In[1]:=
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Apply[Clear, Names["Global`*"]];
Off[General::spell];
Off[General::spell1];

Print["Kinematics"];
omega[t] = {0, 0, theta'[t]};
Print["angular velocity of RB: omega=", omega[t]];
alpha[t] = {0, 0, theta''[t]};
Print["angular acceleration of RB: alpha=", alpha[t]];
xG = (L / 2 - D) * Sin[theta[t]];
yG = - (L / 2 - D) * Cos[theta[t]];
rG = {xG, yG, 0};
Print["position vector of G: rG=", rG];
vG = D[rG, t];
Print["velocity of G: vG=d(rG)/dt=", vG];
aG = D[vG, t];
Print["acceleration of G: aG=d(vG)/dt=", Simplify[aG]];
Print["another way of calculating vG and aG"];
vG = Cross[omega[t], rG];
Print["vG=omega x rG=", vG];
aG = Cross[alpha[t], rG] - omega[t].omega[t] * rG;
Print["aG=alpha x rG - omega.omega rG=", Simplify[aG]];

Print["Forces"];
FO = {FOx, FOy, 0};
Print["reaction force at pin joint O: FO=", FO];
G = {0, -m g, 0};
Print["gravitational force at G: G=", G];
rGO = -rG;
IG = m * L^2 / 12;
Print["mass moment of inertia wrt G: IGz=", IG];
IO = IG + m * (L / 2 - D)^2;
Print["mass moment of inertia wrt O: IOz=IGz+m*(L/2-D)^2=",
  Simplify[IO]];

Print["Method I"];
eqI = Simplify[IO * alpha[t] - Cross[rG, G]];
solutionI = Solve[eqI[[3]] ä 0, theta''[t]][[1]];
Print["moment equation: IO alpha = sum M wrt O = rG x G"];
Print[eqI[[3]], "=0"];
Print["Solution: theta''[t]=", theta''[t] /. Simplify[solutionI]];

Print["Method II"];
eqIIF = Simplify[m * aG - (FO + G)];
Print["force equation: m aG = sum F = FO + G"];
Print["projection on x:"];
Print[eqIIF[[1]], "=0", " (1)"];
Print["projection on y:"];
Print[eqIIF[[2]], "=0", " (2)"];
eqIIM = Simplify[IG * alpha[t] - Cross[rGO, FO]];
Print["moment eqation: IG alpha = sum M wrt G = -rG x FO"];
Print["projection on z: "]
Print[eqIIM[[3]], "=0", " (3)"];
solFOx = Solve[eqIIF[[1]] ä 0, FOx][[1]];
Fx = FOx /. solFOx;
Print["from Eq.(1) => FOx = ", Fx];
solFOy = Solve[eqIIF[[2]] ä 0, FOy][[1]];
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Fy = FOy /. solFOy;
Print["from Eq.(2) => FOy = ", Fy];
solutionII = Solve[(eqIIM[[3]] /. solFOx /. solFOy) ä 0, theta''[t]][[1]];
ddtheta = theta''[t] /. Simplify[solutionII];
Print["from Eqs.(1)(2)(3) => theta''[t] = ", ddtheta];

Print["Initial Conditions"];
Print["at t=0: theta[0]=Pi/2, theta'[0]=omega[0]=0"];
ic = {theta[0] Æ Pi / 2, theta'[0] Æ 0};
ddtheta0 = ddtheta /. {t Æ 0} /. ic;
Print["numerical data: m=12/32.2,L=3,D=.75,g=32.2"];
data = {m Æ 12 / 32.2, L Æ 3, D Æ .75, g Æ 32.2};
Print["theta''[0]=alpha[0]= ", ddtheta0, "=", ddtheta0 /. data, " rad/s^2"];
V = Simplify[Fy /. {t Æ 0} /. ic];
V0 = Simplify[V /. theta''[0] -> ddtheta0];
Print["V=F0y[0] = ", V, "=", V0, "=", V0 /. data, " lb"];
H = Simplify[Fx /. {t Æ 0} /. ic];
Print["H=F0x[0] = ", H, " lb"];

(* Numerical solution of differential equation *)
soldif = NDSolve[
   {(eqI[[3]] /. data) ä 0, theta[0] ä N[Pi / 2], theta'[0] ä 0}, theta[t], {t, 0, 2}];
Plot[Evaluate[theta[t] /. soldif] * 180 / Pi, {t, 0, 2},
  AxesLabel Æ {"t[s]", "theta[deg]"}];
Plot[Evaluate[D[theta[t] /. soldif, t]], {t, 0, 2},
  AxesLabel Æ {"t[s]", "omega[rad/s]"}];
Plot[Evaluate[D[theta[t] /. soldif, {t, 2}]], {t, 0, 2},
  AxesLabel Æ {"t[s]", "alpha[rad/s]"}];

DSolve[theta''[t] + C Sin[theta[t]] ä 0, theta[t], t]
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Kinematics

angular velocity of RB: omega={0, 0, theta¢[t]}

angular acceleration of RB: alpha={0, 0, theta¢¢[t]}

position vector of G: rG={(-D + L/2) Sin[theta[t]], (D - L/2) Cos[theta[t]], 0}

velocity of G: vG=d(rG)/dt={(-D + L/2) Cos[theta[t]] theta¢[t], -(D - L/2) Sin[theta[t]] theta¢[t], 0}

acceleration of G: aG=d(vG)/dt={1/2 (-2 D + L) (-Sin[theta[t]] theta¢[t]^2 + Cos[theta[t]] theta¢¢[t]),
  -1/2 (2 D - L) (Cos[theta[t]] theta¢[t]^2 + Sin[theta[t]] theta¢¢[t]), 0}

another way of calculating vG and aG

vG=omega x rG={-DCos[theta[t]] theta¢[t] + 1/2 LCos[theta[t]] theta¢[t],
  -DSin[theta[t]] theta¢[t] + 1/2 LSin[theta[t]] theta¢[t], 0}

aG=alpha x rG - omega.omega rG={1/2 (2 D - L) (Sin[theta[t]] theta¢[t]^2 - Cos[theta[t]] theta¢¢[t]),
  -1/2 (2 D - L) (Cos[theta[t]] theta¢[t]^2 + Sin[theta[t]] theta¢¢[t]), 0}

Forces

reaction force at pin joint O: FO={FOx, FOy, 0}
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gravitational force at G: G={0, -g m, 0}

mass moment of inertia wrt G: IGz=$\frac{L^2 m}{12}$

mass moment of inertia wrt O: IOz=IGz+m*(L/2-D)^2=$\frac{1}{3}(3 D^2 - 3 D L + L^2) m$

Method I

moment equation: IO alpha = sum M wrt O = rG x G

$\frac{1}{6} m (3 g (-2 D + L) \text{Sin}[\text{theta}[t]] + 2 (3 D^2 - 3 D L + L^2) \text{theta}''[t])=0$

Solution: theta''[t]=$-\frac{3 g (-2 D + L) \text{Sin}[\text{theta}[t]]}{2 (3 D^2 - 3 D L + L^2)}$

Method II

force equation: m aG = sum F = FO + G

projection on x:

$-\text{FOx} + \frac{1}{2}(2 D - L) m (\text{Sin}[\text{theta}[t]] \text{theta}'[t]^2 - \text{Cos}[\text{theta}[t]] \text{theta}''[t])=0$ (1)

projection on y:

$-\text{FOy} + g m - \frac{1}{2}(2 D - L) m (\text{Cos}[\text{theta}[t]] \text{theta}'[t]^2 + \text{Sin}[\text{theta}[t]] \text{theta}''[t])=0$ (2)

moment eqation: IG alpha = sum M wrt G = -rG x FO

projection on z:

$\frac{1}{12}(-6 (2 D - L)(\text{FOx} \text{Cos}[\text{theta}[t]] + \text{FOy} \text{Sin}[\text{theta}[t]]) + L^2 m \text{theta}''[t])=0$ (3)

from Eq.(1) => FOx = $\frac{1}{2}(2 D - L) m (\text{Sin}[\text{theta}[t]] \text{theta}'[t]^2 - \text{Cos}[\text{theta}[t]] \text{theta}''[t])$

from Eq.(2) => FOy = $g m - \frac{1}{2}(2 D - L) m (\text{Cos}[\text{theta}[t]] \text{theta}'[t]^2 + \text{Sin}[\text{theta}[t]] \text{theta}''[t])$

from Eqs.(1)(2)(3) => theta''[t] = $\frac{3 g (2 D - L) \text{Sin}[\text{theta}[t]]}{2 (3 D^2 - 3 D L + L^2)}$

Initial Conditions

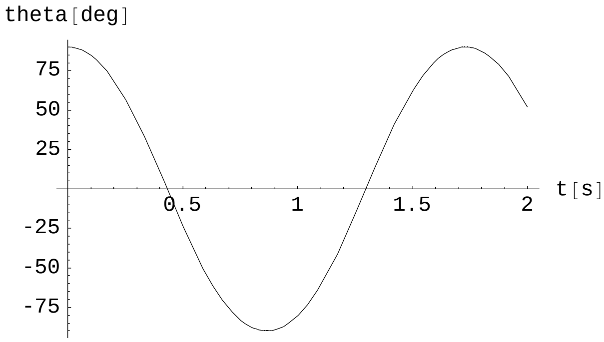
at t=0: theta[0]=Pi/2, theta'[0]=omega[0]=0

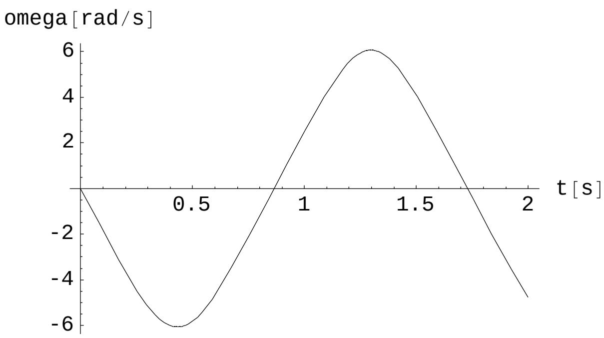
numerical data: m=12/32.2,L=3,D=.75,g=32.2

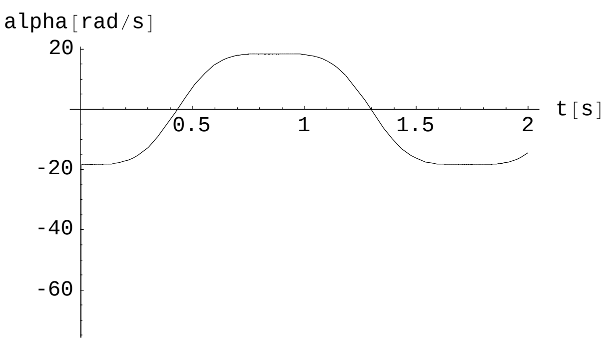
theta''[0]=alpha[0]= $\frac{3 g (2 D - L)}{2 (3 D^2 - 3 D L + L^2)}$=-18.4 rad/s^2

V=F0y[0] = $g m + \frac{1}{2}(-2 D + L) m \text{theta}''[0] = \frac{g L^2 m}{4 (3 D^2 - 3 D L + L^2)}$=6.85714 lb

H=F0x[0] = 0 lb







Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found.

Out[74]=

$$\Big\{\Big\{\text{theta}[t]\ \text{Æ}\ 2\,\text{JacobiAmplitude}\Big[-\frac{1}{2}\sqrt{2\,C\,t^2+t^2\,C[1]+4\,C\,t\,C[2]+2\,t\,C[1]\,C[2]+2\,C\,C[2]^2+C[1]\,C[2]^2},\ \frac{4\,C}{2\,C+C[1]}\Big]\Big\},\ \Big\{\text{theta}[t]\ \text{Æ}\ 2\,\text{JacobiAmplitude}\Big[\frac{1}{2}\sqrt{2\,C\,t^2+t^2\,C[1]+4\,C\,t\,C[2]+2\,t\,C[1]\,C[2]+2\,C\,C[2]^2+C[1]\,C[2]^2},\ \frac{4\,C}{2\,C+C[1]}\Big]\Big\}\Big\}$$