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## 7 Simulation of Kinematic Chains with *Mathematica*<sup>TM</sup>

A planar mechanism will be analyzed and simulated using the *Mathematica*<sup>TM</sup> software. The planar R-RTR-RTR mechanism considered is shown in Fig. 7.1. The driver link is the rigid link 1 (the link  $AB$ ). The following numerical data are given:  $AB = 0.140$  m,  $AC = 0.060$  m,  $AE = 0.250$  m,  $CD = 0.150$  m. The angle of the driver link 1 with the horizontal axis is  $\phi$ .

### 7.1 Position Analysis

The *Mathematica*<sup>TM</sup> commands for the input data are

```
AB=0.140; AC=0.060; AE=0.250; CD=0.150;
```

#### Position analysis for an input angle

The angle of the driver link 1 with the horizontal axis  $\phi = 30^\circ$ . The *Mathematica*<sup>TM</sup> command for the input angle is

```
phi=N[Pi]/6;
```

where **N[expr]** gives the numerical value of **expr** and **Pi** is the constant  $\pi$ , with numerical value approximately equal to 3.14159.

#### *Position of joint A*

A Cartesian reference frame  $xOy$  is selected. The joint  $A$  is in the origin of the reference frame, that is,  $A \equiv O$ ,

$$x_A = 0, y_A = 0. \quad (7.1)$$

#### *Position of joint C*

The coordinates of the joint  $C$  are

$$x_C = 0, y_C = AC = 0.060 \text{ m}. \quad (7.2)$$

#### *Position of joint E*

The coordinates of the joint  $E$  are

$$x_E = 0, y_E = -AE = -0.250 \text{ m}. \quad (7.3)$$

The *Mathematica*<sup>TM</sup> commands for Eqs. (7.1), (7.2), and (7.3) are

```
xA=0; yA=0;
xC=0; yC=AC;
xE=0; yE=-AE;
```

*Position of joint B*

The unknowns are the coordinates of the joint  $B$ ,  $x_B$  and  $y_B$ . Because the joint  $A$  is fixed and the angle  $\phi$  is known, the coordinates of the joint  $B$  are computed from the following expressions

$$\begin{aligned}x_B &= AB \cos \phi = 0.140 \cos 30^\circ = 0.121 \text{ m}, \\y_B &= AB \sin \phi = 0.140 \sin 30^\circ = 0.070 \text{ m}.\end{aligned}\tag{7.4}$$

The *Mathematica*<sup>TM</sup> commands for Eq. (7.4) are

```
xB=AB Cos[phi];
yB=AB Sin[phi];
```

where **phi** is the angle  $\phi$  in radians.

*Position of joint D*

The unknowns are the coordinates of the joint  $D$ ,  $x_D$  and  $y_D$ . Knowing the positions of the joints  $B$  and  $C$ , one can compute the slope  $m$  and the intercept  $b$  of the line  $BC$

$$\begin{aligned}m &= \frac{(y_B - y_C)}{(x_B - x_C)}, \\b &= y_B - m x_B.\end{aligned}\tag{7.5}$$

The *Mathematica*<sup>TM</sup> commands for Eq. (7.5) are

```
m=(yB-yC)/(xB-xC);
b=yB-m xB;
```

The joint  $D$  is located on the line  $BC$ :

$$y_D - m x_D - b = 0.\tag{7.6}$$

Furthermore, the length of the segment  $CD$  is constant:

$$(x_C - x_D)^2 + (y_C - y_D)^2 = CD^2.\tag{7.7}$$

The Eqs. (7.6) and (7.7) with *Mathematica*<sup>TM</sup> commands are

```
eqnD1=(xDsol-xC)^2+(yDsol-yC)^2-CD^2==0;
eqnD2=yDsol-m xDsol-b==0;
```

The Eqs. (7.6) and (7.7) form a system from which the coordinates of the joint  $D$  can be computed. To solve the system of equations, a specific *Mathematica*<sup>TM</sup> command will be used. The command **Solve[eqns, vars]** attempts to solve an equation or set of equations **eqns** for the variables **vars**. For the mechanism

```
solutionD=Solve[{eqnD1,eqnD2},{xDsol,yDsol}];
```

Two sets of solutions are found for the position of the joint  $D$ :

```
xD1=xDsol/.solutionD[[1]];
yD1=yDsol/.solutionD[[1]];
xD2=xDsol/.solutionD[[2]];
yD2=yDsol/.solutionD[[2]];
```

These solutions are located at the intersection of the line  $BC$  with the circle centered in  $C$  and radius  $CD$  (Fig. 7.2), and they have the following numerical values:

$$\begin{aligned}x_{D1} &= -0.149 \text{ m}, & y_{D1} &= 0.047 \text{ m}, \\x_{D2} &= 0.149 \text{ m}, & y_{D2} &= 0.072 \text{ m}.\end{aligned}$$

To determine the correct position of the joint  $D$  for the mechanism, an additional condition is needed. For the first quadrant,  $0 \leq \phi \leq 90^\circ$ , the condition is  $x_D \leq x_C$ .

This condition with *Mathematica*<sup>TM</sup> is

**If[condition, t, f]**, that gives **t** if **condition** evaluates to True, and **f** if it evaluates to False.

For the considered mechanism the following applies:

```
If[xD1<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2];
```

Because  $x_C = 0$  m, the coordinates of the joint  $D$  are

$$\begin{aligned}x_D = x_{D1} &= -0.149 \text{ m,} \\y_D = y_{D1} &= 0.047 \text{ m.}\end{aligned}$$

The numerical solutions for  $B$  and  $D$  are printed using *Mathematica*<sup>TM</sup>:

```
Print["xB = ", xB, " m" ];
Print["yB = ", yB, " m" ];
Print["xD = ", xD, " m" ];
Print["yD = ", yD, " m" ];
```

The *Mathematica*<sup>TM</sup> program for the input angle  $\phi = 30^\circ$  is given in Program 7.1. At the end of the program there are commands to draw the mechanism.

### Position analysis for a complete rotation

For a complete rotation of the driver link  $AB$ ,  $0 \leq \phi \leq 360^\circ$ , a step angle of  $\phi = 60^\circ$  is selected.

To calculate the position analysis for a complete cycle one can use the *Mathematica*<sup>TM</sup> command **For[start, test, incr, body]**. It executes **start**, then repeatedly evaluates **body** and **incr** until **test** fails to give True. For the considered mechanism the following applies:

```
For[phi=0, phi<=2*N[Pi], phi+=N[Pi]/3, Program block];
```

#### Method I

Method I uses constraint conditions for the mechanism for each quadrant. For the mechanism, there are several conditions for the position of the joint  $D$ .

For the angle  $\phi$  located in the first quadrant  $0^\circ \leq \phi \leq 90^\circ$  (Fig. 7.2), and the fourth quadrant  $270^\circ \leq \phi \leq 360^\circ$  (Fig. 7.5), the following relation exists between  $x_D$  and  $x_C$ :

$$x_D \leq x_C.$$

For the angle  $\phi$  located in the second quadrant  $90^\circ < \phi \leq 180^\circ$  (Fig. 7.3), and the third quadrant  $180^\circ < \phi < 270^\circ$  (Fig. 7.4), the following relation

exists between  $x_D$  and  $x_C$ :

$$x_D \geq x_C.$$

The following *Mathematica*<sup>TM</sup> commands are used to determine the correct position of the joint  $D$  for all four quadrants

```
If[0 <= phi <= N[Pi]/2 || 3 N[Pi]/2 <= phi <= 2 N[Pi],  
If[xD1<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2],  
If[xD1>xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2]  
];
```

where `||` is the logical OR function.

The *Mathematica*<sup>TM</sup> program for a complete rotation of the driver link using method I is given in Program 7.2. The graph of the mechanism for a complete rotation of the driver link is given in Fig. 7.6.

### Method II

Another position analysis method for a complete rotation of the driver link uses constraint conditions for the initial value of the angle  $\phi$ . For the mechanism, the correct position of the joint  $D$  is calculated using a simple function, the Euclidian distance between two points  $P$  and  $Q$ :

$$d = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}. \quad (7.8)$$

In *Mathematica*<sup>TM</sup>, the following function is introduced:

```
Dist[xP_,yP_,xQ_,yQ_] := Sqrt[(xP-xQ)^2+(yP-yQ)^2];
```

For the initial angle  $\phi = 0^\circ$ , the constraint is  $x_D \leq x_C$ , so the first position of the joint  $D$ , that is,  $D_0$ , is calculated for the first step  $D = D_0 = D_k$ ,  $k = 0$ . For the next position of the joint,  $D_{k+1}$ , there are two solutions  $D_{k+1}^I$  and  $D_{k+1}^{II}$ ,  $k = 0, 1, 2, \dots$ . In order to choose the correct solution of the joint,  $D_{k+1}$ , it is compared the distances between the old position,  $D_k$ , and each new calculated positions  $D_{k+1}^I$  and  $D_{k+1}^{II}$ . The distances between the known solution  $D_k$  and the new solutions  $D_{k+1}^I$  and  $D_{k+1}^{II}$  are  $d_k^I$  and  $d_k^{II}$ . If the distance to the first solution is less than the distance to the second solution,  $d_k^I < d_k^{II}$ , then the correct answer is  $D_{k+1} = D_{k+1}^I$ , or else  $D_{k+1} = D_{k+1}^{II}$  (Fig. 7.7).

The following *Mathematica*<sup>TM</sup> commands are used to determine the correct position of the joint *D* using a single condition for all four quadrants:

```

increment=0;
For[phi=0, phi<=2*N[Pi], phi+=N[Pi]/3,
...
If[increment==0, If[xD1<xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2],
dist1=Dist[xD1,yD1,xDold,yDold];
dist2=Dist[xD2,yD2,xDold,yDold];
If[dist1<dist2, xD=xD1;yD=yD1, xD=xD2;yD=yD2 ]
];
xDold=xD;
yDold=yD;
increment++;
...
];

```

With this algorithm the correct solution is selected using just one constraint relation for the initial step and then, automatically, the problem is solved. In this way it is not necessary to have different constraints for different quadrants.

The *Mathematica*<sup>TM</sup> program for a complete rotation of the driver link using the second method is given in Program 7.3.

## 7.2 Velocity and Acceleration Analysis

For the considered mechanism (Fig. 7.1) the driver link 1 is rotating with a constant speed of  $n = 50$  rpm. A *Mathematica*<sup>TM</sup> program for velocity and acceleration analysis is presented here.

The *Mathematica*<sup>TM</sup> commands for the angular speed, in rad/s, are

```

n=50; (* rpm *)
omega=n*N[Pi]/30; (* rad/s *)

```

The *Mathematica*<sup>TM</sup> commands for coordinates of the joints *A*, *C*, and *E* are

```

xA=0; yA=0;
xC=0; yC=AC;

```

```
xE=0; yE=-AE;
```

The coordinates of the joint  $B$  ( $B = B_1 = B_2$  on the link 1 or 2) are

$$x_B(t) = AB \cos \phi(t) \quad \text{and} \quad y_B(t) = AB \sin \phi(t).$$

To calculate symbolically the position of the joint  $B$ , the following *Mathematica*<sup>TM</sup> commands are used

```
xB=AB Cos[phi[t]];
yB=AB Sin[phi[t]];
```

where **phi[t]** represents the mathematical function  $\phi(t)$ . The function name is **phi** and it has one argument, the time **t**.

To calculate numerically the position of the joint  $B$ , the symbolic variables need to be substituted with the input data. To apply a transformation rule to a particular expression **expr**, type **expr/.lhs->rhs**. To apply a sequence of rules on each part of the expression **expr**, type **expr/.{lhs1->rhs1, lhs2->rhs2, ...}**.

For the mechanism, the transformation rule represents the initial data:

```
initdata={AB->0.14, AC->0.06, AE->0.25, CD->0.15,
phi[t]->N[Pi]/6, phi'[t]->omega, phi''[t]->0};
```

where **phi'[t]** is the first derivative of **phi** with respect to **t**, and **phi''[t]** is the second derivative of the function.

The command **Print[expr1, expr2, ...]** prints the **expr1**, **expr2**, ..., followed by a new line. To print the solutions of the position vector, the following commands are used:

```
Print["xB = ", xB," = ", xB/.initdata, " m" ];
Print["yB = ", yB," = ", yB/.initdata, " m" ];
```

The linear velocity vector of the joint  $B$  ( $B = B_1 = B_2$ ) is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \dot{x}_B \mathbf{i} + \dot{y}_B \mathbf{j},$$

where

$$\dot{x}_B = \frac{dx_B}{dt} = -AB\dot{\phi} \sin \phi \quad \text{and} \quad \dot{y}_B = \frac{dy_B}{dt} = AB\dot{\phi} \cos \phi,$$

are the components of the velocity vector of  $B$ .

To calculate symbolically the components of the velocity vector using the *Mathematica*<sup>TM</sup> the command  $\mathbf{D}[\mathbf{f}, \mathbf{t}]$  is used, which gives the derivative of  $\mathbf{f}$  with respect to  $\mathbf{t}$ :

```
vBx=D[xB,t];
vBy=D[yB,t];
```

For the mechanism  $\dot{\phi} = \omega = \frac{\pi n}{30} = \frac{\pi(50)}{30}$  rad/s = 5.235 rad/s, the numerical values are

$$\begin{aligned}\dot{x}_B &= -0.140 (5.235) \sin 30^\circ = -0.366 \text{ m/s}, \\ \dot{y}_B &= 0.140 (5.235) \cos 30^\circ = 0.634 \text{ m/s}.\end{aligned}$$

The solutions can be printed using *Mathematica*<sup>TM</sup>:

```
Print["vBx = ", vBx, " = ", vBx/.initdata, " m/s" ];
Print["vBy = ", vBy, " = ", vBy/.initdata, " m/s" ];
```

The linear acceleration vector of the joint  $B$  ( $B = B_1 = B_2$ ) is

$$\mathbf{a}_B = \ddot{x}_B \mathbf{1} + \ddot{y}_B \mathbf{j},$$

where

$$\begin{aligned}\ddot{x}_B &= \frac{d\dot{x}_B}{dt} = -AB\dot{\phi}^2 \cos \phi - AB\ddot{\phi} \sin \phi, \\ \ddot{y}_B &= \frac{d\dot{y}_B}{dt} = -AB\dot{\phi}^2 \sin \phi + AB\ddot{\phi} \cos \phi,\end{aligned}$$

are the components of the acceleration vector of the joint  $B$ .

The *Mathematica*<sup>TM</sup> commands used to calculate symbolically the components of the acceleration vector are

```
aBx=D[vBx,t];
aBy=D[vBy,t];
```

For the considered mechanism the angular acceleration of the link 1 is  $\ddot{\phi} = \dot{\omega} = 0$ . The numerical values of the acceleration of  $B$  are

$$\begin{aligned}\ddot{x}_B &= -0.140 (5.235)^2 \cos 30^\circ = -3.323 \text{ m/s}^2, \\ \ddot{y}_B &= -0.140 (5.235)^2 \sin 30^\circ = -1.919 \text{ m/s}^2.\end{aligned}$$

The solutions printed with *Mathematica*<sup>TM</sup> are

```
Print["aBx = ", aBx," = ", aBx/.initdata, " m/s^2" ];
Print["aBy = ", aBy," = ", aBy/.initdata, " m/s^2" ];
```

The coordinates of the joint  $D$  are  $x_D$  and  $y_D$ . The *Mathematica*<sup>TM</sup> commands used to calculate the position of  $D$  are

```
mBC=(yB-yC)/(xB-xC);
bBC=yB-mBC xB;
eqnD1=(xDsol-xC)^2+(yDsol-yC)^2-CD^2==0;
eqnD2=yDsol-mBC xDsol-nBC==0;
solutionD=Solve[{eqnD1,eqnD2},{xDsol,yDsol}];
```

where  $\mathbf{mBC}$  is the slope and  $\mathbf{bBC}$  is the  $y$ -intercept of the line  $BC$ . Two sets of solutions are found for the position of the joint  $D$  that are functions of the angle  $\phi(t)$  (i.e., functions of time):

```
xD1=xDsol/.solutionD[[1]];
yD1=yDsol/.solutionD[[1]];
xD2=xDsol/.solutionD[[2]];
yD2=yDsol/.solutionD[[2]];
```

To determine the correct position of the joint  $D$  for the mechanism, an additional condition is needed. For the first quadrant,  $0 \leq \phi \leq 90^\circ$ , the condition is  $x_D \leq x_C$ .

This condition using the *Mathematica*<sup>TM</sup> command is:

```
If[xD1/.initdata<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2];
```

The numerical solutions are printed using *Mathematica*<sup>TM</sup>

```
Print["xD = ", xD/.initdata, " m" ];
Print["yD = ", yD/.initdata, " m" ];
```

The linear velocity vector of the joint  $D$  ( $D = D_3 = D_4$  on link 3 or link 4) is

$$\mathbf{v}_D = \mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \dot{x}_D \mathbf{1} + \dot{y}_D \mathbf{j},$$

where

$$\dot{x}_D = \frac{dx_D}{dt} \quad \text{and} \quad \dot{y}_D = \frac{dy_D}{dt},$$

are the components of the velocity vector of the joint  $D$ , respectively, on the  $x$ -axis and the  $y$ -axis.

To calculate symbolically the components of this velocity vector the following *Mathematica*<sup>TM</sup> commands are used:

```
vDx=D[xD,t];
vDy=D[yD,t];
```

For the considered mechanism the numerical values are

$$\dot{x}_D = 0.067 \text{ m/s} \quad \text{and} \quad \dot{y}_D = -0.814 \text{ m/s}.$$

The numerical solutions are printed using *Mathematica*<sup>TM</sup>:

```
Print["vDx = ", vDx/.initdata, " m/s" ];
Print["vDy = ", vDy/.initdata, " m/s" ];
```

The linear acceleration vector of  $D = D_3 = D_4$  is

$$\mathbf{a}_D = \ddot{x}_D \mathbf{i} + \ddot{y}_D \mathbf{j},$$

where

$$\ddot{x}_D = \frac{d\dot{x}_D}{dt} \quad \text{and} \quad \ddot{y}_D = \frac{d\dot{y}_D}{dt}.$$

To calculate symbolically the components of the acceleration vector the following *Mathematica*<sup>TM</sup> commands are used:

```
aDx=D[vDx,t];
aDy=D[vDy,t];
```

The numerical values of the acceleration of  $D$  are

$$\ddot{x}_D = 4.617 \text{ m/s}^2 \quad \text{and} \quad \ddot{y}_D = -1.811 \text{ m/s}^2,$$

and can be printed using *Mathematica*<sup>TM</sup>:

```
Print["aDx = ", aDx/.initdata, " m/s^2" ];
Print["vDy = ", vDy/.initdata, " m/s^2" ];
```

The angle  $\phi_3(t)$  is determined as a function of time  $t$  from the equation of the slope of the line  $BC$ :

$$\tan \phi_3(t) = m_{BC}(t).$$

The *Mathematica*<sup>TM</sup> function **ArcTan[ $\mathbf{z}$ ]** gives the arc tangent of the number  $\mathbf{z}$ . To calculate symbolically the angle  $\phi_3$ ,

```
phi3=ArcTan[mBC];
```

The angular velocity  $\omega_3(t)$  is the derivative with respect to time of the angle  $\phi_3(t)$

$$\omega_3 = \frac{d\phi_3(t)}{dt}.$$

Symbolically, the angular velocity  $\omega_3$  is calculated using *Mathematica*<sup>TM</sup>:

```
omega3=D[phi3,t];
```

The angular acceleration  $\alpha_3(t)$  is the derivative with respect to time of the angular velocity  $\omega_3(t)$ :

$$\alpha_3(t) = \frac{d\omega_3(t)}{dt}.$$

Symbolically, using *Mathematica*<sup>TM</sup>, the angular acceleration  $\alpha_3$  is

```
alpha3=D[omega3,t];
```

The numerical values of the angles, angular velocities, and angular accelerations for the links 2 and 3 are

$$\phi_3 = \phi_2 = 0.082 \text{ rad}, \quad \omega_3 = \omega_2 = 5.448 \text{ rad/s}, \quad \alpha_3 = \alpha_2 = 14.568 \text{ rad/s}^2.$$

The numerical solutions are printed using *Mathematica*<sup>TM</sup>:

```
Print["phi3=phi2= ",phi3/.initdata," rad "];
Print["omega3=omega2= ",omega3/.initdata," rad/s"];
```

```
Print["alpha3=alpha2= ",alpha3/.initdata," rad/s^2"];
```

The angle  $\phi_5(t)$  is determined as a function of time  $t$  from the following equation:

$$\tan \phi_5(t) = \frac{y_D(t) - y_E}{x_D(t) - x_E},$$

and symbolically using *Mathematica*<sup>TM</sup>:

```
phi5=ArcTan[(yD-yE)/(xD-xE)];
```

The angular velocity  $\omega_5(t)$  is the derivative with respect to time of the angle  $\phi_5(t)$

$$\omega_5 = \frac{d\phi_5(t)}{dt}.$$

To calculate symbolically the angular velocity  $\omega_5$  using *Mathematica*<sup>TM</sup>, the following command is used:

```
omega5=D[phi5,t];
```

The angular acceleration  $\alpha_5(t)$  is the derivative with respect to time of the angular velocity  $\omega_5(t)$ :

$$\alpha_5(t) = \frac{d\omega_5(t)}{dt},$$

and it is calculated symbolically with *Mathematica*<sup>TM</sup>:

```
alpha5=D[omega5,t];
```

The numerical values of the angles, angular velocities, and angular accelerations for the links 5 and 4 are

$$\phi_5 = \phi_4 = 2.036 \text{ rad}, \quad \omega_5 = \omega_4 = 0.917 \text{ rad/s}, \quad \alpha_5 = \alpha_4 = -5.771 \text{ rad/s}^2.$$

The numerical solutions printed with *Mathematica*<sup>TM</sup> are

```
Print["phi5=phi4= ",phi5/.initdata," rad "];
Print["omega5=omega4= ",omega5/.initdata," rad/s"];
Print["alpha5=alpha4= ",alpha5/.initdata," rad/s^2"];
```

The *Mathematica*<sup>TM</sup> program for velocity and acceleration analysis is given in Program 7.4.

### 7.3 Contour Equations for Velocities and Accelerations

The same planar R-RTR-RTR mechanism is considered in Fig. 7.8(a). The driver link 1 is rotating with a constant speed of  $n = 50$  rpm. A *Mathematica*<sup>TM</sup> program for velocity and acceleration analysis using the contour equations is presented here.

The mechanism has five moving links and seven full joints. The number of independent contours is

$$n_c = c - n = 7 - 5 = 2,$$

where  $c$  is the number of joints and  $n$  is the number of moving links.

The mechanism has two independent contours. The first contour  $I$  contains the links 0, 1, 2, and 3, while the second contour  $II$  contains the links 0, 3, 4, and 5. The diagram of the mechanism is represented in Fig. 7.8(b). Clockwise paths are chosen for each closed contours  $I$  and  $II$ .

#### First contour analysis

Figure 7.9(a) shows the first independent contour  $I$  with

- rotational joint R between the links 0 and 1 (joint  $A$ );
- rotational joint R between the links 1 and 2 (joint  $B$ );
- translational joint T between the links 2 and 3 (joint  $B$ );
- rotational joint R between the links 3 and 0 (joint  $C$ ).

The angular velocity  $\omega_{10}$  of the driver link is known:

$$\omega_{10} = \omega_1 = \omega = \frac{50\pi}{30} \text{ rad/s} = 5.235 \text{ rad/s}.$$

The origin of the reference frame is the point  $A(0, 0, 0)$ .

For the velocity analysis, the following vectorial equations are used:

$$\begin{aligned} \boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{03} &= \mathbf{0}, \\ \mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{03} + \mathbf{v}_{B32}^T &= \mathbf{0}, \end{aligned} \quad (7.9)$$

where  $\mathbf{r}_{AB} = x_B\mathbf{i} + y_B\mathbf{j}$ ,  $\mathbf{r}_{AC} = x_C\mathbf{i} + y_C\mathbf{j}$ , and

$$\begin{aligned} \boldsymbol{\omega}_{10} &= \omega_{10}\mathbf{k}, \quad \boldsymbol{\omega}_{21} = \omega_{21}\mathbf{k}, \quad \boldsymbol{\omega}_{03} = \omega_{03}\mathbf{k}, \\ \mathbf{v}_{B32}^T &= \mathbf{v}_{32} = v_{32} \cos \phi_2\mathbf{i} + v_{32} \sin \phi_2\mathbf{j}. \end{aligned}$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 7.8(a) and 7.9(a). The numerical computation will then give the correct orientation of the unknown vectors. The components of the vectors  $\mathbf{r}_{AB}$  and  $\mathbf{r}_{AC}$ , and the angle  $\phi_2$  are already known from the position analysis of the mechanism. Equation (7.9) becomes

$$\omega_{10}\mathbf{k} + \omega_{21}\mathbf{k} + \omega_{03}\mathbf{k} = \mathbf{0},$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_B & y_B & 0 \\ 0 & 0 & \omega_{21} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \omega_{03} \end{vmatrix} + v_{32} \cos \phi_2 \mathbf{i} + v_{32} \sin \phi_2 \mathbf{j} = \mathbf{0}. \quad (7.10)$$

In the *Mathematica*<sup>TM</sup> environment, a three-dimensional vector  $\mathbf{v}$  is written as a list of variables  $\mathbf{v}=\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ , where  $\mathbf{x}$ ,  $\mathbf{y}$ , and  $\mathbf{z}$  are the spatial coordinates of the vector  $\mathbf{v}$ . The first component of the vector  $\mathbf{v}$  is  $\mathbf{x}=\mathbf{v}[[1]]$ , the second component is  $\mathbf{y}=\mathbf{v}[[2]]$ , and the third component is  $\mathbf{z}=\mathbf{v}[[3]]$ . For the considered mechanism with *Mathematica*<sup>TM</sup>, the following applies:

```

rB={xB, yB, 0};
rC={xC, yC, 0};
omega10v={0,0,omega};
omega21vSol={0,0,omega21Sol};
omega03vSol={0,0,omega03Sol};
v32vSol={v32Sol Cos[phi2],v32Sol Sin[phi2],0};

```

Equation (7.10) represents a system of three equations and with *Mathematica*<sup>TM</sup> commands gives

```

eqIkv=(omega10v+omega21vSol+omega03vSol)[[3]]==0;
eqIiv=(Cross[rB,omega21vSol]+Cross[rC,omega03vSol]+
v32vSol)[[1]]==0;
eqIjv=(Cross[rB,omega21vSol]+Cross[rC,omega03vSol]+
v32vSol)[[2]]==0;

```

where the command **Cross**[ $\mathbf{a}, \mathbf{b}$ ] gives the vector cross product of the vectors  $\mathbf{a}$  and  $\mathbf{b}$ .

The system of equations can be solved using the *Mathematica*<sup>TM</sup> commands

```
solIvel=Solve[{eqIkV,eqIiv,eqIjv}, {omega21Sol,
omega03Sol,v32Sol}];
```

and the following numerical solutions are obtained:

$$\omega_{21} = 0.212 \text{ rad/s}, \quad \omega_{03} = -5.448 \text{ rad/s}, \quad \text{and } v_{32} = 0.313 \text{ m/s}.$$

To print the numerical values, the following *Mathematica*<sup>TM</sup> commands are used:

```
omega21v=omega21vSol/.solIvel[[1]];
omega03v=omega03vSol/.solIvel[[1]];
v32v=v32vSol/.solIvel[[1]];
Print["omega21 = ",omega21v];
Print["omega03 = ",omega03v];
Print["v32 = ",v32v];
Print["v32r = ",v32Sol/.solIvel[[1]] ];
```

The absolute angular velocities of the links 2 and 3 are

$$\boldsymbol{\omega}_{20} = \boldsymbol{\omega}_{30} = -\boldsymbol{\omega}_{03} = 5.448 \text{ k rad/s}.$$

The absolute linear velocities of the joints *B* and *D* are

$$\begin{aligned} \mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} &= \mathbf{v}_A + \boldsymbol{\omega}_{10} \times \mathbf{r}_{AB} = -0.366 \mathbf{i} + 0.634 \mathbf{j} \text{ m/s}, \\ \mathbf{v}_D = \mathbf{v}_{D_3} = \mathbf{v}_{D_4} &= \mathbf{v}_C + \boldsymbol{\omega}_{30} \times \mathbf{r}_{CD} = 0.067 \mathbf{i} - 0.814 \mathbf{j} \text{ m/s}, \end{aligned}$$

where  $\mathbf{v}_A = \mathbf{0}$  and  $\mathbf{v}_C = \mathbf{0}$ , because the joints *A* and *C* are grounded and

$$\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}.$$

The *Mathematica*<sup>TM</sup> commands for the absolute velocities are

```
omega20v=omega30v=-omega03v;
vBv=Cross[omega10v,rB];
vDv=Cross[omega30v,(rD-rC)];
Print["omega20 = omega30 = ",omega20v];
Print["vB = ",vBv];
Print["vD = ",vDv];
```

For the acceleration analysis, the following vectorial equations are used

$$\begin{aligned} \boldsymbol{\alpha}_{10} + \boldsymbol{\alpha}_{21} + \boldsymbol{\alpha}_{03} &= \mathbf{0}, \\ \mathbf{r}_{AB} \times \boldsymbol{\alpha}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\alpha}_{03} + \mathbf{a}_{B32}^r + \mathbf{a}_{B32}^c - \omega_{10}^2 \mathbf{r}_{AB} - \omega_{20}^2 \mathbf{r}_{BC} &= \mathbf{0}. \end{aligned} \quad (7.11)$$

where

$$\begin{aligned} \boldsymbol{\alpha}_{10} &= \alpha_{10} \mathbf{k}, \quad \boldsymbol{\alpha}_{21} = \alpha_{21} \mathbf{k}, \quad \boldsymbol{\alpha}_{03} = \alpha_{03} \mathbf{k}, \\ \mathbf{a}_{B32}^r &= \mathbf{a}_{32} = a_{32} \cos \phi_2 \mathbf{i} + a_{32} \sin \phi_2 \mathbf{j}, \\ \mathbf{a}_{B32}^c &= \mathbf{a}_{32}^c = 2\boldsymbol{\omega}_{20} \times \mathbf{v}_{32}, \end{aligned}$$

The driver link has a constant angular velocity and  $\alpha_{10} = \dot{\omega}_{10} = 0$ . The acceleration vectors using the *Mathematica*<sup>TM</sup> commands are:

```
alpha10v={0,0,0};
alpha21vSol={0,0,alpha21Sol};
alpha03vSol={0,0,alpha03Sol};
a32vSol={a32Sol Cos[phi2],a32Sol Sin[phi2],0};
```

Equation (7.11) represents a system of three equations and using *Mathematica*<sup>TM</sup> commands gives

```
eqIka=(alpha10v+alpha21vSol+alpha03vSol)[[3]]==0;

eqIia=( Cross[rB,alpha21vSol]+Cross[rC,alpha03vSol]+
a32vSol+2Cross[omega20v,v32v]-(omega10v.omega10v)rB-
(omega20v.omega20v)(rC-rB) )[[1]]==0;

eqIja=( Cross[rB,alpha21vSol]+Cross[rC,alpha03vSol]+
a32vSol+2Cross[omega20v,v32v]-(omega10v.omega10v)rB-
(omega20v.omega20v)(rC-rB) )[[2]]==0;
```

The unknowns in the Eq. (7.18) are  $\alpha_{21}$ ,  $\alpha_{03}$ , and  $a_{32}$ . The system of equations is solved using the *Mathematica*<sup>TM</sup> commands

```
soliacc=Solve[{eqIka,eqIia,eqIja}, {alpha21Sol,
alpha03Sol,a32Sol}];
```

The following numerical solutions are then obtained

$$\alpha_{21} = 14.568 \text{ rad/s}^2, \quad \alpha_{03} = -14.568 \text{ rad/s}^2, \quad \text{and } a_{32} = -0.140 \text{ m/s}^2.$$

To print the numerical values, the following *Mathematica*<sup>TM</sup> commands are used:

```
alpha21v=alpha21vSol/.solIacc[[1]];
alpha03v=alpha03vSol/.solIacc[[1]];
a32v=a32vSol/.solIacc[[1]];
Print["alpha21 = ",alpha21v];
Print["alpha03 = ",alpha03v];
Print["a32 = ",a32v];
Print["a32r = ",a32Sol/.solIacc[[1]]];
```

The absolute angular accelerations of the links 2 and 3 are

$$\boldsymbol{\alpha}_{20} = \boldsymbol{\alpha}_{30} = -\boldsymbol{\alpha}_{03} = 14.568 \text{ k rad/s}^2.$$

The absolute linear accelerations of the joints *B* and *D* are obtained from the following equation:

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{10} \times \mathbf{r}_{AB} - \omega_{10}^2 \mathbf{AB} = -3.323 \mathbf{i} - 1.919 \mathbf{j} \text{ m/s}^2, \\ \mathbf{a}_D &= \mathbf{a}_C + \boldsymbol{\alpha}_{30} \times \mathbf{r}_{CD} - \omega_{30}^2 \mathbf{r}_{CD} = 4.617 \mathbf{i} - 1.811 \mathbf{j} \text{ m/s}^2, \end{aligned}$$

where  $\mathbf{a}_A = \mathbf{0}$  and  $\mathbf{a}_C = \mathbf{0}$ , because the joints *A* and *C* are grounded.

To print the absolute accelerations with *Mathematica*<sup>TM</sup>, the following relations are used:

```
alpha20v=alpha30v=-alpha03v;
aBv=-(omega10v.omega10v)rB;
aDv=Cross[alpha30v,(rD-rC)]-(omega20v.omega20v)(rD-rC);
Print["alpha20 = alpha30 = ",alpha30v];
Print["aB = ",aBv];
Print["aD = ",aDv];
```

### Second contour analysis

Figure 7.10(a) depicts the second independent contour *II*

- rotational joint R between the links 0 and 3 (joint *C*);

- rotational joint R between the links 3 and 4 (joint *D*);
- translational joint T between the links 4 and 5 (joint *D*);
- rotational joint R between the links 5 and 0 (joint *E*).

For the velocity analysis, the following vectorial equations are used

$$\begin{aligned}\boldsymbol{\omega}_{30} + \boldsymbol{\omega}_{43} + \boldsymbol{\omega}_{05} &= \mathbf{0}, \\ \mathbf{r}_{AC} \times \boldsymbol{\omega}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{05} + \mathbf{v}_{D54}^r &= \mathbf{0},\end{aligned}\quad (7.12)$$

where  $\mathbf{r}_{AD} = x_D \mathbf{i} + y_D \mathbf{j}$ ,  $\mathbf{r}_{AE} = x_E \mathbf{i} + y_E \mathbf{j}$ , and

$$\begin{aligned}\boldsymbol{\omega}_{30} &= \omega_{30} \mathbf{k}, \quad \boldsymbol{\omega}_{43} = \omega_{43} \mathbf{k}, \quad \boldsymbol{\omega}_{05} = \omega_{05} \mathbf{k}, \\ \mathbf{v}_{D54}^r &= \mathbf{v}_{54} = v_{54} \cos \phi_4 \mathbf{i} + v_{54} \sin \phi_4 \mathbf{j}.\end{aligned}$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 7.8(a) and 7.10(a). The numerical computation will then give the correct orientation of the unknown vectors. The components of the vectors  $\mathbf{r}_{AD}$  and  $\mathbf{r}_{AE}$ , and the angle  $\phi_4$  are already known from the position analysis of the mechanism.

The unknown vectors with *Mathematica*<sup>TM</sup> commands are

```
omega43vSol={0,0,omega43Sol};
omega05vSol={0,0,omega05Sol};
v54vSol={v54Sol Cos[phi4],v54Sol Sin[phi4],0};
```

Equation (7.12) becomes

$$\begin{aligned}\omega_{30} \mathbf{k} + \omega_{43} \mathbf{k} + \omega_{05} \mathbf{k} &= \mathbf{0}, \\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \omega_{30} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_D & y_D & 0 \\ 0 & 0 & \omega_{43} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_E & y_E & 0 \\ 0 & 0 & \omega_{05} \end{vmatrix} + \\ v_{54} \cos \phi_4 \mathbf{i} + v_{54} \sin \phi_4 \mathbf{j} &= \mathbf{0}.\end{aligned}\quad (7.13)$$

Equation (7.13) projected onto the “fixed” reference frame *Oxyz* gives

$$\begin{aligned}\omega_{30} + \omega_{43} + \omega_{05} &= 0, \\ y_C \omega_{30} + y_D \omega_{43} + y_E \omega_{05} + v_{54} \cos \phi_4 &= 0, \\ -x_C \omega_{30} - x_D \omega_{43} - x_E \omega_{05} + v_{54} \sin \phi_4 &= 0.\end{aligned}\quad (7.14)$$

The above system of equations using the following *Mathematica*<sup>TM</sup> commands becomes

```
eqIIkv=(omega30v+omega43vSol+omega05vSol)[[3]]==0;
eqIIiv=(Cross[rC,omega30v]+Cross[rD,omega43vSol]+
Cross[rE,omega05vSol]+v54vSol)[[1]]==0;
eqIIjv=(Cross[rC,omega30v]+Cross[rD,omega43vSol]+
Cross[rE,omega05vSol]+v54vSol)[[2]]==0;
```

Equation (7.14) represents an algebraic system of three equations with three unknowns:  $\omega_{43}$ ,  $\omega_{05}$ , and  $v_{54}$ . The system is solved using the *Mathematica*<sup>TM</sup> commands

```
solIIvel=Solve[{eqIIkv,eqIIiv,eqIIjv}, {omega43Sol,
omega05Sol,v54Sol}];
```

The following numerical solutions are obtained:

$$\omega_{43} = -4.531 \text{ rad/s}, \quad \omega_{05} = -0.917 \text{ rad/s}, \quad \text{and} \quad v_{54} = 0.757 \text{ m/s}.$$

To print the numerical values with *Mathematica*<sup>TM</sup>, the following commands are used:

```
omega43v=omega43vSol/.solIIvel[[1]];
omega05v=omega05vSol/.solIIvel[[1]];
v54v=v54vSol/.solIIvel[[1]];
Print["omega43 = ",omega43v];
Print["omega05 = ",omega05v];
Print["v54 = ",v54v];
Print["v54r=",v54Sol/.solIIvel[[1]] ];
```

The absolute angular velocities of the links 4 and 5 are

$$\omega_{40} = \omega_{50} = -\omega_{05} = 0.917 \text{ k rad/s}, \quad (7.15)$$

and with *Mathematica*<sup>TM</sup> commands, they are

```
omega40v=omega50v=-omega05v;
```

```
Print["omega40 = omega50 = ",omega50v];
```

For the acceleration analysis, the following vectorial equations are used:

$$\begin{aligned}\boldsymbol{\alpha}_{30} + \boldsymbol{\alpha}_{43} + \boldsymbol{\alpha}_{05} &= \mathbf{0}, \\ \mathbf{r}_{AC} \times \boldsymbol{\alpha}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\alpha}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\alpha}_{05} + \mathbf{a}_{D54}^r + \mathbf{a}_{B54}^c - \omega_{30}^2 \mathbf{r}_{CD} - \omega_{40}^2 \mathbf{r}_{DE} &= \mathbf{0}.\end{aligned}\quad (7.16)$$

where

$$\begin{aligned}\boldsymbol{\alpha}_{30} &= \alpha_{30} \mathbf{k}, \quad \boldsymbol{\alpha}_{43} = \alpha_{43} \mathbf{k}, \quad \boldsymbol{\alpha}_{05} = \alpha_{05} \mathbf{k}, \\ \mathbf{a}_{B54}^r &= \mathbf{a}_{54} = a_{54} \cos \phi_4 \mathbf{i} + a_{54} \sin \phi_4 \mathbf{j}, \\ \mathbf{a}_{B54}^c &= 2\boldsymbol{\omega}_{40} \times \mathbf{v}_{54}.\end{aligned}$$

The unknown acceleration vectors using the *Mathematica*<sup>TM</sup> commands are

```
alpha43vSol={0,0,alpha43Sol};  
alpha05vSol={0,0,alpha05Sol};  
a54vSol={a54Sol Cos[phi4],a54Sol Sin[phi4],0};
```

Equation (7.16) becomes

$$\begin{aligned}\alpha_{30} \mathbf{k} + \alpha_{43} \mathbf{k} + \alpha_{05} \mathbf{k} &= \mathbf{0}, \\ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \alpha_{30} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_D & y_D & 0 \\ 0 & 0 & \alpha_{43} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_E & y_E & 0 \\ 0 & 0 & \alpha_{05} \end{vmatrix} + \\ a_{54} \cos \phi_4 \mathbf{i} + a_{54} \sin \phi_4 \mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \omega_{40} \\ v_{54} \cos \phi_4 & v_{54} \sin \phi_4 & 0 \end{vmatrix} - \\ \omega_{30}^2 [(x_D - x_C) \mathbf{i} + (y_D - y_C) \mathbf{j}] - \\ \omega_{40}^2 [(x_E - x_D) \mathbf{i} + (y_E - y_D) \mathbf{j}] &= \mathbf{0}.\end{aligned}\quad (7.17)$$

Equation (7.17) can be rewritten as

$$\begin{aligned}\alpha_{30} + \alpha_{43} + \alpha_{05} &= 0, \\ y_C \alpha_{30} + y_D \alpha_{43} + y_E \alpha_{05} + a_{54} \cos \phi_4 - 2\omega_{40} v_{54} \sin \phi_4 - \\ \omega_{30}^2 (x_D - x_C) - \omega_{40}^2 (x_E - x_D) &= 0, \\ -x_C \alpha_{30} - x_D \alpha_{43} - x_E \alpha_{05} + a_{54} \sin \phi_4 + 2\omega_{40} v_{54} \cos \phi_4 - \\ \omega_{30}^2 (y_D - y_C) - \omega_{40}^2 (y_E - y_D) &= 0.\end{aligned}\quad (7.18)$$

The contour acceleration equations using *Mathematica*<sup>TM</sup> commands are

```

eqIIka=(alpha30v+alpha43vSol+alpha05vSol)[[3]]==0;

eqIIia=( Cross[rC,alpha30v]+Cross[rD,alpha43vSol]+
Cross[rE,alpha05vSol]+a54vSol+2Cross[omega40v,v54v]-
(omega30v.omega30v)(rD-rC)-
(omega40v.omega40v)(rE-rD) )[[1]]==0;

eqIIja=( Cross[rC,alpha30v]+Cross[rD,alpha43vSol]+
Cross[rE,alpha05vSol]+a54vSol+2Cross[omega40v,v54v]-
(omega30v.omega30v)(rD-rC)-
(omega40v.omega40v)(rE-rD) )[[2]]==0;

```

The unknowns in Eq. (7.18) are  $\alpha_{43}$ ,  $\alpha_{05}$ , and  $a_{54}$ . To solve the system, the following *Mathematica*<sup>TM</sup> command is used:

```

solIIacc=Solve[{eqIIka,eqIIia,eqIIja}, {alpha43Sol,
alpha05Sol,a54Sol}];

```

The following numerical solutions are obtained:

$$\alpha_{43} = -20.339 \text{ rad/s}^2, \quad \alpha_{05} = 5.771 \text{ rad/s}^2, \quad \text{and} \quad a_{54} = 3.411 \text{ m/s}^2.$$

The *Mathematica*<sup>TM</sup> commands are

```

alpha43v=alpha43vSol/.solIIacc[[1]];
alpha05v=alpha05vSol/.solIIacc[[1]];
a54v=a54vSol/.solIIacc[[1]];
Print["alpha43 = ",alpha43v];
Print["alpha05 = ",alpha05v];
Print["a54 = ",a54v];
Print["a54r=",a54Sol/.solIIacc[[1]]];

```

The absolute angular accelerations of the links 4 and 5 are

$$\alpha_{40} = \alpha_{50} = -\alpha_{05} = -5.771 \text{ k rad/s}^2,$$

and with *Mathematica*<sup>TM</sup> they are

```
alpha40v=alpha50v=-alpha05v;
Print["alpha40 = alpha50 = ",alpha50v];
```

The Mathematica<sup>TM</sup> program for the velocity and acceleration analysis using the contour method is given in Program 7.5.

## 7.4 Dynamic Force Analysis

In this section the motor moment  $\mathbf{M}_m$  required for the dynamic equilibrium of the considered mechanism, shown in Fig. 7.11(a), is calculated. The joint reaction forces are also calculated. The widths of the links 1, 3, and 5 are  $AB = 0.140$  m,  $FD = 0.400$  m, and respectively,  $EG = 0.500$  m. The height of the links 1, 3, and 5 is  $h = 0.010$  m. The width of the links 2 and 4 is  $w_{Slider} = 0.050$  m, and the height is  $h_{Slider} = 0.020$  m. All five moving links are rectangular prisms with the depth  $d = 0.001$  m. The angle of the driver link is  $\phi = \frac{\pi}{6}$  rad and the angular velocity is  $n = 50$  rpm. The external moment applied on link 5 is opposed to the motion of the link. Because  $\omega_5 = 0.917$  k rad/s, the external moment vector will be  $\mathbf{M}_e = -100$  k N·m. The density of the material is  $\rho_{Steel} = \rho = 8000$  kg/m<sup>3</sup>. The gravitational acceleration is  $g = 9.807$  m/s<sup>2</sup>. The center of mass locations of the links  $i = 1, 2, \dots, 5$  are designated by  $C_i(x_{C_i}, y_{C_i}, 0)$ .

The input data are introduced using a Mathematica<sup>TM</sup> rule:

```
rule={AB->0.14,AC->0.06,AE->0.25,CD->0.15,FD->0.4,
EG->0.5,h->0.01,d->0.001,hSlider->0.02,wSlider->0.05,
rho->8000,g->9.807,Me->-100.,
phi[t]->N[Pi]/6,phi'[t]->omega,phi''[t]->0};
```

where  $\omega = n * N[Pi] / 30$ .

### Inertia forces and moments

To calculate the inertia moment  $\mathbf{M}_i$  and the total force  $\mathbf{F}_i$  for the link  $i = 1, 2, \dots, 5$ , the mass  $m_i$ , the acceleration of the center of mass  $\mathbf{a}_{C_i}$ , the gravity force  $\mathbf{G}_i$ , and the mass moment of inertia  $I_{C_i}$  are needed.

*Link 1*

The mass of the link is

$$m_1 = \rho AB h d.$$

The position, velocity, and acceleration for the center of mass  $C_1$  are

$$\mathbf{r}_{C_1} = \mathbf{r}_B/2, \quad \mathbf{v}_{C_1} = \mathbf{v}_B/2, \quad \text{and} \quad \mathbf{a}_{C_1} = \mathbf{a}_B/2.$$

The inertia force is

$$\mathbf{F}_{in1} = -m_1 \mathbf{a}_{C_1}.$$

The gravitational force is

$$\mathbf{G}_1 = -m_1 g \mathbf{k}.$$

The total force on link 1 at the mass center  $C_1$  is

$$\mathbf{F}_1 = \mathbf{F}_{in1} + \mathbf{G}_1.$$

The mass moment of inertia is

$$I_{C_1} = m_1 (AB^2 + h^2)/12.$$

The moment of inertia is

$$\mathbf{M}_1 = \mathbf{M}_{in1} = -I_{C_1} \boldsymbol{\alpha}_1.$$

To calculate and print the numerical values of the total force  $\mathbf{F}_1$  and the moment  $\mathbf{M}_1$ , the following *Mathematica*<sup>TM</sup> commands are used:

```

m1=rho AB h d /.rule;
rC1=rB/2; vC1=vB/2; aC1=aB/2;
Fin1=-m1 aC1 /.rule;
G1={0,-m1*g,0} /.rule;
F1=(Fin1+G1) /.rule;
IC1=m1 (AB^2+h^2)/12 /.rule;
M1=Min1=-IC1 alpha1 /.rule;
Print["F1 = ",F1];
Print["M1 = ",M1];

```

*Link 2*

The mass of the link is

$$m_2 = \rho h_{Slider} w_{Slider} d.$$

The position, velocity, and acceleration for the center of mass  $C_2$  are

$$\mathbf{r}_{C_2} = \mathbf{r}_B, \quad \mathbf{v}_{C_2} = \mathbf{v}_B, \quad \text{and} \quad \mathbf{a}_{C_2} = \mathbf{a}_B.$$

The inertia force is

$$\mathbf{F}_{in2} = -m_2 \mathbf{a}_{C_2}.$$

The gravitational force is

$$\mathbf{G}_2 = -m_2 g \mathbf{k}.$$

The total force on slider 2 at  $B$  is

$$\mathbf{F}_2 = \mathbf{F}_{in2} + \mathbf{G}_2.$$

The mass moment of inertia is

$$I_{C_2} = m_2 (h_{Slider}^2 + w_{Slider}^2)/12.$$

The moment of inertia is

$$\mathbf{M}_2 = \mathbf{M}_{in2} = -I_{C_2} \boldsymbol{\alpha}_2.$$

The *Mathematica*<sup>TM</sup> commands for the total force  $\mathbf{F}_2$  and the moment  $\mathbf{M}_2$  are

```

m2=rho hSlider wSlider d /.rule;
rC2=rB; vC2=vB; aC2=aB;
Fin2=-m2 aC2 /.rule;
G2={0,-m2*g,0} /.rule;
F2=(Fin2+G2) /.rule;
IC2=m2 (hSlider^2+wSlider^2)/12 /.rule;
M2=Min2=-IC2 alpha2 /.rule;
Print["F2 = ",F2];
Print["M2 = ",M2];

```

*Link 3*

The mass of the link is

$$m_3 = \rho FD h d.$$

The position, velocity, and acceleration for the center of mass  $C_3$  are

$$x_{C_3} = x_C + (FD/2 - CD) \cos \phi_3, \quad y_{C_3} = y_C + (FD/2 - CD) \sin \phi_3,$$

$$\mathbf{r}_{C_3} = x_{C_3} \mathbf{i} + y_{C_3} \mathbf{j}, \quad \mathbf{v}_{C_3} = \dot{x}_{C_3} \mathbf{i} + \dot{y}_{C_3} \mathbf{j}, \quad \text{and} \quad \mathbf{a}_{C_3} = \ddot{x}_{C_3} \mathbf{i} + \ddot{y}_{C_3} \mathbf{j}.$$

The inertia force is

$$\mathbf{F}_{in3} = -m_3 \mathbf{a}_{C_3}.$$

The gravitational force is

$$\mathbf{G}_3 = -m_3 g \mathbf{k}.$$

The total force at  $C_3$  is

$$\mathbf{F}_3 = \mathbf{F}_{in3} + \mathbf{G}_3.$$

The mass moment of inertia is

$$I_{C_3} = m_3 (FD^2 + h^2)/12.$$

The total moment on link 3 is

$$\mathbf{M}_3 = \mathbf{M}_{in3} = -I_{C_3} \boldsymbol{\alpha}_3.$$

The force  $\mathbf{F}_3$  and the moment  $\mathbf{M}_3$  with Mathematica<sup>TM</sup> are

```

m3=rho FD h d /.rule;
xC3=xC+(FD/2-CD) Cos[phi3];
yC3=yC+(FD/2-CD) Sin[phi3];
rC3={xC3,yC3,0};
vC3=D[rC3,t];
aC3=D[D[rC3,t],t];
Fin3=-m3 aC3 /.rule;
G3={0,-m3*g,0} /.rule;
F3=(Fin3+G3) /.rule;
IC3=m3 (FD^2+h^2)/12 /.rule;
M3=Min3=-IC3 alpha3 /.rule;
Print["F3 = ",F3];
Print["M3 = ",M3];

```

*Link 4*

The mass of the link is

$$m_4 = \rho h_{Slider} w_{Slider} d.$$

The position, velocity, and acceleration for the center of mass  $C_4$  are

$$\mathbf{r}_{C_4} = \mathbf{r}_D, \quad \mathbf{v}_{C_4} = \mathbf{v}_D, \quad \text{and} \quad \mathbf{a}_{C_4} = \mathbf{a}_D.$$

The inertia force is

$$\mathbf{F}_{in4} = -m_4 \mathbf{a}_{C_4}.$$

The gravitational force is

$$\mathbf{G}_4 = -m_4 g \mathbf{k}.$$

The total force on slider 4 at  $D$  is

$$\mathbf{F}_4 = \mathbf{F}_{in4} + \mathbf{G}_4.$$

The mass moment of inertia is

$$I_{C_4} = m_4(h_{Slider}^2 + w_{Slider}^2)/12.$$

The moment of inertia is

$$\mathbf{M}_4 = \mathbf{M}_{in4} = -I_{C_4} \boldsymbol{\alpha}_4.$$

To calculate and print the numerical values of the total force  $\mathbf{F}_4$  and the moment  $\mathbf{M}_4$ , the following *Mathematica*<sup>TM</sup> commands are used:

```

m4=rho hSlider wSlider d /.rule;
rC4=rD; vC4=vD; aC4=aD;
Fin4=-m4 aC4 /.rule;
G4={0,-m4*g,0} /.rule;
F4=(Fin4+G4) /.rule;
IC4=m4 (hSlider^2+wSlider^2)/12 /.rule;
M4=Min4=-IC4 alpha4 /.rule;
Print["F4 = ",F4];
Print["M4 = ",M4];

```

*Link 5*

The mass of the link is

$$m_5 = \rho EG h d.$$

The position, velocity, and acceleration for the center of mass  $C_5$  are

$$x_{C_5} = (EG/2) \cos \phi_5, \quad y_{C_5} = (EG/2) \sin \phi_5,$$

$$\mathbf{r}_{C_5} = x_{C_5} \mathbf{i} + y_{C_5} \mathbf{j}, \quad \mathbf{v}_{C_5} = \dot{x}_{C_5} \mathbf{i} + \dot{y}_{C_5} \mathbf{j}, \quad \text{and} \quad \mathbf{a}_{C_5} = \ddot{x}_{C_5} \mathbf{i} + \ddot{y}_{C_5} \mathbf{j}.$$

The inertia force is

$$\mathbf{F}_{in5} = -m_5 \mathbf{a}_{C_5}.$$

The gravitational force is

$$\mathbf{G}_5 = -m_5 g \mathbf{k}.$$

The total force on link 5 at  $C_5$  is

$$\mathbf{F}_5 = \mathbf{F}_{in5} + \mathbf{G}_5.$$

The mass moment of inertia is

$$I_{C_5} = m_5 (EG^2 + h^2)/12.$$

The moment of inertia is

$$\mathbf{M}_5 = \mathbf{M}_{in5} = -I_{C_5} \boldsymbol{\alpha}_5.$$

The total force  $\mathbf{F}_5$  and the moment  $\mathbf{M}_5$  with *Mathematica*<sup>TM</sup> are

```

m5=rho EG h d /.rule;
xC5=EG/2 Cos[phi5];
yC5=EG/2 Sin[phi5];
rC5={xC5,yC5,0};
vC5=D[rC5,t];
aC5=D[D[rC5,t],t];
Fin5=-m5 aC5 /.rule;
G5={0,-m5*g,0} /.rule;
F5=(Fin5+G5) /.rule;
IC5=m5 (EG^2+h^2)/12 /.rule;
M5=Min5=-IC5 alpha5 /.rule;
M5e={0,0,Me} /.rule;

```

```
Print["F5 = ",F5];
Print["M5 = ",M5];
```

The numerical values are

$$\begin{aligned}\mathbf{F}_1 &= 0.018\mathbf{i} - 0.099\mathbf{j} \text{ N}, \quad \mathbf{M}_1 = \mathbf{0} \text{ N} \cdot \text{m}, \\ \mathbf{F}_2 &= 0.026\mathbf{i} - 0.063\mathbf{j} \text{ N}, \quad \mathbf{M}_2 = -0.00002\mathbf{k} \text{ N} \cdot \text{m}, \\ \mathbf{F}_3 &= 0.049\mathbf{i} - 0.333\mathbf{j} \text{ N}, \quad \mathbf{M}_3 = -0.00621\mathbf{k} \text{ N} \cdot \text{m}, \\ \mathbf{F}_4 &= -0.036\mathbf{i} - 0.063\mathbf{j} \text{ N}, \quad \mathbf{M}_4 = 0.00001\mathbf{k} \text{ N} \cdot \text{m}, \\ \mathbf{F}_5 &= -0.055\mathbf{i} - 0.410\mathbf{j} \text{ N}, \quad \mathbf{M}_5 = 0.00481\mathbf{k} \text{ N} \cdot \text{m}.\end{aligned}$$

### Joint reaction forces

The diagram representing the mechanism is shown in Fig. 7.11(b). It has two contours 0-1-2-3-0 and 0-3-4-5-0.

Reaction force  $\mathbf{F}_{05}$

The rotation joint  $E_R$  between the links 0 and 5 is replaced with the unknown reaction force  $\mathbf{F}_{05}$  (Fig. 7.12)

$$\mathbf{F}_{05} = F_{05x}\mathbf{i} + F_{05y}\mathbf{j}.$$

With *Mathematica*<sup>TM</sup>, the force  $\mathbf{F}_{05}$  is written as

```
F05Sol={F05xSol,F05ySol,0};
```

Following the path  $I$ , as shown in Fig. 7.12, a force equation is written for the translation joint  $D_T$ . The projection of all forces, that act on the link 5, onto the sliding direction  $\mathbf{r}_{DE}$  is zero:

$$\sum \mathbf{F}^{(5)} \cdot \mathbf{r}_{DE} = (\mathbf{F}_5 + \mathbf{F}_{05}) \cdot \mathbf{r}_{DE} = 0, \quad (7.19)$$

where  $\mathbf{r}_{DE} = \mathbf{r}_{AE} - \mathbf{r}_{AD}$ .

Equation (7.19) with *Mathematica*<sup>TM</sup> becomes

```
rDE=(rE-rD)/.rule;
eqER1=(F5+F05Sol).rDE==0;
```

where the command **a.b** gives the scalar product of the vectors **a** and **b**. Continuing on the path *I*, a moment equation is written for the rotation joint  $D_R$ :

$$\sum \mathbf{M}_D^{(4\&5)} = \mathbf{r}_{DE} \times \mathbf{F}_{05} + \mathbf{r}_{DC_5} \times \mathbf{F}_5 + \mathbf{M}_4 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}, \quad (7.20)$$

where  $\mathbf{r}_{DC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AD}$ .

Equation (7.20) with *Mathematica*<sup>TM</sup> gives

```
rDC5=(rC5-rD)/.rule;
eqER2=(Cross[rDE,F05Sol]+Cross[rDC5,F5]+
M4+M5+M5e)[[3]]==0;
```

The system of two equations is solved using *Mathematica*<sup>TM</sup> command

```
solF05=Solve[{eqER1,eqER2},{F05xSol,F05ySol}];
```

The following numerical solution is obtained:

$$\mathbf{F}_{05} = 268.127\mathbf{i} + 135.039\mathbf{j} \text{ N.}$$

Reaction force  $\mathbf{F}_{45}$

The translation joint  $D_T$  between the links 4 and 5 is replaced with the unknown reaction force  $\mathbf{F}_{45}$  (Fig. 7.13):

$$\mathbf{F}_{45} = -\mathbf{F}_{54} = F_{45x}\mathbf{i} + F_{45y}\mathbf{j}.$$

The position of the application point  $P$  of the force  $\mathbf{F}_{45}$  is unknown

$$\mathbf{r}_{AP} = x_P\mathbf{i} + y_P\mathbf{j},$$

where  $x_P$  and  $y_P$  are the plane coordinates of the point  $P$ .

The force  $\mathbf{F}_{45}$  and its point of application  $P$  with *Mathematica*<sup>TM</sup> is written as

```
F45Sol={F45xSol,F45ySol,0};
```

```
rPSol={xPSol,yPSol,0};
```

Following the path *I* (Fig. 7.13), a moment equation is written for the rotation joint  $E_R$

$$\sum \mathbf{M}_E^{(5)} = \mathbf{r}_{EP} \times \mathbf{F}_{45} + \mathbf{r}_{EC_5} \times \mathbf{F}_5 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}, \quad (7.21)$$

where  $\mathbf{r}_{EP} = \mathbf{r}_{AP} - \mathbf{r}_{AE}$ , and  $\mathbf{r}_{EC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AE}$ .

One can write Eq. (7.21) using the *Mathematica*<sup>TM</sup> commands

```
rEP=(rPSol-rE)/.rule;
rEC5=(rC5-rE)/.rule;
eqDT1=(Cross[rEP,F45Sol]+Cross[rEC5,F5]+
M5+M5e)[[3]]==0;
```

Following the path *II* (Fig. 7.13), a moment equation is written for the rotation joint  $D_R$ :

$$\sum \mathbf{M}_D^{(4)} = \mathbf{r}_{DP} \times \mathbf{F}_{54} + \mathbf{M}_4 = \mathbf{0}, \quad (7.22)$$

where  $\mathbf{r}_{DP} = \mathbf{r}_{AP} - \mathbf{r}_{AD}$  and  $\mathbf{F}_{54} = -\mathbf{F}_{45}$ .

Equation (7.22) with *Mathematica*<sup>TM</sup> is

```
rDP=(rPSol-rD)/.rule;
eqDT2=(Cross[rDP,F54Sol]+M4)[[3]]==0;
```

The direction of the unknown joint force  $\mathbf{F}_{45}$  is perpendicular to the sliding direction  $\mathbf{r}_{DE}$

$$\mathbf{F}_{45} \cdot \mathbf{r}_{DE} = 0, \quad (7.23)$$

and using *Mathematica*<sup>TM</sup> command

```
eqDT3=F45Sol.rDE==0;
```

The application point  $P$  of the force  $\mathbf{F}_{45}$  is located on the direction  $DE$ , that is

$$\frac{y_D - y_E}{x_D - x_E} = \frac{y_P - y_E}{x_P - x_E}. \quad (7.24)$$

One can write Eq. (7.24) using the *Mathematica*<sup>TM</sup> commands

```
eqDT4=((yD-yE)/(xD-xE)/.rule)==
((yPSol-yE)/(xPSol-xE)/.rule);
```

The system of four equations is solved using the *Mathematica*<sup>TM</sup> command

```
solF45=Solve[{eqDT1,eqDT2,eqDT3,eqDT4},
{F45xSol,F45ySol,xPSol,yPSol}];
```

The following numerical solutions are obtained:

$$\mathbf{F}_{45} = -268.072\mathbf{i} - 134.628\mathbf{j} \text{ N and } \mathbf{r}_{AP} = -0.149\mathbf{i} + 0.047\mathbf{j} \text{ m.}$$

Reaction force  $\mathbf{F}_{34}$

The rotation joint  $D_R$  between the links 3 and 4 is replaced with the unknown reaction force  $\mathbf{F}_{34}$  (Fig. 7.14):

$$\mathbf{F}_{34} = -\mathbf{F}_{34} = F_{34x}\mathbf{i} + F_{34y}\mathbf{j},$$

and with *Mathematica*<sup>TM</sup>

```
F34Sol={F34xSol,F34ySol,0};
```

Following the path  $I$ , a force equation can be written for the translation joint  $D_T$ . The projection of all forces, that act on the link 4, onto the sliding direction  $ED$  is zero:

$$\sum \mathbf{F}^{(4)} \cdot \mathbf{ED} = (\mathbf{F}_4 + \mathbf{F}_{34}) \cdot \mathbf{r}_{ED} = 0, \quad (7.25)$$

where  $\mathbf{r}_{ED} = \mathbf{r}_{AD} - \mathbf{r}_{AE}$ .

Equation (7.25) using *Mathematica*<sup>TM</sup> gives

```
rED=(rD-rE)/.rule;
eqDR1=(F4+F34Sol).rED==0;
```

Continuing on the path  $I$  (Fig. 7.14), a moment equation is written for the rotation joint  $E_R$ :

$$\sum \mathbf{M}_E^{(4\&5)} = \mathbf{r}_{ED} \times \mathbf{F}_{34} + \mathbf{r}_{EC_4} \times \mathbf{F}_4 + \mathbf{r}_{EC_5} \times \mathbf{F}_5 + \mathbf{M}_4 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}. \quad (7.26)$$

where  $\mathbf{r}_{EC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AE}$ , and  $\mathbf{r}_{EC_4} = \mathbf{r}_{AC_4} - \mathbf{r}_{AE}$ .  
Equation (7.26) with *Mathematica*<sup>TM</sup> becomes

```
rEC5=(rC5-rE)/.rule;
rEC4=(rC4-rE)/.rule;
eqDR2=(Cross[rEC4,F4]+Cross[rEC5,F5]+
Cross[rED,F34Sol]+M4+M5+M5e)[[3]]==0;
```

The system of two equations is solved using the *Mathematica*<sup>TM</sup> commands

```
solF34=Solve[{eqDR1,eqDR2},{F34xSol,F34ySol}];
```

The following numerical solution is obtained:

$$\mathbf{F}_{34} = -268.035\mathbf{i} - 134.564\mathbf{j} \text{ N.}$$

Reaction force  $\mathbf{F}_{03}$

The rotation joint  $C_R$  between the links 0 and 3 is replaced with the unknown reaction force  $\mathbf{F}_{03}$  (Fig. 7.15):

$$\mathbf{F}_{03} = F_{03x}\mathbf{i} + F_{03y}\mathbf{j}.$$

With *Mathematica*<sup>TM</sup> the force  $\mathbf{F}_{03}$  is written as

```
F03Sol={F03xSol,F03ySol,0};
```

Following the path  $I$  (Fig. 7.15), a force equation is written for the translation joint  $B_T$ . The projection of all forces, that act on the link 3, onto the sliding direction  $CD$  is zero:

$$\sum \mathbf{F}^{(3)} \cdot \mathbf{r}_{CD} = (\mathbf{F}_{03} + \mathbf{F}_{43} + \mathbf{F}_3) \cdot \mathbf{r}_{CD} = 0, \quad (7.27)$$

where  $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$ .

Equation (7.27) with *Mathematica*<sup>TM</sup> commands is

```
rCD=(rD-rC)/.rule;
eqCR1=(F03Sol+F43+F3).rCD==0;
```

Continuing on the path *II* (Fig. 7.15), a moment equation is written for the rotation joint  $B_R$ :

$$\sum \mathbf{M}_B^{(3\&2)} = \mathbf{r}_{BC_3} \times \mathbf{F}_3 + \mathbf{r}_{BC} \times \mathbf{F}_{03} + \mathbf{r}_{BD} \times \mathbf{F}_{43} + \mathbf{M}_3 + \mathbf{M}_2 = \mathbf{0}, \quad (7.28)$$

where  $\mathbf{r}_{BC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AB}$ ,  $\mathbf{r}_{BC} = \mathbf{r}_{AC} - \mathbf{r}_{AB}$ , and  $\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$ .  
With *Mathematica*<sup>TM</sup> Eq. (7.28) gives

```
rBC3=(rC3-rB)/.rule;
rBC=(rC-rB)/.rule;
rBD=(rD-rB)/.rule;
eqCR2=(Cross[rBC3,F3]+Cross[rBC,F03Sol]+
Cross[rBD,F43]+M2+M3)[[3]]==0;
```

To solve the system of two equations the *Mathematica*<sup>TM</sup> command is used:

```
solF03=Solve[{eqCR1,eqCR2}, {F03xSol,F03ySol}];
```

The following numerical solution is obtained:

$$\mathbf{F}_{03} = -256.71\mathbf{i} - 272.141\mathbf{j} \text{ N.}$$

Reaction force  $\mathbf{F}_{23}$

The translation joint  $B_T$  between the links 2 and 3 is replaced with the unknown reaction force  $\mathbf{F}_{23}$  (Fig. 7.16):

$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\mathbf{i} + F_{23y}\mathbf{j}.$$

The position of the application point  $Q$  of the force  $\mathbf{F}_{23}$  is unknown:

$$\mathbf{r}_{AQ} = x_Q\mathbf{i} + y_Q\mathbf{j},$$

where  $x_Q$  and  $y_Q$  are the plane coordinates of the point  $Q$ .

The force  $\mathbf{F}_{23}$  and its point of application  $Q$  are written in *Mathematica*<sup>TM</sup> as

```
F34Sol={F34xSol,F34ySol,0};
rQSol={xQSol,yQSol,0};
```

Following the path *I* (Fig. 7.16), a moment equation is written for the rotation joint  $C_R$ :

$$\sum \mathbf{M}_C^{(3)} = \mathbf{r}_{CQ} \times \mathbf{F}_{23} + \mathbf{r}_{CC_3} \times \mathbf{F}_3 + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_3 = \mathbf{0}, \quad (7.29)$$

where  $\mathbf{r}_{CQ} = \mathbf{r}_{AQ} - \mathbf{r}_{AC}$ ,  $\mathbf{r}_{CC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AC}$ , and  $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$ . Using *Mathematica*<sup>TM</sup>, Eq. (7.29) is written as

```
rCQ=(rQsol-rC)/.rule;
rCC3=(rC3-rC)/.rule;
rCD=(rD-rC)/.rule;
eqBT1=(Cross[rCQ,F23sol]+Cross[rCC3,F3]+
Cross[rCD,F43]+M3)[[3]]==0;
```

Following the path *II* (Fig. 7.16), a moment equation is written for the rotation joint  $B_R$ :

$$\sum \mathbf{M}_B^{(2)} = \mathbf{r}_{BQ} \times \mathbf{F}_{32} + \mathbf{M}_2 = \mathbf{0}, \quad (7.30)$$

where  $\mathbf{r}_{BQ} = \mathbf{r}_{AQ} - \mathbf{r}_{AB}$ .

Equation (7.30) with *Mathematica*<sup>TM</sup> becomes

```
rBQ=(rQsol-rB)/.rule;
eqBT2=(Cross[rBQ,F32sol]+M2)[[3]]==0;
```

The direction of the unknown joint force  $\mathbf{F}_{23}$  is perpendicular to the sliding direction  $BC$ . The following relation is written:

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0,$$

or with *Mathematica*<sup>TM</sup>, it is

```
eqBT3=F23sol.rBC==0;
```

The application point  $Q$  of the force  $\mathbf{F}_{23}$  is located on the direction  $BC$ , that is

$$\frac{y_C - y_B}{x_C - x_B} = \frac{y_C - y_Q}{x_C - x_Q}. \quad (7.31)$$

Equation (7.31) with *Mathematica*<sup>TM</sup> gives

```
eqBT4=((yC-yB)/(xC-xB)/.rule)==
((yC-yQSol)/(xC-xQSol)/.rule);
```

The system of four equations is solved using the *Mathematica*<sup>TM</sup> command

```
solF23=Solve[{eqBT1,eqBT2,eqBT3,eqBT4},
{F23xSol,F23ySol,xQSol,yQSol}];
```

The following numerical solutions are obtained:

$$\mathbf{F}_{23} = -11.374\mathbf{i} + 137.91\mathbf{j} \text{ N} \quad \text{and} \quad \mathbf{r}_{AQ} = 0.121\mathbf{i} + 0.070\mathbf{j} \text{ m.}$$

Reaction force  $\mathbf{F}_{12}$

The rotation joint  $B_R$  between the links 1 and 2 is replaced with the unknown reaction force  $\mathbf{F}_{12}$  (Fig. 7.17):

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x}\mathbf{i} + F_{12y}\mathbf{j}.$$

With *Mathematica*<sup>TM</sup> it is written as:

```
F12Sol={F12xSol,F12ySol,0};
```

Following the path  $I$  (Fig. 7.17), a force equation is written for the translation joint  $B_T$ . The projection of all forces, that act on the link 2, onto the sliding direction  $BC$  is zero:

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{F}_2) \cdot \mathbf{r}_{BC} = 0. \quad (7.32)$$

Using *Mathematica*<sup>TM</sup> it is written as:

```
rBC=(rC-rB)/.rule;
eqBR1=(F12Sol+F2).rBC==0;
```

Continuing on the path  $I$ , a moment equation is written for the rotation joint  $C_R$ :

$$\begin{aligned} \sum \mathbf{M}_C^{(2\&3)} &= \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC_2} \times \mathbf{F}_2 + \mathbf{r}_{CC_3} \times \mathbf{F}_3 + \\ &\mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_2 + \mathbf{M}_3 = \mathbf{0}, \end{aligned} \quad (7.33)$$

where  $\mathbf{r}_{CB} = \mathbf{r}_{AB} - \mathbf{r}_{AC}$ ,  $\mathbf{r}_{CC_2} = \mathbf{r}_{AC_2} - \mathbf{r}_{AC}$ ,  $\mathbf{r}_{CC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AC}$ , and  $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$ .

Using the *Mathematica*<sup>TM</sup>, commands Eq. (7.33) gives

```

rCB=(rB-rC)/.rule;
rCC2=(rC2-rC)/.rule;
rCC3=(rC3-rC)/.rule;
rCD=(rD-rC)/.rule;
eqBR2=(Cross[rCB,F12Sol]+Cross[rCC2,F2]+
Cross[rCC3,F3]+Cross[rCD,F43]+M2+M3)[[3]]==0;

```

The system of two equations is solved using the *Mathematica*<sup>TM</sup> command:

```

solF12=Solve[{eqBR1,eqBR2},{F12xSol,F12ySol}];

```

and the following numerical solution is obtained:

$$\mathbf{F}_{12} = -11.401\mathbf{i} + 137.974\mathbf{j} \text{ N.}$$

The motor moment  $\mathbf{M}_m$

The motor moment needed for the dynamic equilibrium of the mechanism is  $\mathbf{M}_m = M_m \mathbf{k}$  (Fig. 7.18) and with *Mathematica*<sup>TM</sup> it is

```

M1mSol={0,0,MmSol};

```

Following the path *I* (Fig. 7.18), a moment equation is written for the rotation joint  $A_R$ :

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{r}_{AC_1} \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M}_m = \mathbf{0}. \quad (7.34)$$

Equation (7.34) is solved using the *Mathematica*<sup>TM</sup> commands:

```

eqMA=(Cross[rAB,F21]+Cross[rAC1,F1]+M1+
M1mSol)[[3]]==0;
solMm=Solve[eqMA,MmSol];
Mm=MmSol/.solMm[[1]];
M1m={0,0,Mm};

```

```
Print["Mm = ",Mm];
```

The numerical solution is

$$\mathbf{M}_m = 17.533 \mathbf{k} \quad \text{N} \cdot \text{m}.$$

Reaction force  $\mathbf{F}_{01}$

The rotation joint  $A_R$  between the links 0 and 1 is replaced with the unknown reaction force  $\mathbf{F}_{01}$  (Fig. 7.19):

$$\mathbf{F}_{01} = -\mathbf{F}_{10} = F_{01x}\mathbf{i} + F_{01y}\mathbf{j},$$

With *Mathematica*<sup>TM</sup> it is written as:

```
F01Sol={F01xSol,F01ySol,0};
```

Following the path  $I$  (Fig. 7.19), a moment equation is written for the rotation joint  $B_R$ :

$$\sum \mathbf{M}_B^{(1)} = \mathbf{r}_{BA} \times \mathbf{F}_{01} + \mathbf{r}_{BC_1} \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M}_m = \mathbf{0}, \quad (7.35)$$

where  $\mathbf{r}_{BA} = -\mathbf{r}_{AB}$ , and  $\mathbf{r}_{BC_1} = \mathbf{r}_{AC_1} - \mathbf{r}_{AB}$ .

Equation (7.35) using the *Mathematica*<sup>TM</sup> commands gives

```
rBA=-rB/.rule;
rBC1=(rC1-rB)/.rule;
eqAR1=(Cross[rBA,F01Sol]+Cross[rBC1,F1]+M1+
M1m)[[3]]==0;
```

Continuing on the path  $I$  (Fig. 7.19), a force equation is written for the translation joint  $B_T$ . The projection of all forces, that act on the links 1 and 2, onto the sliding direction  $BC$  is zero:

$$\sum \mathbf{F}^{(1\&2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{01} + \mathbf{F}_1 + \mathbf{F}_2) \cdot \mathbf{r}_{BC} = 0, \quad (7.36)$$

or with *Mathematica*<sup>TM</sup> it is

```
eqAR2=(F01Sol+F1+F2).rBC==0;
```

The system of two equations is solved using the *Mathematica*<sup>TM</sup> command

```
solF01=Solve[{eqAR1,eqAR2},{F01xSol,F01ySol}];
```

The following numerical solution is obtained

$$\mathbf{F}_{01} = -11.419\mathbf{i} + 138.073\mathbf{j} \text{ N.}$$

The *Mathematica*<sup>TM</sup> program for the dynamic force analysis is presented in Program 7.6.

## 7.5 Problems

- 7.1 Referring to Example 3.1 (Fig. 3.11), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.2 Referring to Example 3.2 (Fig. 3.12), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.3 Referring to Example 3.3 (Fig. 3.15), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.4 Referring to Problem 3.4 (Fig. 3.19), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.5 Referring to Problem 3.5 (Fig. 3.20), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.5 Referring to Problem 3.11 (Fig. 3.26), write a *Mathematica*<sup>TM</sup> program for the position analysis of the mechanism.
- 7.7 Referring to Example 4.1 (Fig. 4.7), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism.
- 7.8 Referring to Example 4.2 (Fig. 4.8), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism.
- 7.9 Referring to Problem 4.1 [Fig. 3.16(a)], write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism.
- 7.10 Referring to Problem 5.1 [Fig. 3.16(a)], write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism using the contour equations method.
- 7.11 Referring to Problem 4.3 (Fig. 4.10), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism.
- 7.12 Referring to Problem 5.3 (Fig. 3.10), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism using the contour equations method.
- 7.13 Referring to Problem 4.4 (Fig. 3.19), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism.

- 7.14 Referring to Problem 5.4 (Fig. 3.19), write a *Mathematica*<sup>TM</sup> program for the velocity and acceleration analysis of the mechanism using the contour equations method.
- 7.15 Referring to Problem 6.3 (Fig. 4.10), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.
- 7.16 Referring to Problem 6.16 (Fig. 3.31), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.
- 7.17 Referring to Problem 6.18 (Fig. 3.33), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.
- 7.18 Referring to Problem 6.24 (Fig. 3.11), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.
- 7.19 Referring to Problem 6.25 (Fig. 3.12), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.
- 7.20 Referring to Problem 6.26 (Fig. 3.15), write a *Mathematica*<sup>TM</sup> program for the equilibrium moment and the joint forces of the mechanism.

## **7.6 Programs**

## References

- [1] P. Antonescu, *Mechanisms*, Printech, Bucharest, 2003.
- [2] P. Appell, *Traité de Mécanique Rationnelle*, Gautier-Villars, Paris, 1941.
- [3] I.I. Artobolevski, *Mechanisms in Modern Engineering Design*, MIR, Moscow, 1977.
- [4] M. Atanasiu, *Mecanica*, EDP, Bucharest, 1973.
- [5] H. Baruh, *Analytical Dynamics*, WCB/McGraw-Hill, Boston, 1999.
- [6] A. Bedford and W. Fowler, *Dynamics*, Addison Wesley, Menlo Park, 1999.
- [7] A. Bedford and W. Fowler, *Statics*, Addison Wesley, Menlo Park, 1999.
- [8] M.I. Buculei, *Mechanisms*, University of Craiova Press, Craiova, 1976.
- [9] M.I. Buculei, D. Bagnaru, G. Nanu, D.B. Marghitu, *Analysis of Mechanisms with Bars*, Scrisul romanesc, Craiova, 1986.
- [10] T. Demian et al., *Mechanisms - problems*, EDP, Bucharest, 1969.
- [11] A.G. Erdman, and G.N. Sandor, *Mechanisms Design*, Prentice-Hall, Upper Saddle River, 1984.
- [12] A. Ertas and J.C. Jones, *The Engineering Design Process*, John Wiley & Sons, New York, 1996.
- [13] F. Freudenstein, "An Application of Boolean Algebra to the Motion of Epicyclic Drives," *Transaction of the ASME, Journal of Engineering for Industry*, pp.176-182, 1971.
- [14] J.H. Ginsberg, *Advanced Engineering Dynamics*, Cambridge University Press, Cambridge, 1995.
- [15] D.T. Greenwood, *Principles of Dynamics*, Prentice-Hall, Englewood Cliffs, 1998.
- [16] A.S. Hall, Jr., A.R. Holowenko, and H.G. Laughlin, *Theory and problems of machine design*, McGraw-Hill, New York, 1961.

- [17] R.C. Hibbeler, *Engineering Mechanics - Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1995.
- [18] R.C. Juvinall and K.M. Marshek, *Fundamentals of Machine Component Design*, John Wiley & Sons, New York, 1983.
- [19] T.R. Kane, *Analytical Elements of Mechanics*, Vol. 1, Academic Press, New York, 1959.
- [20] T.R. Kane, *Analytical Elements of Mechanics*, Vol. 2, Academic Press, New York, 1961.
- [21] T.R. Kane and D.A. Levinson, "The Use of Kane's Dynamical Equations in Robotics", *MIT International Journal of Robotics Research*, No. 3, pp. 3-21, 1983.
- [22] T.R. Kane, P.W. Likins, and D.A. Levinson, *Spacecraft Dynamics*, McGraw-Hill, New York, 1983.
- [23] T.R. Kane and D.A. Levinson, *Dynamics*, McGraw-Hill, New York, 1985.
- [24] J.T. Kimbrell, *Kinematics Analysis and Synthesis*, McGraw-Hill, New York, 1991.
- [25] R. Maeder, *Programming in Mathematica*, Addison-Wesley Publishing Company, Redwood City, California, 1990.
- [26] N.H. Madsen, *Statics and Dynamics*, class notes, [www.eng.auburn.edu/users/nmadsen/](http://www.eng.auburn.edu/users/nmadsen/), 2004.
- [27] N.I. Manolescu, F. Kovacs, and A. Oranescu, *The Theory of Mechanisms and Machines*, EDP, Bucharest, 1972.
- [28] D.B. Marghitu, *Mechanical Engineer's Handbook*, Academic Press, San Diego, California, 2001.
- [29] D.B. Marghitu and M.J. Crocker, *Analytical Elements of Mechanisms*, Cambridge University Press, Cambridge, 2001.
- [30] D.B. Marghitu and E.D. Stoenescu, *Kinematics and Dynamics of Machines and Machine Design*, class notes, [www.eng.auburn.edu/users/marghitu/](http://www.eng.auburn.edu/users/marghitu/), 2004.

- [31] J.L. Meriam and L.G. Kraige, *Engineering Mechanics: Dynamics*, John Wiley & Sons, New York, 1997.
- [32] D.J. McGill and W.W. King, *Engineering Mechanics: Statics and an Introduction to Dynamics*, PWS Publishing Company, Boston, 1995.
- [33] R.L. Mott, *Machine elements in mechanical design*, Prentice Hall, Upper Saddle River, New Jersey, 1999.
- [34] D.H. Myszka, *Machines and Mechanisms*, Prentice-Hall, Upper Saddle River, New Jersey, 1999.
- [35] R.L. Norton, *Machine Design*, Prentice-Hall, Upper Saddle River, New Jersey, 1996.
- [36] R.L. Norton, *Design of Machinery*, McGraw-Hill, New York, 1999.
- [37] W.C. Orthwein, *Machine Component Design*, West Publishing Company, St. Paul, 1990.
- [38] L.A. Pars, *A treatise on analytical dynamics*, Wiley, New York, 1965.
- [39] R.M. Pehan, *Dynamics of Machinery*, McGraw-Hill, New York, 1967.
- [40] I. Popescu, *Mechanisms*, University of Craiova Press, Craiova, 1990.
- [41] I. Popescu and C. Ungureanu, *Structural Synthesis and Kinematics of Mechanisms with Bars*, Universitaria Press, Craiova, 2000.
- [42] I. Popescu and D.B. Marghitu, "Dyad Classification for Mechanisms," *World Conference on Integrated Design and Process Technology*, Austin, Texas, December 3-5, 2003.
- [43] I. Popescu, E.D. Stoenescu, and D.B. Marghitu, "Analysis of Spatial Kinematic Chains Using the System Groups," *8th International Congress on Sound and Vibration*, St. Petersburg, Russia, July 5-8, 2004.
- [44] M. Radoi and E. Deciu, *Mecanica*, EDP, Bucharest, 1981.
- [45] F. Reuleaux, *The Kinematics of Machinery*, Dover, New York, 1963.
- [46] C.A. Rubin, *The Student Edition of Working Model*, Addison-Wesley Publishing Company, Reading, Massachusetts, 1995.

- [47] I.H. Shames, *Engineering Mechanics - Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1997.
- [48] J.E. Shigley and C.R. Mischke, *Mechanical Engineering Design*, McGraw-Hill, New York, 1989.
- [49] J.E. Shigley and J.J. Uicker, *Theory of Machines and Mechanisms*, McGraw-Hill, New York, 1995.
- [50] R.W. Soutas-Little and D.J. Inman, *Engineering Mechanics: Statics and Dynamics*, Prentice-Hall, Upper Saddle River, New Jersey, 1999.
- [51] A. Stan and M. Grumarescu, *Mechanics Problems*, EDP, Bucharest, 1973.
- [52] A. Stoenescu, A. Ripianu, and M. Atanasiu, *Theoretical Mechanics Problems*, EDP, Bucharest, 1965.
- [53] A. Stoenescu and G. Silas, *Theoretical Mechanics*, ET, Bucharest, 1957.
- [54] E.D. Stoenescu, *Dynamics and Synthesis of Kinematic Chains with Impact and Clearance*, Ph.D. Dissertation, Mechanical Engineering, Auburn University, 2005.
- [55] L. W. Tsai, *Mechanism Design: Enumeration of Kinematic Structures According to Function*, CRC Press, Boca Raton, Florida, 2001.
- [56] R. Voinea, D. Voiculescu, and V. Ceausu, *Mecanica*, EDP, Bucharest, 1983.
- [57] K.J. Waldron and G.L. Kinzel, *Kinematics, Dynamics, and Design of Machinery*, John Wiley&Sons, New York, 1999.
- [58] C.E. Wilson and J.P. Sadler, *Kinematics and Dynamics of Machinery*, Harper Collins College Publishers, 1991.
- [59] C.W. Wilson, *Computer integrated machine design*, Prentice Hall, Inc., Upper Saddle River, New Jersey, 1997.
- [60] S. Wolfram, *Mathematica*, Wolfram Media/Cambridge University Press, Cambridge, 1999.

- [61] \* \* \* , *The theory of mechanisms and machines (Teoria mehanizmov i masin)*, Vassaia scola, Minsc, 1970.
- [62] \* \* \* , *Working Model 2D, Users Manual*, Knowledge Revolution, San Mateo, California, 1996.

## Figure captions

Figure 7.1. R-RTR-RTR mechanism.

Figure 7.2. Solutions for the position of the joint  $D$  for  $0 \leq \phi \leq 90^\circ$  ( $x_D \leq x_C$ ).

Figure 7.3. Solutions for the position of the joint  $D$  for  $90^\circ < \phi \leq 180^\circ$  ( $x_D \geq x_C$ ).

Figure 7.4. Solutions for the position of the joint  $D$  for  $180^\circ < \phi < 270^\circ$  ( $x_D \geq x_C$ ).

Figure 7.5. Solutions for the position of the joint  $D$  for  $270^\circ \leq \phi \leq 360^\circ$  ( $x_D \leq x_C$ ).

Figure 7.6. Graph of the mechanism for a complete rotation,  $0 \leq \phi \leq 360^\circ$ .

Figure 7.7. Distance condition for position analysis:  $d_k^I < d_k^{II} \Rightarrow D_{k+1} = D_{k+1}^I$ .

Figure 7.8. (a) R-RTR-RTR mechanism; (b) contour diagram.

Figure 7.9. First independent contour.

Figure 7.10. Second independent contour.

Figure 7.11. Forces and moments for R-RTR-RTR mechanism.

Figure 7.12. Rotation joint  $E_R$  and reaction force  $\mathbf{F}_{05}$ .

Figure 7.13. Translation joint  $D_T$  and reaction force  $\mathbf{F}_{45}$ .

Figure 7.14. Rotation joint  $D_R$  and reaction force  $\mathbf{F}_{34}$ .

Figure 7.15. Rotation joint  $C_R$  and reaction force  $\mathbf{F}_{03}$ .

Figure 7.16. Translation joint  $B_T$  and reaction force  $\mathbf{F}_{23}$ .

Figure 7.17. Rotation joint  $B_R$  and reaction force  $\mathbf{F}_{12}$ .

Figure 7.18. Dynamic equilibrium moment  $\mathbf{M}_m$ .

Figure 7.19. Rotation joint  $A_R$  and reaction force  $\mathbf{F}_{01}$ .