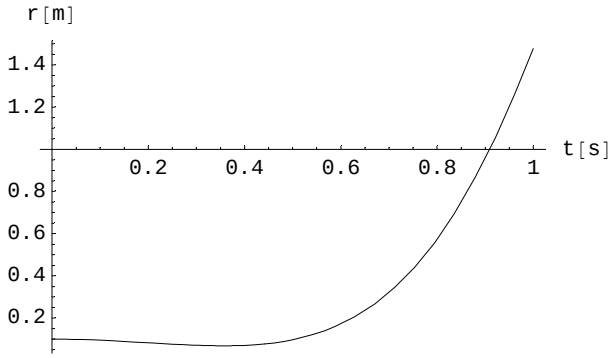
$$\frac{1}{6}\,(3\,g\,L\,m\,\text{Cos}[\text{theta}[t]] + 2\,L^2\,m\,\text{theta}''[t] - 6\,(-IA\,\text{theta}''[t] + m\,\text{Csc}[\text{theta}[t]]\,r[t]\,(-2\,\text{Sin}[\text{theta}[t]]\,r'[t]\,\text{theta}'[t] + \text{Cos}[\text{theta}[t]]\,r''[t] + r[t]\,(-\text{Cos}[\text{theta}[t]]\,\text{theta}'[t]^2 - \text{Sin}[\text{theta}[t]]\,\text{theta}''[t])))) == 0$$

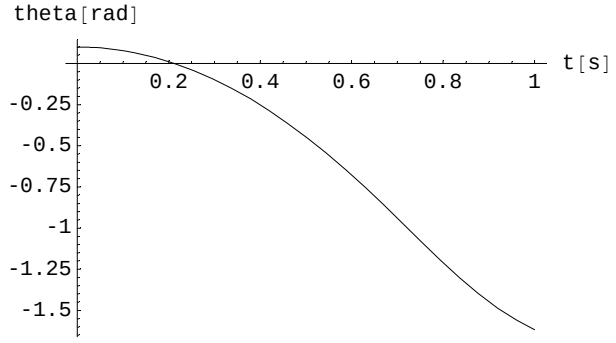
$$g\,m + m\,(2\,\text{Cos}[\text{theta}[t]]\,r'[t]\,\text{theta}'[t] + \text{Sin}[\text{theta}[t]]\,r''[t] + r[t]\,(-\text{Sin}[\text{theta}[t]]\,\text{theta}'[t]^2 + \text{Cos}[\text{theta}[t]]\,\text{theta}''[t])) + m\,\text{Cot}[\text{theta}[t]]\,(-2\,\text{Sin}[\text{theta}[t]]\,r'[t]\,\text{theta}'[t] + \text{Cos}[\text{theta}[t]]\,r''[t] + r[t]\,(-\text{Cos}[\text{theta}[t]]\,\text{theta}'[t]^2 - \text{Sin}[\text{theta}[t]]\,\text{theta}''[t])) == 0$$

numerical application

$$5.\,\text{Cos}[\text{theta}[t]] + r[t]\,(2.\,r'[t]\,\text{theta}'[t] - 1.\,\text{Cot}[\text{theta}[t]]\,r''[t]) + 1.33333\,\text{theta}''[t] + r[t]^2\,(1.\,\text{Cot}[\text{theta}[t]]\,\text{theta}'[t]^2 + 1.\,\text{theta}''[t]) == 0$$

$$10. + 0.\,\text{Cos}[\text{theta}[t]]\,r'[t]\,\text{theta}'[t] + 1.\,\text{Csc}[\text{theta}[t]]\,r''[t] == 1.\,\text{Csc}[\text{theta}[t]]\,r[t]\,\text{theta}'[t]^2$$





- Graphics **-**

r[.5] = 0.0971902 m

theta[.5] = −0.444956 rad