

<i>I.3 Position Analysis</i>	0
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## 3 Position Analysis

### 3.1 Absolute Cartesian Method

The position analysis of a kinematic chain requires the determination of the joint positions and/or the position of the center of gravity (CG) of the link. A planar link with the end nodes  $A$  and  $B$  is considered in Fig. 3.1. Let  $(x_A, y_A)$  be the coordinates of joint  $A$  with respect to the reference frame  $xOy$ , and  $(x_B, y_B)$  be the coordinates of joint  $B$  with the same reference frame. Using the theorem of Pythagoras the following relation can be written:

$$(x_B - x_A)^2 + (y_B - y_A)^2 = AB^2 = L_{AB}^2, \quad (3.1)$$

where  $L_{AB}$  is the length of the link  $AB$ .

Let  $\phi$  be the angle of the link  $AB$  with the horizontal axis  $Ox$ . Then, the slope  $m$  of the link  $AB$  is defined as

$$m = \tan \phi = \frac{y_B - y_A}{x_B - x_A}. \quad (3.2)$$

Let  $b$  be the intercept of  $AB$  with the vertical axis  $Oy$ . Using the slope  $m$  and the  $y$  intercept  $b$ , the equation of the straight link (line), in the plane, is

$$y = mx + b, \quad (3.3)$$

where  $x$  and  $y$  are the coordinates of any point on this link.

Two lines are perpendicular to each other if and only if the slope of one is the negative reciprocal of the slope of the other. Thus, if  $m$  and  $n$  are the slopes of two perpendicular lines

$$m = -\frac{1}{n} \quad \text{and} \quad mn = -1. \quad (3.4)$$

If two distinct points  $A(x_A, y_A)$  and  $B(x_B, y_B)$  are on a straight line then the equation of the straight line can be written in the forms

$$\frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A} \quad \text{and} \quad \begin{vmatrix} x & y & 1 \\ x_A & y_A & 1 \\ x_B & y_B & 1 \end{vmatrix} = 0. \quad (3.5)$$

Given two points  $P(x_P, y_P)$  and  $Q(x_Q, y_Q)$  and a real number  $k$ ,  $k \in \mathbf{R} - \{-1\}$ , the coordinates of a point  $R(x_R, y_R)$  on the line segment  $PQ$ , whose

distance from  $P$  bears the distance from  $R$  to  $Q$  the ratio  $k$  ( $PR = k RQ$ ), are

$$x_R = \frac{x_P + k x_Q}{1 + k} \quad \text{and} \quad y_R = \frac{y_P + k y_Q}{1 + k}. \quad (3.6)$$

The symbol  $\in$  means “belongs to”.

For  $k = 1$  the above formulas become

$$x_R = \frac{x_P + x_Q}{2} \quad \text{and} \quad y_R = \frac{y_P + y_Q}{2}. \quad (3.7)$$

These give the coordinates of the midpoint of the interval from  $P$  to  $Q$ .

For  $k > 0$  the point  $R$  is interior to the segment  $PQ$  and for  $k < 0$  the point  $R$  is exterior to the segment  $PQ$ .

For a link with a translational joint (Fig. 3.2) the sliding direction ( $\Delta$ ) is given by the equation

$$x \cos \alpha + y \sin \alpha - p = 0, \quad (3.8)$$

where  $p$  is the distance from the origin  $O$  to the sliding line ( $\Delta$ ). The position function for the joint  $A(x_A, y_A)$  is

$$x_A \cos \alpha + y_A \sin \alpha - p = \pm d, \quad (3.9)$$

where  $d$  is the distance from  $A$  to the sliding line. The relation between the joint  $A$  and a point  $B$  on the sliding direction,  $B \in (\Delta)$ , is

$$(x_A - x_B) \sin \beta + (y_A - y_B) \cos \beta = \pm d, \quad (3.10)$$

where  $\beta = \alpha + \frac{\pi}{2}$ .

If  $Ax + By + C = 0$  is the linear equation of the line ( $\Delta$ ) then the distance  $d$  is (Fig. 3.2):

$$d = \frac{|Ax_A + By_A + C|}{\sqrt{A^2 + B^2}}. \quad (3.11)$$

For a driver link in rotational motion [Fig. 3.3(a)] the following relations can be written

$$x_B = x_A + L_{AB} \cos \phi \quad \text{and} \quad y_B = y_A + L_{AB} \sin \phi. \quad (3.12)$$

From Fig. 3.3(b), for a driver link in translational motion, one can have

$$\begin{aligned}x_B &= x_A + s \cos \phi + L_1 \cos(\phi + \alpha), \\y_B &= y_A + s \sin \phi + L_1 \sin(\phi + \alpha).\end{aligned}\quad (3.13)$$

For the RRR dyad (Fig. 3.4) there are two quadratic equations of the form

$$\begin{aligned}(x_A - x_C)^2 + (y_A - y_C)^2 &= AC^2 = L_{AC}^2 = L_2^2, \\(x_B - x_C)^2 + (y_B - y_C)^2 &= BC^2 = L_{BC}^2 = L_3^2,\end{aligned}\quad (3.14)$$

where the coordinates of the joint  $C$ ,  $x_C$  and  $y_C$ , are the unknowns. With  $x_C$  and  $y_C$  determined, the angles  $\phi_2$  and  $\phi_3$  are computed from the relations

$$\tan \phi_2 = \frac{y_C - y_A}{x_C - x_A} \quad \text{and} \quad \tan \phi_3 = \frac{y_C - y_B}{x_C - x_B}.\quad (3.15)$$

The following relations can be written for the RRT dyad [Fig. 3.5(a)]:

$$\begin{aligned}(x_A - x_C)^2 + (y_A - y_C)^2 &= AC^2 = L_{AC}^2 = L_2^2, \\(x_C - x_B) \sin \alpha - (y_C - y_B) \cos \alpha &= \pm h.\end{aligned}\quad (3.16)$$

From the two previous equations the two unknowns  $x_C$  and  $y_C$  are computed. Figure 3.5(b) depicts the particular case for the RRT dyad when  $L_3 = h = 0$  and the position equations are

$$(x_A - x_C)^2 + (y_A - y_C)^2 = L_2^2 \quad \text{and} \quad \tan \alpha = \frac{y_C - y_B}{x_C - x_B}.\quad (3.17)$$

For the RTR dyad [Fig. 3.6(a)] the known data are: the positions of the joint  $A$  and  $B$ ,  $x_A, y_A, x_B, y_B$ , the angle  $\alpha$ , and the length  $L_2$  ( $h = L_2 \sin \alpha$ ). There are four unknowns in the position of  $C(x_C, y_C)$  and in the equation for the sliding line ( $\Delta$ ):  $y = mx + b$ . The unknowns in the sliding line  $m$  and  $b$  are computed from the relations

$$L_2 \sin \alpha = \frac{|m x_A - y_A + b|}{\sqrt{m^2 + 1}} \quad \text{and} \quad y_B = m x_B + b.\quad (3.18)$$

The coordinates of the joint  $C$  can be obtained using the equations

$$(x_A - x_C)^2 + (y_A - y_C)^2 = L_2^2 \quad \text{and} \quad y_C = m x_C + b.\quad (3.19)$$

In Fig. 3.6(b) the particular case when  $L_1 = h = 0$  is shown, the position equation is

$$\tan \phi_2 = \tan \phi_3 = \frac{y_A - y_B}{x_A - x_B}. \quad (3.20)$$

To compute the coordinates of joint  $C$  for the TRT dyad (Fig. 3.7) two equations can be written:

$$\begin{aligned} (x_C - x_A) \sin \alpha - (y_C - y_A) \cos \alpha &= \pm d, \\ (x_C - x_B) \sin \beta - (y_C - y_B) \cos \beta &= \pm h. \end{aligned} \quad (3.21)$$

The input data are  $x_A, y_A, x_B, y_B, \alpha, \beta, d, h$  and the output data are  $x_C, y_C$ .

Consider the mechanism shown in Fig. 3.8. The angle of link 1 with the horizontal axis  $Ax$  is  $\phi$ ,  $\phi = \angle(AB, Ax)$ , and it is known. The following dimensions are given:  $AB = l_1$ ,  $CD = l_3$ ,  $CE = l_4$ ,  $AD = d$ , and  $h$  is the distance from the slider 5 to the horizontal axis  $Ax$ .

Next, the positions of the joints and the angles of the links will be calculated.

The origin of the system is at  $A$ ,  $A \equiv O$ ,  $x_A = y_A = 0$ . The coordinates of the rotational joint at  $B$  are

$$x_B = l_1 \cos \phi \quad \text{and} \quad y_B = l_1 \sin \phi.$$

The coordinates of the rotational joint at  $D$  are

$$x_D = d \quad \text{and} \quad y_D = 0.$$

For the dyad  $DBC$  (RTR) the angle  $\phi_2 = \phi_3$  of link 2 or link 3 with the horizontal axis is calculated from the equation

$$\tan \phi_2 = \tan \phi_3 = \frac{y_B - y_D}{x_B - x_D} = \frac{l_1 \sin \phi}{l_1 \cos \phi - d}. \quad (3.22)$$

The joints  $C(x_C, y_C)$  and  $D$  are on link 3 (straight line  $DBC$ ) and

$$\tan \phi_3 = \frac{y_C - y_D}{x_C - x_D} = \frac{y_C}{x_C - d}. \quad (3.23)$$

Equations (3.22) and (3.23) give

$$\frac{y_B - y_D}{x_B - x_D} = \frac{y_C - y_D}{x_C - x_D} \quad \text{or} \quad \frac{l_1 \sin \phi}{l_1 \cos \phi - d} = \frac{y_C}{x_C - d}. \quad (3.24)$$

The length of link 3 is  $CD = l_3$  (constant) and the distance from  $C$  to  $D$  is

$$(x_C - x_D)^2 + (y_C - y_D)^2 = l_3^2 \quad \text{or} \quad (x_C - d)^2 + y_C^2 = l_3^2. \quad (3.25)$$

The coordinates  $x_C$  and  $y_C$  of joint  $C$  result from Eq. (3.24) and Eq. (3.25).

Because of the quadratic equation, two solutions are obtained for  $x_C$  and  $y_C$ . For continuous motion of the mechanism there are constraint relations for choosing the correct solution:  $x_C < x_B < x_D$  and  $y_C > 0$ .

For the last dyad  $CEE$  (RRT) a position function can be written for joint  $E$  ( $CE = l_4 = \text{constant}$ ) as:

$$(x_C - x_E)^2 + (y_C - h)^2 = l_4^2.$$

It results in values  $x_{E1}$  and  $x_{E2}$  and the solution  $x_E > x_C$  will be selected for continuous motion of the mechanism.

The angle  $\phi_4$  of link 4 with the horizontal axis is obtained from

$$\tan \phi_4 = \frac{y_C - y_E}{x_C - x_E} = \frac{y_C - h}{x_C - x_E}. \quad (3.26)$$

## 3.2 Vector Loop Method

First the independent closed loops are identified. A vector equation corresponding to each independent loop is established. The vector equation gives rise to two scalar equations, one for the horizontal axis  $x$ , and one for the vertical axis  $y$ .

For an open kinematic chain (Fig. 3.9) with general joints (pin joints, slider joints, etc.), a vector loop equation can be considered:

$$\mathbf{r}_A + \mathbf{r}_1 + \dots + \mathbf{r}_n = \mathbf{r}_B, \quad (3.27)$$

or

$$\sum_{k=1}^n \mathbf{r}_k = \mathbf{r}_B - \mathbf{r}_A. \quad (3.28)$$

The vectorial Eq. (3.28) can be projected on the reference frame  $xOy$ :

$$\sum_{k=1}^n r_k \cos \phi_k = x_B - x_A \quad \text{and} \quad \sum_{k=1}^n r_k \sin \phi_k = y_B - y_A. \quad (3.29)$$

**RRR Dyad**

The input data are: the position of  $A$  is  $(x_A, y_A)$ , the position of  $B$  is  $(x_B, y_B)$ , the length of  $AC$  is  $L_{AC} = L_2$ , and the length of  $BC$  is  $L_{BC} = L_3$  (Fig. 3.4). The unknown data are: the position of  $C(x_C, y_C)$ , and the angles  $\phi_2$  and  $\phi_3$ . The position equation for the RRR dyad is  $\mathbf{r}_{AC} + \mathbf{r}_{CB} = \mathbf{r}_B - \mathbf{r}_A$ , or

$$\begin{aligned} L_2 \cos \phi_2 + L_3 \cos(\phi_3 + \pi) &= x_B - x_A, \\ L_2 \sin \phi_2 + L_3 \sin(\phi_3 + \pi) &= y_B - y_A. \end{aligned} \quad (3.30)$$

The angles  $\phi_2$  and  $\phi_3$  can be computed from Eq. (3.30). The position of  $C$  can be computed using the known angle  $\phi_2$ :

$$x_C = x_A + L_2 \cos \phi_2 \quad \text{and} \quad y_C = y_A + L_2 \sin \phi_2. \quad (3.31)$$

**RRT Dyad**

The input data are: the position of  $A$  is  $(x_A, y_A)$ , the position of  $B$  is  $(x_B, y_B)$ , the length of  $AC$  is  $L_2$ , the length of  $CD$  is  $L_3$ , and the angles  $\alpha$  and  $\beta$  are constants [Fig. 3.5(a)]. The unknown data are: the position of  $C(x_C, y_C)$ , the angle  $\phi_2$ , and the distance  $r = DB$ .

The vectorial equation for this kinematic chain is  $\mathbf{r}_{AC} + \mathbf{r}_{CD} + \mathbf{r}_{DB} = \mathbf{r}_B - \mathbf{r}_A$ , or

$$\begin{aligned} L_2 \cos \phi_2 + L_3 \cos(\alpha + \beta + \pi) + r \cos(\alpha + \pi) &= x_B - x_A, \\ L_2 \sin \phi_2 + L_3 \sin(\alpha + \beta + \pi) + r \sin(\alpha + \pi) &= y_B - y_A. \end{aligned} \quad (3.32)$$

One can compute  $r$  and  $\phi_2$  from Eq. (3.32). The position of  $C$  can be found with Eq. (3.31).

*Particular case*  $L_3 = 0$  [Fig. 3.5(b)]. In this case Eq. (3.32) can be written as

$$\begin{aligned} L_2 \cos \phi_2 + r \cos(\alpha + \pi) &= x_B - x_A, \\ L_2 \sin \phi_2 + r \sin(\alpha + \pi) &= y_B - y_A. \end{aligned} \quad (3.33)$$

**RTR Dyad**

The input data are: the position of  $A$  is  $(x_A, y_A)$ , the position of  $B$  is  $(x_B, y_B)$ , the length of  $AC$  is  $L_2$ , and the angle  $\alpha$  is constant [Fig. 3.6(a)].

The unknown data are: the distance  $r = CB$  and the angles  $\phi_2$  and  $\phi_3$ . The vectorial loop equation can be written as  $\mathbf{r}_{AC} + \mathbf{r}_{CB} = \mathbf{r}_B - \mathbf{r}_A$ , or

$$\begin{aligned} L_2 \cos \phi_2 + r \cos(\alpha + \phi_2 + \pi) &= x_B - x_A, \\ L_2 \sin \phi_2 + r \sin(\alpha + \phi_2 + \pi) &= y_B - y_A. \end{aligned} \quad (3.34)$$

One can compute  $r$  and  $\phi_2$  from Eq. (3.34). The angle  $\phi_3$  can be written as:

$$\phi_3 = \phi_2 + \alpha. \quad (3.35)$$

*Particular case*  $L_2 = 0$  [Fig. 3.6(b)]. In this case from Eqs. (3.34) and (3.35) one can obtain

$$r \cos \phi_3 = x_B - x_A \quad \text{and} \quad r \sin \phi_3 = y_B - y_A. \quad (3.36)$$

The method is illustrated through the following example. Figure 3.10(a) shows a four-bar mechanism (R-RRR mechanism) with link lengths  $r_0$ ,  $r_1$ ,  $r_2$ , and  $r_3$ . Find the angles  $\phi_2$  and  $\phi_3$  as functions of the driver link angle  $\phi = \phi_1$ .

The links are denoted as vectors  $\mathbf{r}_0$ ,  $\mathbf{r}_1$ ,  $\mathbf{r}_2$  and  $\mathbf{r}_3$ , ( $|\mathbf{r}_i| = r_i$ ,  $i = 0, 1, 2, 3$ ), and the angles are measured counterclockwise from the  $x$  axis [Fig. 3.10(b)]. For the closed loop  $ABCD$ , a vectorial equation can be written as:

$$\mathbf{r}_0 + \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = \mathbf{0}. \quad (3.37)$$

By projecting the above vectorial equation onto  $x$  and  $y$ , two scalar equations are obtained:

$$r_0 + r_1 \cos \phi_1 + r_2 \cos \phi_2 - r_3 \cos \phi_3 = 0, \quad (3.38)$$

and

$$r_1 \sin \phi_1 + r_2 \sin \phi_2 - r_3 \sin \phi_3 = 0. \quad (3.39)$$

Equations (3.38) and (3.39) represent a set of nonlinear equations in two unknowns,  $\phi_2$  and  $\phi_3$ . The solution of these two equations solves the position analysis.

Rearranging Eqs. (3.38) and (3.39):

$$r_2 \cos \phi_2 = (r_3 \cos \phi_3 - r_0) - r_1 \cos \phi_1, \quad (3.40)$$

and

$$r_2 \sin \phi_2 = r_3 \sin \phi_3 - r_1 \sin \phi_1. \quad (3.41)$$

Squaring both sides of the above equations and adding

$$\begin{aligned} r_2^2 &= r_0^2 + r_1^2 + r_3^2 - 2r_3 \cos \phi_3 (r_0 + r_1 \cos \phi_1) \\ &\quad - 2r_1 r_3 \sin \phi_1 \sin \phi_3 + 2r_0 r_1 \cos \phi_1, \end{aligned}$$

or

$$a \sin \phi_3 + b \cos \phi_3 = c, \quad (3.42)$$

where

$$\begin{aligned} a &= \sin \phi_1, \quad b = \cos \phi_1 + (r_0/r_1), \quad \text{and} \\ c &= (r_0/r_3) \cos \phi_1 + [(r_0^2 + r_1^2 + r_3^2 - r_2^2)/(2r_1 r_3)]. \end{aligned} \quad (3.43)$$

Using the relations

$$\sin \phi_3 = 2 \tan(\phi_3/2)[1 + \tan^2(\phi_3/2)],$$

and

$$\cos \phi_3 = [1 - \tan^2(\phi_3/2)]/[1 + \tan^2(\phi_3/2)], \quad (3.44)$$

in Eq. (3.42), the following relation is obtained:

$$(b + c) \tan^2(\phi_3/2) - 2a \tan(\phi_3/2) + (c - b) = 0,$$

which gives

$$\tan(\phi_3/2) = (a \pm \sqrt{a^2 + b^2 - c^2})/(b + c). \quad (3.45)$$

Thus, for each given value of  $\phi_1$  and the length of the links, two distinct values of the angle  $\phi_3$  are obtained:

$$\begin{aligned} \phi_{3(1)} &= 2 \tan^{-1}[(a + \sqrt{a^2 + b^2 - c^2})/(b + c)], \\ \phi_{3(2)} &= 2 \tan^{-1}[(a - \sqrt{a^2 + b^2 - c^2})/(b + c)]. \end{aligned} \quad (3.46)$$

The two values of  $\phi_3$  correspond to the two different positions of the mechanism.

The angle  $\phi_2$  can be eliminated from Eqs. (3.38) and (3.39) to give  $\phi_1$  in a similar way to that just described.

### 3.3 Examples

**Example 3.1.** Figure 3.11(a) shows a quick-return shaper mechanism. Given the lengths  $AB = 0.20$  m,  $AD = 0.40$  m,  $CD = 0.70$  m,  $CE = 0.30$  m, and the input angle  $\phi = \phi_1 = 45^\circ$ , obtain the positions of all the other joints. The distance from the slider 5 to the horizontal axis  $Ax$  is  $y_E = 0.35$  m.

Solution.

The coordinates of the joint  $B$  are

$$\begin{aligned}x_B &= AB \cos \phi = 0.20 \cos 45^\circ = 0.141 \text{ m}, \\y_B &= AB \sin \phi = 0.20 \sin 45^\circ = 0.141 \text{ m}.\end{aligned}$$

The vector diagram Fig. 3.11(b) is drawn by representing the RTR ( $BBD$ ) dyad. The vector equation, corresponding to this loop, is written as:

$$\mathbf{r}_B + \mathbf{r} - \mathbf{r}_D = \mathbf{0} \quad \text{or} \quad \mathbf{r} = \mathbf{r}_D - \mathbf{r}_B,$$

where  $\mathbf{r} = \mathbf{r}_{BD}$  and  $|\mathbf{r}| = r$ . Projecting the above vectorial equation on  $x$  and  $y$  axes, two scalar equations are obtained:

$$\begin{aligned}r \cos(\pi + \phi_3) &= x_D - x_B = -0.141 \text{ m}, \\r \sin(\pi + \phi_3) &= y_D - y_B = -0.541 \text{ m}.,\end{aligned}$$

The angle  $\phi_3$  is obtained by solving the system equations

$$\tan \phi_3 = \frac{y_D - y_B}{x_D - x_B} = \frac{0.541}{0.141} \implies \phi_3 = 75.36^\circ.$$

The distance  $r$  is

$$r = \frac{x_D - x_B}{\cos(\pi + \phi_3)} = 0.56 \text{ m}.$$

The coordinates of the joint  $C$  are

$$\begin{aligned}x_C &= CD \sin \phi_3 = 0.17 \text{ m}, \\y_C &= AB \cos \phi_3 - AD = 0.26 \text{ m}.\end{aligned}$$

For the next dyad RRT ( $CEE$ ) [Fig. 3.11(c)] one can write

$$\begin{aligned}CE \cos(\pi - \phi_4) &= x_E - x_C, \\CE \sin(\pi - \phi_4) &= y_E - y_C.\end{aligned}$$

Solving this system, the unknowns  $\phi_4$  and  $x_E$  are obtained:

$$\phi_4 = 165.9^\circ \quad \text{and} \quad x_E = -0.114 \text{ m.}$$

**Example 3.2:** R-RTR-RRT mechanism. The planar R-RTR-RRT mechanism is considered in Fig. 3.12. The driver is the rigid link 1 (the element  $AB$ ) and makes an angle  $\phi = \phi_1 = \pi/6$  with the horizontal. The length of the links are  $AB=0.02$  m,  $BC=0.03$  m, and  $CD=0.06$  m. The following dimensions are given:  $AE=0.05$  m and  $L_a=0.02$  m. Find the positions of the joints and the angles of the links.

Solution.

*Position of joint A:*

A cartesian reference frame  $xOyz$  with the unit vectors  $[\mathbf{i}, \mathbf{j}, \mathbf{k}]$  is selected, as shown in Fig. 3.12. Since joint  $A$  is in the origin of the reference system  $A \equiv O$  then

$$x_A = y_A = 0.$$

*Position of joint E:*

The the coordinates of joint  $E$  are

$$x_E = -AE = -0.05 \text{ m} \quad \text{and} \quad y_E = 0.$$

*Position of joint B:*

Because the joint  $A$  is fixed and the angle  $\phi$  is known, the coordinates of joint  $B$  are computed with

$$\begin{aligned} x_B &= AB \cos \phi = 0.02 \cos \pi/6 = 0.017 \text{ m,} \\ y_B &= AB \sin \phi = 0.02 \sin \pi/6 = 0.010 \text{ m.} \end{aligned}$$

*Position of joint C:*

Joints  $E$ ,  $B$ , and  $C$  are located on the same straight line,  $EBC$ . The slope of this straight line is

$$m = \frac{y_B - y_E}{x_B - x_E} = \frac{y_C - y_E}{x_C - x_E} \quad \text{or} \quad \frac{0.010}{0.017 - (-0.05)} = \frac{y_C}{x_C - (-0.05)}. \quad (3.47)$$

The length of the link  $BC$  is constant and a quadratic equation can be written as:

$$\begin{aligned} (x_C - x_B)^2 + (y_C - y_B)^2 &= BC^2 \quad \text{or} \\ (x_C - 0.017)^2 + (y_C - 0.01)^2 &= 0.03^2. \end{aligned} \quad (3.48)$$

Solving Eq. (3.47) and Eq. (3.48) two sets of solutions are found for the position of joint  $C$ . These solutions are

$$\begin{aligned}x_{C_1} &= -0.012 \text{ m}, & y_{C_1} &= 0.005 \text{ m}, \\x_{C_2} &= 0.046 \text{ m}, & y_{C_2} &= 0.014 \text{ m}.\end{aligned}$$

The points  $C_1$  and  $C_2$  are the intersections of the circle of radius  $BC$  (with its center at  $B$ ), with the straight line  $EC$ , as shown in Fig. 3.13. To determine the position of joint  $C$  for this position of the mechanism ( $\phi = \pi/6$ ), an additional constraint condition is needed:  $x_C > x_B$ . With this constraint the coordinates of joint  $C$  have the following numerical values:

$$x_C = x_{C_2} = 0.046 \text{ m} \quad \text{and} \quad y_C = y_{C_2} = 0.014 \text{ m}.$$

*Position of joint D:*

The  $x$ -coordinate of  $D$  is  $x_D = L_a = 0.02 \text{ m}$ . The length of the link  $CD$  is constant and a quadratic equation can be written:

$$\begin{aligned}(x_D - x_C)^2 + (y_D - y_C)^2 &= CD^2 \quad \text{or} \\(0.02 - 0.046)^2 + (y_D - 0.014)^2 &= 0.06^2.\end{aligned}\tag{3.49}$$

Solving Eq. (3.49), two sets of solutions are found for the position of the joint  $D$ . These solutions are

$$y_{D_1} = -0.039 \text{ m} \quad \text{and} \quad y_{D_2} = 0.067 \text{ m}.$$

The points  $D_1$  and  $D_2$  are the intersections of the circle of radius  $CD$  (with its center at  $C$ ) with the vertical line  $x = L_a$ , as shown in Fig. 3.14. To determine the correct position of joint  $D$  for the angle  $\phi = \pi/6$ , an additional constraint condition is needed:  $y_D < y_C$ . With this constraint the coordinates of joint  $D$  are

$$x_D = 0.02 \text{ m} \quad \text{and} \quad y_D = y_{D_1} = -0.039 \text{ m}.$$

*Angle  $\phi_2$ :*

The angle of link 2 (or link 3) with the horizontal axis is calculated from the slope of the straight line  $EB$ :

$$\phi_2 = \phi_3 = \arctan \frac{y_B - y_E}{x_B - x_E} = \arctan \frac{0.010}{0.017 - (-0.050)} = 0.147 \text{ rad} = 8.449^\circ.$$

Angle  $\phi_4$ :

The angle of link 4 with the horizontal axis is obtained from the slope of the straight line  $CD$ :

$$\phi_4 = \arctan \frac{y_C - y_D}{x_C - x_D} = \arctan \frac{0.014 + 0.039}{0.046 - 0.020} = 1.104 \text{ rad} = 63.261^\circ.$$

**Example 3.3:** R-TRR-RRT Mechanism.

The mechanism is shown in Fig. 3.15. The following data are given:  $AC = 0.100$  m,  $BC = 0.300$  m,  $BD = 0.900$  m, and  $L_a = 0.100$  m. If the angle of link 1 with the horizontal axis is  $\phi = 45^\circ$ , find the positions of joint  $D$ .

Solution.

*Position of joint A:*

A cartesian reference frame with the origin at  $A$  is selected. The coordinates of the joint  $A$  are

$$x_A = y_A = 0.$$

*Position of joint C:*

The coordinates of joint  $C$  are

$$x_C = AC = 0.100 \text{ m} \quad \text{and} \quad y_C = 0.$$

*Position of joint B:*

The slope of the line  $AB$  is

$$\tan \phi = \frac{y_B}{x_B} \quad \text{or} \quad \tan 45^\circ = \frac{y_B}{x_B}. \quad (3.50)$$

The length of the link  $BC$  is constant and the following equation can be written:

$$(x_B - x_C)^2 + (y_B - y_C)^2 = BC^2 \quad \text{or} \quad (x_B - 0.1)^2 + y_B^2 = 0.3^2. \quad (3.51)$$

Equations (3.50) and (3.51) form a system of two equations with the unknowns  $x_B$  and  $y_B$ . The following numerical results are obtained:

$$\begin{aligned} x_{B_1} &= -0.156 \text{ m}, & y_{B_1} &= -0.156 \text{ m}, \\ x_{B_2} &= 0.256 \text{ m}, & y_{B_2} &= 0.256 \text{ m}. \end{aligned}$$

To determine the correct position of the joint  $B$  for the angle  $\phi = 45^\circ$ , an additional constraint condition is needed:  $x_B > x_C$ . With this constraint the coordinates of joint  $B$  are

$$x_B = x_{B_2} = 0.256 \text{ m} \quad \text{and} \quad y_B = y_{B_2} = 0.256 \text{ m}.$$

*Position of joint  $D$ :*

The slider 5 has a translational motion in the horizontal direction and  $y_D = L_a$ . There is only one unknown,  $x_D$ , for joint  $D$ . The following expression can be written:

$$\begin{aligned} (x_B - x_D)^2 + (y_B - y_D)^2 &= BD^2 \quad \text{or} \\ (0.256 - x_D)^2 + (0.256 - 0.1)^2 &= 0.9^2 \end{aligned} \quad (3.52)$$

Solving Eq. (3.52), two numerical values are obtained:

$$x_{D_1} = -0.630 \text{ m}, \quad x_{D_2} = 1.142 \text{ m}. \quad (3.53)$$

For continuous motion of the mechanism, a geometric constraint  $x_D > x_B$  has to be selected. Using this relation the coordinates of joint  $D$  are

$$x_D = 1.142 \text{ m} \quad \text{and} \quad y_D = 0.100 \text{ m}.$$

### 3.4 Problems

- 3.1 Find the analytical expression of any point  $P$  of the elipsograph mechanism in Fig. 2.53.
- 3.2 The following data are given for the four-bar mechanism shown in Fig. 3.16:  $AB = CD = 0.04$  m and  $AD = BC = 0.09$  m. Find the trajectory of the point  $M$  located on the link  $BC$ , for the case a)  $BM = MC$ , and b)  $MC = 2BM$ .
- 3.3 The planar four-bar mechanism depicted in Fig. 3.17 has dimensions  $AB = 0.03$  m,  $BC = 0.065$  m,  $CD = 0.05$  m,  $BM = 0.09$  m, and  $CM = 0.12$  m. Find the trajectory described by the point  $M$ .
- 3.4 The mechanism shown in Fig. 3.18 has dimensions  $AB = 0.03$  m,  $BC = 0.12$  m,  $CD = 0.12$  m,  $DE = 0.07$  m,  $CF = 0.17$  m,  $R_1 = 0.04$  m,  $R_4 = 0.08$  m,  $L_a = 0.025$  m, and  $L_b = 0.105$  m. Find the trajectory of the joint  $C$ .
- 3.5 The planar R-RRR-RRT mechanism considered is depicted in Fig. 3.19. The driver link is the rigid link 1 (the element  $AB$ ). The following data are given:  $AB=0.150$  m,  $BC=0.400$  m,  $CD=0.370$  m,  $CE=0.230$  m,  $EF=CE$ ,  $L_a=0.300$  m,  $L_b=0.450$  m, and  $L_c=CD$ . The angle of the driver link 1 with the horizontal axis is  $\phi = \phi_1 = 45^\circ$ . Find the positions of the joints and the angles of the links.
- 3.6 The R-RRR-RTT mechanism is shown in Fig. 3.20. The following data are given:  $AB=0.080$  m,  $BC=0.350$  m,  $CD=0.150$  m,  $CE=0.200$  m,  $L_a=0.200$  m,  $L_b=0.350$  m,  $L_c=0.040$  m. The angle of the driver element (link  $AB$ ) with the horizontal axis is  $\phi = 135^\circ$ . Determine the positions of the joints and the angles of the links.
- 3.7 The mechanism shown in Fig. 3.21 has the following dimensions:  $AB = 40$  mm,  $AD = 150$  mm,  $BC = 100$  mm,  $CE = 30$  mm,  $EF = 120$  mm, and  $a = 90$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.
- 3.8 The dimensions for the mechanism shown in Fig. 3.22 are:  $AB = 250$  mm,  $BD = 670$  mm,  $DE = 420$  mm,  $AE = 640$  mm,  $BC =$

- 240 mm,  $CD = 660$  mm,  $CF = 850$  mm, and  $b = 170$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.
- 3.9 The mechanism in Fig. 3.23 has the dimensions:  $AB = 120$  mm,  $AC = 60$  mm,  $BD = 240$  mm,  $DE = 330$  mm,  $EF = 190$  mm,  $L_a = 300$  mm, and  $L_b = 70$  mm. The angle of the driver link 1 with the horizontal is  $\phi = \phi_1 = 150^\circ$ . Find the positions of the joints and the angles of the links.
- 3.10 The dimensions for the mechanism shown in Fig. 3.24 are:  $AB = 100$  mm,  $BC = 260$  mm,  $AD = 240$  mm,  $CD = 140$  mm,  $DE = 80$  mm,  $EF = 250$  mm, and  $L_a = 20$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 45^\circ$ . Find the positions of the joints and the angles of the links.
- 3.11 The mechanism in Fig. 3.25 has the dimensions:  $AB = 150$  mm,  $AC = 450$  mm,  $BD = 700$  mm,  $L_a = 100$  mm, and  $L_b = 200$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 120^\circ$ . Find the positions of the joints and the angles of the links.
- 3.12 Figure 3.26 shows a mechanism with the following dimensions:  $AB = 180$  mm,  $BD = 700$  mm, and  $L_a = 210$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 135^\circ$ . Find the positions of the joints and the angles of the links.
- 3.13 The mechanism in Fig. 3.27 has the dimensions:  $AB = 100$  mm,  $AC = 240$  mm,  $BD = 400$  mm,  $DE = 250$  mm,  $EF = 135$  mm,  $L_a = 35$  mm, and  $L_b = 170$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 150^\circ$ . Find the positions of the joints and the angles of the links.
- 3.14 Figure 3.28 shows a mechanism with the following dimensions:  $AB = 120$  mm,  $BC = 450$  mm,  $CD = DE = 180$  mm,  $EF = 300$  mm,  $L_a = 450$  mm,  $L_b = 150$  mm, and  $L_c = 140$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 120^\circ$ . Find the positions of the joints and the angles of the links.
- 3.15 Figure 3.29 shows a mechanism with the following dimensions:  $AB = 140$  mm,  $BC = 650$  mm,  $CE = 250$  mm,  $CD = 400$  mm,  $EF =$

350 mm,  $L_a = 370$  mm,  $L_b = 550$  mm, and  $L_c = 700$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 150^\circ$ . Find the positions of the joints and the angles of the links.

- 3.16 Figure 3.30 shows a mechanism with the following dimensions:  $AB = 60$  mm,  $BC = 160$  mm,  $CF = 150$  mm,  $CD = 60$  mm,  $DE = 180$  mm,  $L_a = 210$  mm,  $L_b = 120$  mm, and  $L_c = 65$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.
- 3.17 Figure 3.31 shows a mechanism with the following dimensions:  $AB = 20$  mm,  $BC = 50$  mm,  $AD = 25$  mm, and  $BE = 60$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 60^\circ$ . Find the positions of the joints and the angles of the links.
- 3.18 The dimensions of the mechanism shown in Fig. 3.32 are:  $AB = 150$  mm,  $BC = 300$  mm,  $BE = 600$  mm,  $CE = 850$  mm,  $CD = 330$  mm,  $EF = 1200$  mm,  $L_a = 350$  mm,  $L_b = 200$  mm, and  $L_c = 100$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 120^\circ$ . Find the positions of the joints and the angles of the links.
- 3.19 The dimensions of the mechanism shown in Fig. 3.33 are:  $AB = 150$  mm,  $AC = 220$  mm,  $CD = 280$  mm,  $DE = 200$  mm, and  $L_a = 230$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 60^\circ$ . Find the positions of the joints and the angles of the links.
- 3.20 The dimensions of the mechanism shown in Fig. 3.34 are:  $AB = 200$  mm,  $AC = 60$  mm,  $CD = 200$  mm, and  $DE = 500$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 45^\circ$ . Find the positions of the joints and the angles of the links.
- 3.21 The dimensions of the mechanism shown in Fig. 3.35 are:  $AB = 120$  mm,  $AC = 200$  mm,  $CD = 380$  mm, and  $b = 450$  mm. The angle of the driver link 1 with the horizontal is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.
- 3.22 The dimensions of the mechanism shown in Fig. 3.36 are:  $AB = 160$  mm,  $AC = 90$  mm, and  $CD = 160$  mm. The angle of driver

link 1 with the horizontal axis is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.

3.23 The dimensions of the mechanism shown in Fig. 3.37 are:  $AB = 100$  mm,  $AC = 280$  mm,  $BD = L_a = 470$  mm, and  $DE = 220$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 30^\circ$ . Find the positions of the joints and the angles of the links.

3.24 The dimensions of the mechanism shown in Fig. 3.38 are:  $AB = 250$  mm,  $AD = 700$  mm,  $BC = 300$  mm, and  $a = 650$  mm. The angle of driver link 1 with the horizontal axis is  $\phi = \phi_1 = 145^\circ$ . Find the positions of the joints and the angles of the links.

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## Figure captions

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Figure 3.34 Mechanism for Problem 3.20.

Figure 3.35 Mechanism for Problem 3.21.

Figure 3.36 Mechanism for Problem 3.22.

Figure 3.37 Mechanism for Problem 3.23.

Figure 3.38 Mechanism for Problem 3.24.