Contents

7 Simulation of Kinematic Chains with *Mathematica*^{TM}

A planar mechanism will be analyzed and simulated using the *Mathematica*^{TM} software. The planar R-RTR-RTR mechanism considered is shown in Fig. 7.1. The driver link is the rigid link 1 (the link AB). The following numerical data are given: $AB = 0.140$ m, $AC = 0.060$ m, $AE = 0.250$ m, $CD = 0.150$ m. The angle of the driver link 1 with the horizontal axis ϕ.

7.1 Position Analysys

The *Mathematica*^{TM} commands for the input data are

```
AB=0.140;  AC=0.060;   AE=0.250;   CD=0.150;
```

Position analysis for an input angle

The angle of the driver link 1 with the horizontal axis $\phi = 30°$. The *Mathematica*^{TM} command for the input angle is

```
phi=N[Pi]/6;
```

where **N[expr]** gives the numerical value of **expr** and **Pi** is the constant π, with numerical value approximately equal to 3.14159.

Position of joint A

A Cartesian reference frame xOy is selected. The joint A is in the origin of the reference frame, that is, $A \equiv O$,

$$x_A = 0,\ y_A = 0. \tag{7.1}$$

Position of joint C

The coordinates of the joint C are

$$x_C = 0,\ y_C = AC = 0.060 \text{ m}. \tag{7.2}$$

Position of joint E

The coordinates of the joint E are

$$x_E = 0,\ y_E = -AE = -0.250 \text{ m}. \tag{7.3}$$

The *Mathematica*^{TM} commands for Eqs. (7.1), (7.2), and (7.3) are

```
xA=0; yA=0;
xC=0; yC=AC;
xE=0; yE=-AE;
```

Position of joint B

The unknowns are the coordinates of the joint B, x_B and y_B. Because the joint A is fixed and the angle ϕ is known, the coordinates of the joint B are computed from the following expressions

$$x_B = AB\cos\phi = 0.140\cos 30^\circ = 0.121 \text{ m},$$
$$y_B = AB\sin\phi = 0.140\sin 30^\circ = 0.070 \text{ m}. \tag{7.4}$$

The *Mathematica*^{TM} commands for Eq. (7.4) are

```
xB=AB Cos[phi];
yB=AB Sin[phi];
```

where **phi** is the angle ϕ in radians.

Position of joint D

The unknowns are the coordinates of the joint D, x_D and y_D. Knowing the positions of the joints B and C, one can compute the slope m and the intercept b of the line BC

$$m = \frac{(y_B - y_C)}{(x_B - x_C)},$$
$$b = y_B - m\ x_B. \tag{7.5}$$

The *Mathematica*^{TM} commands for Eq. (7.5) are

```
m=(yB-yC)/(xB-xC);
b=yB-m xB;
```

The joint D is located on the line BC

$$y_D - m\ x_D - b = 0. \tag{7.6}$$

Furthermore, the length of the segment CD is constant

$$(x_C - x_D)^2 + (y_C - y_D)^2 = CD^2. \tag{7.7}$$

The Eqs. (7.6) and (7.7) with *Mathematica*^{TM} commands are

```
eqnD1=(xDsol-xC)^2+(yDsol-yC)^2-CD^2==0;
eqnD2=yDsol-m xDsol-b==0;
```

The Eqs. (7.6) and (7.7) form a system from which the coordinates of the joint D can be computed. To solve the system of equations, a specific *Mathematica*^{TM} command will be used. The command **Solve[eqns, vars]** attempts to solve an equation or set of equations **eqns** for the variables **vars**. For the mechanism

```
solutionD=Solve[{eqnD1,eqnD2},{xDsol,yDsol}];
```

Two sets of solutions are found for the position of the joint D

```
xD1=xDsol/.solutionD[[1]];
yD1=yDsol/.solutionD[[1]];
xD2=xDsol/.solutionD[[2]];
yD2=yDsol/.solutionD[[2]];
```

These solutions are located at the intersection of the line BC with the circle centered in C and radius CD (Fig. 7.2) and they have the following numerical values

$$x_{D1} = -0.149 \text{ m}, \; y_{D1} = 0.047 \text{ m},$$
$$x_{D2} = 0.149 \text{ m}, \; y_{D2} = 0.072 \text{ m}.$$

To determine the correct position of the joint D for the mechanism, an additional condition is needed. For the first quadrant, $0 \leq \phi \leq 90°$, the condition is $x_D \leq x_C$.
This condition with *Mathematica*^{TM} is
If[condition, t, f], that gives **t** if **condition** evaluates to True, and **f** if it evaluates to False.
For the considered mechanism

```
If[xD1<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2];
```

Because $x_C = 0$ m, the coordinates of the joint D are

$$x_D = x_{D1} = -0.149\,\text{m},$$
$$y_D = y_{D1} = 0.047\,\text{m}.$$

The numerical solutions for B and D are printed using *Mathematica*^{TM}

```
Print["xB = ", xB, " m" ];
Print["yB = ", yB, " m" ];
Print["xD = ", xD, " m" ];
Print["yD = ", yD, " m" ];
```

The *Mathematica*^{TM} program for the input angle $\phi = 30°$ is given in Program 7.1. The program contains at the end the commands to draw the mechanism.

Position analysis for a complete rotation

For a complete rotation of the driver link AB, $0 \leq \phi \leq 360°$, a step angle of $\phi = 60°$ is selected.
To calculate the position analysis for a complete cycle one can use the *Mathematica*^{TM} command **`For[start, test, incr, body]`**. It executes **`start`**, then repeatedly evaluates **`body`** and **`incr`** until **`test`** fails to give True. In the case of the mechanism

```
For[phi=0, phi<=2*N[Pi], phi+=N[Pi]/3, Program block];
```

Method I

Method I uses constraint conditions for the mechanism for each quadrant. For the mechanism, there are several conditions for the position of the joint D.
For the angle ϕ located in the first quadrant $0° \leq \phi \leq 90°$ (Fig. 7.2), and the fourth quadrant $270° \leq \phi \leq 360°$ (Fig. 7.5), the following relation exists between x_D and x_C

$$x_D \leq x_C.$$

For the angle ϕ located in the second quadrant $90° < \phi \leq 180°$ (Fig. 7.3), and the third quadrant $180° < \phi < 270°$ (Fig. 7.4), the following relation exists between x_D and x_C

$$x_D \geq x_C.$$

The following *Mathematica*^{TM} commands are used to determine the correct position of the joint D for all four quadrants

```
If[0 <= phi <= N[Pi]/2 || 3 N[Pi]/2 <= phi <= 2 N[Pi],
If[xD1<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2],
If[xD1>=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2]
];
```

where `||` is the logical OR function.

The *Mathematica*^{TM} program for a complete rotation of the driver link using the method I is given in Program 7.2. The graph of the mechanism for a complete rotation of the driver link is given in Fig. 7.6.

Method II

Another position analysis method for a complete rotation of the driver link uses constraint conditions for the initial value of the angle ϕ. For the mechanism, the correct position of the joint D is calculated using a simple function, the euclidian distance between two points P and Q

$$d = \sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2}. \quad (7.8)$$

In *Mathematica*^{TM}, the following function is introduced

```
Dist[xP_,yP_,xQ_,yQ_]:=Sqrt[(xP-xQ)^2+(yP-yQ)^2];
```

For the initial angle $\phi = 0°$, the constraint is $x_D \leq x_C$, so the first position of the joint D, that is, D_0, is calculated for the first step $D = D_0 = D_k$, $k = 0$. For the next position of the joint, D_{k+1}, there are two solutions D^I_{k+1} and D^{II}_{k+1}, $k = 0, 1, 2, ...$. In order to choose the correct solution of the joint, D_{k+1}, it is compared the distances between the old position, D_k, and each new calculated positions D^I_{k+1} and D^{II}_{k+1}. The distances between the known solution D_k and the new solutions D^I_{k+1} and D^{II}_{k+1} are d^I_k and d^{II}_k. If the distance to the first solution is less than the distance to the second solution, $d^I_k < d^{II}_k$, then the correct answer is $D_{k+1} = D^I_{k+1}$, else $D_{k+1} = D^{II}_{k+1}$ (Fig. 7.7).

The following *Mathematica*^{TM} commands are used to determine the correct position of the joint D using a single condition for all four quadrants

```
increment=0;
For[phi=0, phi<=2*N[Pi], phi+=N[Pi]/3,
...
If[increment==0, If[xD1<xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2],
dist1=Dist[xD1,yD1,xDold,yDold];
dist2=Dist[xD2,yD2,xDold,yDold];
If[dist1<dist2, xD=xD1;yD=yD1, xD=xD2;yD=yD2 ]
];
xDold=xD;
yDold=yD;
increment++;
...
];
```

Using this algorithm the correct solution is selected using just one constraint relation for the initial step and then automatically the problem is solved. In this way it is not necessary to have different constraints for different quadrants.

The *Mathematica*^{TM} program for a complete rotation of the driver link using the second method is given in Program 7.3.

7.2 Velocity and Acceleration Analysis

For the considered mechanism (Fig. 7.1) the driver link 1 is rotating with a constant speed of $n = 50$ rpm. Next a *Mathematica*^{TM} program for velocity and acceleration analysis will be presented.

The *Mathematica*^{TM} commands for the angular speed, in rad/s, are

```
n=50; (* rpm *)
omega=n*N[Pi]/30; (* rad/s *)
```

The *Mathematica*^{TM} commands for coordinates of the joints A, C and E are

```
xA=0; yA=0;
xC=0; yC=AC;
```

```
xE=0; yE=-AE;
```

The coordinates of the joint B ($B = B_1 = B_2$ on the link 1 or 2) are

$$x_B(t) = AB \cos\phi(t) \quad \text{and} \quad y_B(t) = AB \sin\phi(t).$$

To calculate symbolically the position of the joint B, the following *Mathematica^{TM}* commands are used

```
xB=AB Cos[phi[t]];
yB=AB Sin[phi[t]];
```

where **phi[t]** represents the mathematical function $\phi(t)$. The function name is **phi** and it has one argument the time **t**.
To calculate numerically the position of the joint B, the symbolical variables needs to be substituted with the input data. To apply a transformation rule to a particular expression **expr**, type **expr/.lhs->rhs**. To apply a sequence of rules on each part of the expression **expr**, type **expr/.{lhs1->rhs1, lhs2->rhs2, ...}**.
For the mechanism, the transformation rule represents the initial data of the mechanism

```
initdata={AB->0.14, AC->0.06, AE->0.25, CD->0.15,
phi[t]->N[Pi]/6, phi'[t]->omega, phi''[t]->0};
```

where **phi'[t]** is the first derivative of **phi** with respect to **t**, and **phi''[t]** is the second derivative of the function.
The command **Print[expr1, expr2, ...]** prints the **expr1, expr2, ...**, followed by a new line. To print the solutions of the position vector, the following commands are used

```
Print["xB = ", xB," = ", xB/.initdata, " m" ];
Print["yB = ", yB," = ", yB/.initdata, " m" ];
```

The linear velocity vector of the joint B ($B = B_1 = B_2$) is

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \dot{x}_B\boldsymbol{\imath} + \dot{y}_B\boldsymbol{\jmath},$$

where

$$\dot{x}_B = \frac{dx_B}{dt} = -AB\dot{\phi}\sin\phi \quad \text{and} \quad \dot{y}_B = \frac{dy_B}{dt} = AB\dot{\phi}\cos\phi,$$

are the components of the velocity vector of B.
To calculate symbolically the components of the velocity vector using the *Mathematica^{TM}* the command **D[f, t]** is used, which gives the derivative of **f** with respect to **t**

```
vBx=D[xB,t];
vBy=D[yB,t];
```

For the mechanism, $\dot{\phi} = \omega = \frac{\pi n}{30} = \frac{\pi(50)}{30}$ rad/s = 5.235 rad/s, and the numerical values are

$$\dot{x}_B = -0.140\ (5.235)\sin 30^\circ = -0.366\,\text{m/s},$$
$$\dot{y}_B = 0.140\ (5.235)\cos 30^\circ = 0.634\,\text{m/s}.$$

The solutions can be printed using *Mathematica^{TM}*

```
Print["vBx = ", vBx," = ", vBx/.initdata, " m/s" ];
Print["vBy = ", vBy," = ", vBy/.initdata, " m/s" ];
```

The linear acceleration vector of the joint B ($B = B_1 = B_2$) is

$$\mathbf{a}_B = \ddot{x}_B\mathbf{\imath} + \ddot{y}_B\mathbf{\jmath},$$

where

$$\ddot{x}_B = \frac{d\dot{x}_B}{dt} = -AB\dot{\phi}^2\cos\phi - AB\ddot{\phi}\sin\phi,$$
$$\ddot{y}_B = \frac{d\dot{y}_B}{dt} = -AB\dot{\phi}^2\sin\phi + AB\ddot{\phi}\cos\phi,$$

are the components of the acceleration vector of the joint B.
The *Mathematica^{TM}* commands used to calculate symbolically the components of the acceleration vector are

```
aBx=D[vBx,t];
```

```
aBy=D[vBy,t];
```

For the considered mechanism the angular acceleration of the link 1 is $\ddot{\phi} = \dot{\omega} = 0$. The numerical values of the acceleration of B are

$$\ddot{x}_B = -0.140\ (5.235)^2 \cos 30^\circ = -3.323\,\text{m/s}^2,$$
$$\ddot{y}_B = -0.140\ (5.235)^2 \sin 30^\circ = -1.919\,\text{m/s}^2.$$

The solutions printed with *Mathematica^{TM}* are

```
Print["aBx = ", aBx," = ", aBx/.initdata, " m/s^2" ];
Print["aBy = ", aBy," = ", aBy/.initdata, " m/s^2" ];
```

The coordinates of the joint D are x_D and y_D. The *Mathematica^{TM}* commands used to calculate the position of D are

```
mBC=(yB-yC)/(xB-xC);
bBC=yB-mBC xB;
eqnD1=(xDsol-xC)^2+(yDsol-yC)^2-CD^2==0;
eqnD2=yDsol-mBC xDsol-nBC==0;
solutionD=Solve[{eqnD1,eqnD2},{xDsol,yDsol}];
```

where **mBC** is the slope and **bBC** is the y-intercept of the line BC.
Two sets of solutions are found for the position of the joint D, that are functions of the angle $\phi(t)$, i.e. functions of time.

```
xD1=xDsol/.solutionD[[1]];
yD1=yDsol/.solutionD[[1]];
xD2=xDsol/.solutionD[[2]];
yD2=yDsol/.solutionD[[2]];
```

To determine the correct position of the joint D for the mechanism, an additional condition is needed. For the first quadrant, $0 \leq \phi \leq 90^\circ$, the condition is $x_D \leq x_C$.
This condition using the *Mathematica^{TM}* command is

```
If[xD1/.initdata<=xC, xD=xD1;yD=yD1, xD=xD2;yD=yD2];
```

The numerical solutions are printed using $Mathematica^{TM}$

```
Print["xD = ", xD/.initdata, " m" ];
Print["yD = ", yD/.initdata, " m" ];
```

The linear velocity vector of the joint D ($D = D_3 = D_4$ on link 3 or link 4) is

$$\mathbf{v}_D = \mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \dot{x}_D\mathbf{\imath} + \dot{y}_D\mathbf{\jmath},$$

where

$$\dot{x}_D = \frac{dx_D}{dt} \quad \text{and} \quad \dot{y}_D = \frac{dy_D}{dt}.$$

are the components of the velocity vector of the joint D on the x-axis and, respectively, on the y-axis.
To calculate symbolically the components of this velocity vector the following *Mathematica* ^{TM} commands are used

```
vDx=D[xD,t];
vDy=D[yD,t];
```

For the considered mechanism the numerical values are

$$\dot{x}_D = 0.067\,\text{m/s and } \dot{y}_D = -0.814\,\text{m/s}.$$

The numerical solutions are printed using $Mathematica^{TM}$

```
Print["vDx = ", vDx/.initdata, " m/s" ];
Print["vDy = ", vDy/.initdata, " m/s" ];
```

The linear acceleration vector of $D = D_3 = D_4$ is

$$\mathbf{a}_D = \ddot{x}_D\mathbf{\imath} + \ddot{y}_D\mathbf{\jmath},$$

where

$$\ddot{x}_D = \frac{d\dot{x}_D}{dt} \quad \text{and} \quad \ddot{y}_B = \frac{d\dot{y}_B}{dt}.$$

To calculate symbolically the components of the acceleration vector the following $Mathematica^{TM}$ commands are used

```
aDx=D[vDx,t];
```

```
aDy=D[vDy,t];
```

The numerical values of the acceleration of D are

$$\ddot{x}_D = 4.617\,\text{m/s}^2 \quad \text{and} \quad \ddot{y}_D = -1.811\,\text{m/s}^2,$$

and can be printed using *Mathematica*^{TM}

```
Print["aDx = ", aDx/.initdata, " m/s^2" ];
Print["vDy = ", vDy/.initdata, " m/s^2" ];
```

The angle $\phi_3(t)$ is determined as a function of time t from the equation of the slope of the line BC

$$\tan\phi_3(t) = m_{BC}(t).$$

The *Mathematica*^{TM} function **ArcTan[z]**, gives the arc tangent of the number **z**. To calculate symbolically the angle ϕ_3

```
phi3=ArcTan[mBC];
```

The angular velocity $\omega_3(t)$ is the derivative with respect to time of the angle $\phi_3(t)$

$$\omega_3 = \frac{d\phi_3(t)}{dt}.$$

Symbolically the angular velocity ω_3 is calculated using *Mathematica*^{TM}

```
omega3=D[phi3,t];
```

The angular acceleration $\alpha_3(t)$ is the derivative with respect to time of the angular velocity $\omega_3(t)$

$$\alpha_3(t) = \frac{d\omega_3(t)}{dt}.$$

Symbolically, using *Mathematica*^{TM}, the angular acceleration α_3 is

```
alpha3=D[omega3,t];
```

The numerical values of the angles, angular velocities and angular accelerations for the link 2 and link 3 are

$$\phi_3 = \phi_2 = 0.082 \text{ rad}, \quad \omega_3 = \omega_2 = 5.448 \text{ rad/s}, \quad \alpha_3 = \alpha_2 = 14.568 \text{ rad/s}^2.$$

The numerical solutions are printed using *Mathematica*^{TM}

```
Print["phi3=phi2= ",phi3/.initdata," rad "];
Print["omega3=omega2= ",omega3/.initdata," rad/s"];
Print["alpha3=alpha2= ",alpha3/.initdata," rad/s^2"];
```

The angle $\phi_5(t)$ is determined as a function of time t from the following equation

$$\tan \phi_5(t) = \frac{y_D(t) - y_E}{x_D(t) - x_E},$$

and symbolically using *Mathematica*^{TM}

```
phi5=ArcTan[(yD-yE)/(xD-xE)];
```

The angular velocity $\omega_5(t)$ is the derivative with respect to time of the angle $\phi_5(t)$

$$\omega_5 = \frac{d\phi_5(t)}{dt}.$$

To calculate symbolically the angular velocity ω_5 using *Mathematica*^{TM} the following command is used

```
omega5=D[phi5,t];
```

The angular acceleration $\alpha_5(t)$ is the derivative with respect to time of the angular velocity $\omega_5(t)$

$$\alpha_5(t) = \frac{d\omega_5(t)}{dt},$$

and symbolically with *Mathematica*^{TM}

```
alpha5=D[omega5,t];
```

The numerical values of the angles, angular velocities and angular accelerations for the link 5 and link 4 are

$$\phi_5 = \phi_4 = 2.036 \text{ rad}, \quad \omega_5 = \omega_4 = 0.917 \text{ rad/s}, \quad \alpha_5 = \alpha_4 = -5.771 \text{ rad/s}^2.$$

The numerical solutions printed with *Mathematica*^{TM} are

```
Print["phi5=phi4= ",phi5/.initdata," rad "];
Print["omega5=omega4= ",omega5/.initdata," rad/s"];
Print["alpha5=alpha4= ",alpha5/.initdata," rad/s^2"];
```

The *Mathematica*^{TM} program for the velocity and acceleration analysis is given in Program 7.4.

7.3 Contour Equations for Velocities and Accelerations

The same planar R-RTR-RTR mechanism is considered in Fig. 7.8(a). The driver link 1 is rotating with a constant speed of $n = 50$ rpm. A *Mathematica*^{TM} program for velocity and acceleration analysis using the contour equations will be presented.

The mechanism has five moving links and seven full joints. The number of independent contours is

$$n_c = c - n = 7 - 5 = 2,$$

where c is the number of joints and n is the number of moving links.

The mechanism has two independent contours. The first contour I contains the links 0, 1, 2 and 3, while the second contour II contains the links 0, 3, 4 and 5. The diagram of the mechanism is represented in Fig. 7.8(b). Clockwise paths are chosen for each closed contours I and II.

First contour analysis

Figure 7.9(a) shows the first independent contour I with

- rotational joint R between the links 0 and 1 (joint A);
- rotational joint R between the links 1 and 2 (joint B);
- translational joint T between the links 2 and 3 (joint B);
- rotational joint R between the links 3 and 0 (joint C).

The angular velocity ω_{10} of the driver link is known

$$\omega_{10} = \omega_1 = \omega = \frac{50\pi}{30} \text{ rad/s} = 5.235 \text{ rad/s}.$$

The origin of the reference frame is the point $A(0,0,0)$.
For the velocity analysis, the following vectorial equations are used

$$\boldsymbol{\omega}_{10} + \boldsymbol{\omega}_{21} + \boldsymbol{\omega}_{03} = \mathbf{0},$$
$$\mathbf{r}_{AB} \times \boldsymbol{\omega}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\omega}_{03} + \mathbf{v}^r_{B32} = \mathbf{0}, \tag{7.9}$$

where $\mathbf{r}_{AB} = x_B\mathbf{\imath} + y_B\mathbf{\jmath}$, $\mathbf{r}_{AC} = x_C\mathbf{\imath} + y_C\mathbf{\jmath}$, and

$$\boldsymbol{\omega}_{10} = \omega_{10}\mathbf{k},\ \boldsymbol{\omega}_{21} = \omega_{21}\mathbf{k},\ \boldsymbol{\omega}_{03} = \omega_{03}\mathbf{k},$$
$$\mathbf{v}^r_{B32} = \mathbf{v}_{32} = v_{32}\cos\phi_2\mathbf{\imath} + v_{32}\sin\phi_2\mathbf{\jmath}.$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 7.8(a) and 7.9(a). Then the numerical computation will give the correct orientation of the unknown vectors. The components of the vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{AC}$, and the angle ϕ_2 are already known from the position analysis of the mechanism. Equation (7.9) becomes

$$\omega_{10}\mathbf{k} + \omega_{21}\mathbf{k} + \omega_{03}\mathbf{k} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_B & y_B & 0 \\ 0 & 0 & \omega_{21} \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \omega_{03} \end{vmatrix} + v_{32}\cos\phi_2\mathbf{\imath} + v_{32}\sin\phi_2\mathbf{\jmath} = \mathbf{0}. \tag{7.10}$$

In the *Mathematica^{TM}* environment, a three-dimensional vector **v** is written as a list of variables `v={x, y, z}` where `x`, `y`, and `z` are the spatial coordinates of the vector **v**. The first component of the vector **v** is `x=v[[1]]`, the second component is `y=v[[2]]`, and the third component is `z=v[[3]]`.
For the considered mechanism with *Mathematica^{TM}*

```
rB={xB, yB, 0};
rC={xC, yC, 0};
omega10v={0,0,omega};
omega21vSol={0,0,omega21Sol};
omega03vSol={0,0,omega03Sol};
v32vSol={v32Sol Cos[phi2],v32Sol Sin[phi2],0};
```

Equation (7.10) represents a system of three equations and with *Mathematica^{TM}* commands gives

```
eqIkv=(omega10v+omega21vSol+omega03vSol)[[3]]==0;
```

```
  eqIiv=(Cross[rB,omega21vSol]+Cross[rC,omega03vSol]+
v32vSol)[[1]]==0;
  eqIjv=(Cross[rB,omega21vSol]+Cross[rC,omega03vSol]+
v32vSol)[[2]]==0;
```

where the command **Cross[a,b]** gives the vector cross product of the vectors **a** and **b**.
The system of equations can be solved using the *Mathematica*^{TM} commands

```
  solIvel=Solve[{eqIkv,eqIiv,eqIjv}, {omega21Sol,
omega03Sol,v32Sol}];
```

The following numerical solutions are obtained

$$\omega_{21} = 0.212 \text{ rad/s}, \ \omega_{03} = -5.448 \text{ rad/s}, \text{ and } v_{32} = 0.313 \text{ m/s}.$$

To print the numerical values the following *Mathematica*^{TM} commands are used

```
  omega21v=omega21vSol/.solIvel[[1]];
  omega03v=omega03vSol/.solIvel[[1]];
  v32v=v32vSol/.solIvel[[1]];
  Print["omega21 = ",omega21v];
  Print["omega03 = ",omega03v];
  Print["v32 = ",v32v];
  Print["v32r = ",v32Sol/.solIvel[[1]] ];
```

The absolute angular velocities of the links 2 and 3 are

$$\boldsymbol{\omega}_{20} = \boldsymbol{\omega}_{30} = -\boldsymbol{\omega}_{03} = 5.448 \ \mathbf{k} \text{ rad/s}.$$

The absolute linear velocities of the joints B and D are

$$\mathbf{v}_B = \mathbf{v}_{B_1} = \mathbf{v}_{B_2} = \mathbf{v}_A + \boldsymbol{\omega}_{10} \times \mathbf{r}_{AB} = -0.366 \ \boldsymbol{\imath} + 0.634 \ \boldsymbol{\jmath} \text{ m/s},$$
$$\mathbf{v}_D = \mathbf{v}_{D_3} = \mathbf{v}_{D_4} = \mathbf{v}_C + \boldsymbol{\omega}_{30} \times \mathbf{r}_{CD} = 0.067 \ \boldsymbol{\imath} - 0.814 \ \boldsymbol{\jmath} \text{ m/s},$$

where $\mathbf{v}_A = \mathbf{0}$ and $\mathbf{v}_C = \mathbf{0}$ because the joints A and C are grounded and

$$\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}.$$

The *Mathematica^{TM}* commands for the absolute velocities are

```
omega20v=omega30v=-omega03v;
vBv=Cross[omega10v,rB];
vDv=Cross[omega30v,(rD-rC)];
Print["omega20 = omega30 = ",omega20v];
Print["vB = ",vBv];
Print["vD = ",vDv];
```

For the acceleration analysis, the following vectorial equations are used

$$\boldsymbol{\alpha}_{10} + \boldsymbol{\alpha}_{21} + \boldsymbol{\alpha}_{03} = \mathbf{0},$$
$$\mathbf{r}_{AB} \times \boldsymbol{\alpha}_{21} + \mathbf{r}_{AC} \times \boldsymbol{\alpha}_{03} + \mathbf{a}^r_{B32} + \mathbf{a}^c_{B32} - \omega^2_{10}\mathbf{r}_{AB} - \omega^2_{20}\mathbf{r}_{BC} = \mathbf{0}. \quad (7.11)$$

where

$$\boldsymbol{\alpha}_{10} = \alpha_{10}\mathbf{k},\ \boldsymbol{\alpha}_{21} = \boldsymbol{\alpha}_{21}\mathbf{k},\ \boldsymbol{\alpha}_{03} = \boldsymbol{\alpha}_{03}\mathbf{k},$$
$$\mathbf{a}^r_{B32} = \mathbf{a}_{32} = a_{32}\cos\phi_2\mathbf{\imath} + a_{32}\sin\phi_2\mathbf{\jmath},$$
$$\mathbf{a}^c_{B32} = \mathbf{a}^c_{32} = 2\boldsymbol{\omega}_{20} \times \mathbf{v}_{32},$$

The driver link has a constant angular velocity and $\alpha_{10} = \dot{\omega}_{10} = 0$.
The acceleration vectors using the *Mathematica^{TM}* commands are

```
alpha10v={0,0,0};
alpha21vSol={0,0,alpha21Sol};
alpha03vSol={0,0,alpha03Sol};
a32vSol={a32Sol Cos[phi2],a32Sol Sin[phi2],0};
```

Equation (7.11) represents a system of three equations and using *Mathematica^{TM}* commands gives

```
eqIka=(alpha10v+alpha21vSol+alpha03vSol)[[3]]==0;

eqIia=( Cross[rB,alpha21vSol]+Cross[rC,alpha03vSol]+
a32vSol+2Cross[omega20v,v32v]-(omega10v.omega10v)rB-
(omega20v.omega20v)(rC-rB) )[[1]]==0;

eqIja=( Cross[rB,alpha21vSol]+Cross[rC,alpha03vSol]+
a32vSol+2Cross[omega20v,v32v]-(omega10v.omega10v)rB-
```

```
(omega20v.omega20v)(rC-rB) )[[2]]==0;
```

The unknowns in the Eq. (7.18) are α_{21}, α_{03} and a_{32}. The system of equations is solved using the *Mathematica*^{TM} commands

```
  solIacc=Solve[{eqIka,eqIia,eqIja}, {alpha21Sol,
alpha03Sol,a32Sol}];
```

The following numerical solutions are obtained

$$\alpha_{21} = 14.568 \text{ rad/s}^2, \; \alpha_{03} = -14.568 \text{ rad/s}^2, \text{ and } a_{32} = -0.140 \text{ m/s}^2.$$

To print the numerical values the following *Mathematica*^{TM} commands are used

```
  alpha21v=alpha21vSol/.solIacc[[1]];
  alpha03v=alpha03vSol/.solIacc[[1]];
  a32v=a32vSol/.solIacc[[1]];
  Print["alpha21 = ",alpha21v];
  Print["alpha03 = ",alpha03v];
  Print["a32 = ",a32v];
  Print["a32r = ",a32Sol/.solIacc[[1]]];
```

The absolute angular accelerations of the links 2 and 3 are

$$\boldsymbol{\alpha}_{20} = \boldsymbol{\alpha}_{30} = -\boldsymbol{\alpha}_{03} = 14.568 \; \mathbf{k} \text{ rad/s}^2.$$

The absolute linear accelerations of the joints B and D are obtained from the following equation

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha}_{10} \times \mathbf{r}_{AB} - \omega_{10}^2 \mathbf{AB} = -3.323 \; \boldsymbol{\imath} - 1.919 \; \boldsymbol{\jmath} \text{ m/s}^2,$$
$$\mathbf{a}_D = \mathbf{a}_C + \boldsymbol{\alpha}_{30} \times \mathbf{r}_{CD} - \omega_{30}^2 \mathbf{r}_{CD} = 4.617 \; \boldsymbol{\imath} - 1.811 \; \boldsymbol{\jmath} \text{ m/s}^2,$$

where $\mathbf{a}_A = \mathbf{0}$ and $\mathbf{a}_C = \mathbf{0}$ because the joints A and C are grounded.

To print the absolute accelerations with *Mathematica*^{TM} the following relations are used

```
  alpha20v=alpha30v=-alpha03v;
  aBv=-(omega10v.omega10v)rB;
```

```
aDv=Cross[alpha30v,(rD-rC)]-(omega20v.omega20v)(rD-rC);
Print["alpha20 = alpha30 = ",alpha30v];
Print["aB = ",aBv];
Print["aD = ",aDv];
```

Second contour analysis

Figure 7.10(a) depicts the second independent contour II

- rotational joint R between the links 0 and 3 (joint C);
- rotational joint R between the links 3 and 4 (joint D);
- translational joint T between the links 4 and 5 (joint D);
- rotational joint R between the links 5 and 0 (joint E).

For the velocity analysis, the following vectorial equations are used

$$\begin{aligned}&\boldsymbol{\omega}_{30} + \boldsymbol{\omega}_{43} + \boldsymbol{\omega}_{05} = \mathbf{0},\\ &\mathbf{r}_{AC} \times \boldsymbol{\omega}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\omega}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\omega}_{05} + \mathbf{v}^r_{D54} = \mathbf{0},\end{aligned} \tag{7.12}$$

where $\mathbf{r}_{AD} = x_D\mathbf{\imath} + y_D\mathbf{\jmath}$, $\mathbf{r}_{AE} = x_E\mathbf{\imath} + y_E\mathbf{\jmath}$, and

$$\begin{aligned}&\boldsymbol{\omega}_{30} = \omega_{30}\mathbf{k},\ \boldsymbol{\omega}_{43} = \omega_{43}\mathbf{k},\ \boldsymbol{\omega}_{05} = \omega_{05}\mathbf{k},\\ &\mathbf{v}^r_{D54} = \mathbf{v}_{54} = v_{54}\cos\phi_4\mathbf{\imath} + v_{54}\sin\phi_4\mathbf{\jmath}.\end{aligned}$$

The sign of the relative angular velocities is selected as positive as shown in Figs. 7.8(a) and 7.10(a). Then the numerical computation will give the correct orientation of the unknown vectors. The components of the vectors $\mathbf{r}_{AD}$ and $\mathbf{r}_{AE}$, and the angle ϕ_4 are already known from the position analysis of the mechanism.

The unknown vectors with *Mathematica^{TM}* commands are

```
omega43vSol={0,0,omega43Sol};
omega05vSol={0,0,omega05Sol};
v54vSol={v54Sol Cos[phi4],v54Sol Sin[phi4],0};
```

Equtions (7.12) becomes

$$\begin{aligned}&\omega_{30}\mathbf{k} + \omega_{43}\mathbf{k} + \omega_{05}\mathbf{k} = \mathbf{0},\\ &\begin{vmatrix}\mathbf{\imath} & \mathbf{\jmath} & \mathbf{k}\\ x_C & y_C & 0\\ 0 & 0 & \omega_{30}\end{vmatrix} + \begin{vmatrix}\mathbf{\imath} & \mathbf{\jmath} & \mathbf{k}\\ x_D & y_D & 0\\ 0 & 0 & \omega_{43}\end{vmatrix} + \begin{vmatrix}\mathbf{\imath} & \mathbf{\jmath} & \mathbf{k}\\ x_E & y_E & 0\\ 0 & 0 & \omega_{05}\end{vmatrix} +\\ &v_{32}\cos\phi_4\mathbf{\imath} + v_{32}\sin\phi_4\mathbf{\jmath} = \mathbf{0}.\end{aligned} \tag{7.13}$$

The Eq. (7.13) projected onto the "fixed" reference frame $Oxyz$ gives

$$\begin{aligned} &\omega_{30} + \omega_{43} + \omega_{05} = 0, \\ &y_C\omega_{30} + y_D\omega_{43} + y_E\omega_{05} + v_{54}\cos\phi_4 = 0, \\ &-x_C\omega_{30} - x_D\omega_{43} - x_E\omega_{05} + v_{54}\sin\phi_4 = 0. \end{aligned} \tag{7.14}$$

The above system of equations using the following *Mathematica^{TM}* commands becomes

```
eqIIkv=(omega30v+omega43vSol+omega05vSol)[[3]]==0;
  eqIIiv=(Cross[rC,omega30v]+Cross[rD,omega43vSol]+
Cross[rE,omega05vSol]+v54vSol)[[1]]==0;
  eqIIjv=(Cross[rC,omega30v]+Cross[rD,omega43vSol]+
Cross[rE,omega05vSol]+v54vSol)[[2]]==0;
```

The Eq. (7.14) represents an algebraic system of three equations with three unknowns ω_{43}, ω_{05} and v_{54}. The system is solved using the *Mathematica^{TM}* commands

```
solIIvel=Solve[{eqIIkv,eqIIiv,eqIIjv}, {omega43Sol,
omega05Sol,v54Sol}];
```

The following numerical solutions are obtained

$$\omega_{43} = -4.531 \text{ rad/s},\ \omega_{05} = -0.917 \text{ rad/s},\ \text{and } v_{54} = 0.757 \text{ m/s}.$$

To print the numerical values with *Mathematica^{TM}* the following commands are used

```
omega43v=omega43vSol/.solIIvel[[1]];
omega05v=omega05vSol/.solIIvel[[1]];
v54v=v54vSol/.solIIvel[[1]];
Print["omega43 = ",omega43v];
Print["omega05 = ",omega05v];
Print["v54 = ",v54v];
Print["v54r=",v54Sol/.solIIvel[[1]] ];
```

The absolute angular velocities of the links 4 and 5 are

$$\boldsymbol{\omega}_{40} = \boldsymbol{\omega}_{50} = -\boldsymbol{\omega}_{05} = 0.917\ \mathbf{k} \text{ rad/s}, \tag{7.15}$$

and with *Mathematica^{TM}* commands

```
omega40v=omega50v=-omega05v;
Print["omega40 = omega50 = ",omega50v];
```

For the acceleration analysis, the following vectorial equations are used

$$\boldsymbol{\alpha}_{30} + \boldsymbol{\alpha}_{43} + \boldsymbol{\alpha}_{05} = \mathbf{0}, \tag{7.16}$$
$$\mathbf{r}_{AC} \times \boldsymbol{\alpha}_{30} + \mathbf{r}_{AD} \times \boldsymbol{\alpha}_{43} + \mathbf{r}_{AE} \times \boldsymbol{\alpha}_{05} + \mathbf{a}^r_{D54} + \mathbf{a}^c_{B54} - \omega^2_{30}\mathbf{r}_{CD} - \omega^2_{40}\mathbf{r}_{DE} = \mathbf{0}.$$

where

$$\boldsymbol{\alpha}_{30} = \alpha_{30}\mathbf{k},\ \boldsymbol{\alpha}_{43} = \boldsymbol{\alpha}_{43}\mathbf{k},\ \boldsymbol{\alpha}_{05} = \boldsymbol{\alpha}_{05}\mathbf{k},$$
$$\mathbf{a}^r_{B54} = \mathbf{a}_{54} = a_{54}\cos\phi_4\mathbf{\imath} + a_{54}\sin\phi_4\mathbf{\jmath},$$
$$\mathbf{a}^c_{B54} = 2\boldsymbol{\omega}_{40} \times \mathbf{v}_{54}.$$

The unknown acceleration vectors using the *Mathematica^{TM}* commands are

```
alpha43vSol={0,0,alpha43Sol};
alpha05vSol={0,0,alpha05Sol};
a54vSol={a54Sol Cos[phi4],a54Sol Sin[phi4],0};
```

Equation (7.16) becomes

$$\alpha_{30}\mathbf{k} + \alpha_{43}\mathbf{k} + \alpha_{05}\mathbf{k} = \mathbf{0},$$
$$\begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_C & y_C & 0 \\ 0 & 0 & \alpha_{30} \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_D & y_D & 0 \\ 0 & 0 & \alpha_{43} \end{vmatrix} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ x_E & y_E & 0 \\ 0 & 0 & \alpha_{05} \end{vmatrix} +$$
$$a_{54}\cos\phi_4\mathbf{\imath} + a_{54}\sin\phi_4\mathbf{\jmath} + \begin{vmatrix} \mathbf{\imath} & \mathbf{\jmath} & \mathbf{k} \\ 0 & 0 & \omega_{40} \\ v_{54}\cos\phi_4 & v_{54}\sin\phi_4 & 0 \end{vmatrix} -$$
$$\omega^2_{30}[(x_D - x_C)\mathbf{\imath} + (y_D - y_C)\mathbf{\jmath}] -$$
$$\omega^2_{40}[(x_E - x_D)\mathbf{\imath} + (y_E - y_D)\mathbf{\jmath}] = \mathbf{0}. \tag{7.17}$$

The Eq. (7.17) can be rewritten as

$$\alpha_{30} + \alpha_{43} + \alpha_{05} = 0,$$
$$y_C\alpha_{30} + y_D\alpha_{43} + y_E\alpha_{05} + a_{54}\cos\phi_4 - 2\omega_{40}v_{54}\sin\phi_4 -$$

$$\omega_{30}^2(x_D - x_C) - \omega_{40}^2(x_E - x_D) = 0,$$
$$-x_C\alpha_{30} - x_D\alpha_{43} - x_E\alpha_{05} + a_{54}\sin\phi_4 + 2\omega_{40}v_{54}\cos\phi_4 -$$
$$\omega_{30}^2(y_D - y_C) - \omega_{40}^2(y_E - y_D) = 0. \tag{7.18}$$

The contour acceleration equations using *Mathematica^{TM}* commands are

```
eqIIka=(alpha30v+alpha43vSol+alpha05vSol)[[3]]==0;

eqIIia=( Cross[rC,alpha30v]+Cross[rD,alpha43vSol]+
Cross[rE,alpha05vSol]+a54vSol+2Cross[omega40v,v54v]-
(omega30v.omega30v)(rD-rC)-
(omega40v.omega40v)(rE-rD) )[[1]]==0;

eqIIja=( Cross[rC,alpha30v]+Cross[rD,alpha43vSol]+
Cross[rE,alpha05vSol]+a54vSol+2Cross[omega40v,v54v]-
(omega30v.omega30v)(rD-rC)-
(omega40v.omega40v)(rE-rD) )[[2]]==0;
```

The unknowns in the Eq. (7.18) are α_{43}, α_{05} and a_{54}. To solve the system the following *Mathematica^{TM}* command is used

```
solIIacc=Solve[{eqIIka,eqIIia,eqIIja}, {alpha43Sol,
alpha05Sol,a54Sol}];
```

The following numerical solutions are obtained

$$\alpha_{43} = -20.339 \text{ rad/s}^2,\ \alpha_{05} = 5.771 \text{ rad/s}^2,\ \text{and } a_{54} = 3.411 \text{ m/s}^2.$$

The *Mathematica^{TM}* commands are

```
alpha43v=alpha43vSol/.solIIacc[[1]];
alpha05v=alpha05vSol/.solIIacc[[1]];
a54v=a54vSol/.solIIacc[[1]];
Print["alpha43 = ",alpha43v];
Print["alpha05 = ",alpha05v ];
Print["a54 = ",a54v];
Print["a54r=",a54Sol/.solIIacc[[1]]];
```

The absolute angular accelerations of the links 4 and 5 are

$$\boldsymbol{\alpha}_{40} = \boldsymbol{\alpha}_{50} = -\boldsymbol{\alpha}_{05} = -5.771 \ \mathbf{k} \ \text{rad/s}^2,$$

and with *Mathematica^{TM}*

```
alpha40v=alpha50v=-alpha05v;
Print["alpha40 = alpha50 = ",alpha50v];
```

The *Mathematica^{TM}* program for the velocity and acceleration analysis using the contour method is given in Program 7.5.

7.4 Dynamic Force Aanalysis

In this section, the motor moment $\mathbf{M}_m$ required for the dynamic equilibrium of the considered mechanism, shown in Fig. 7.11(a), is calculated. The joint reaction forces are also calculated. The widths of the links 1, 3, and 5 are $AB = 0.140$ m, $FD = 0.400$ m, and respectively, $EG = 0.500$ m. The height of the links 1, 3, and 5 is $h = 0.010$ m. The width of the links 2 and 4 is $w_{Slider} = 0.050$ m, and the height is $h_{Slider} = 0.020$ m. All the five moving links are rectangular prisms with the depth $d = 0.001$ m. The angle of the driver link is $\phi = \dfrac{\pi}{6}$ rad and the angular velocity is $n = 50$ rpm. The external moment applied on the link 5 is opposed to the motion of the link. Because $\boldsymbol{\omega}_5 = 0.917 \ \mathbf{k}$ rad/s, the external moment vector will be $\mathbf{M}_e = -100 \ \mathbf{k}$ N·m. The density of the material is $\rho_{Steel} = \rho = 8000$ kg/m^3. The gravitational acceleration is $g = 9.807$ m/s^2. The center of mass locations of the links $i = 1, 2, ..., 5$ are designated by $C_i(x_{C_i}, y_{C_i}, 0)$.

The input data are introduced using a *Mathematica^{TM}* rule

```
rule={AB->0.14,AC->0.06,AE->0.25,CD->0.15,FD->0.4,
EG->0.5,h->0.01,d->0.001,hSlider->0.02,wSlider->0.05,
rho->8000,g->9.807,Me->-100.,
phi[t]->N[Pi]/6,phi'[t]->omega,phi''[t]->0};
```

where `omega=n*N[Pi]/30`.

Inertia forces and moments

To calculate the inertia moment $\mathbf{M}_i$ and the total force $\mathbf{F}_i$ for the link $i = 1, 2, ..., 5$, the mass m_i, the acceleration of the center of mass $\mathbf{a}_{C_i}$, the gravity force $\mathbf{G}_i$, and the mass moment of inertia I_{C_i} are needed.

Link 1

The mass is of the link is

$$m_1 = \rho\ AB\ h\ d.$$

The position, velocity, and acceleration for the center of mass C_1 are

$$\mathbf{r}_{C_1} = \mathbf{r}_B/2,\ \mathbf{v}_{C_1} = \mathbf{v}_B/2,\ \text{and } \mathbf{a}_{C_1} = \mathbf{a}_B/2.$$

The inertia force is

$$\mathbf{F}_{in1} = -m_1\ \mathbf{a}_{C_1}.$$

The gravitational force is

$$\mathbf{G}_1 = -m_1\ g\ \mathbf{k}.$$

The total force on link 1 at the mass center C_1 is

$$\mathbf{F}_1 = \mathbf{F}_{in1} + \mathbf{G}_1.$$

The mass moment of inertia is

$$I_{C_1} = m_1\ (AB^2 + h^2)/12.$$

The moment of inertia is

$$\mathbf{M}_1 = \mathbf{M}_{in1} = -I_{C_1}\ \boldsymbol{\alpha}_1.$$

To calculate and print the numerical values of the total force $\mathbf{F}_1$ and the moment $\mathbf{M}_1$ the following *Mathematica*^{TM} commands are used

```
m1=rho AB h d /.rule;
rC1=rB/2; vC1=vB/2; aC1=aB/2;
Fin1=-m1 aC1 /.rule;
G1={0,-m1*g,0} /.rule;
F1=(Fin1+G1) /.rule;
IC1=m1 (AB^2+h^2)/12 /.rule;
M1=Min1=-IC1 alpha1 /.rule;
Print["F1 = ",F1];
Print["M1 = ",M1];
```

Link 2
The mass of the link 2 is

$$m_2 = \rho \; h_{Slider} \; w_{Slider} \; d.$$

The position, velocity, and acceleration for the center of mass C_2 are

$$\mathbf{r}_{C_2} = \mathbf{r}_B, \; \mathbf{v}_{C_2} = \mathbf{v}_B, \; \text{and } \mathbf{a}_{C_2} = \mathbf{a}_B.$$

The inertia force is

$$\mathbf{F}_{in2} = -m_2 \; \mathbf{a}_{C_2}.$$

The gravitational force is

$$\mathbf{G}_2 = -m_2 \; g \; \mathbf{k}.$$

The total force on slider 2 at B is

$$\mathbf{F}_2 = \mathbf{F}_{in2} + \mathbf{G}_2.$$

The mass moment of inertia is

$$I_{C_2} = m_2 \; (h^2_{Slider} + w^2_{Slider})/12.$$

The moment of inertia is

$$\mathbf{M}_2 = \mathbf{M}_{in2} = -I_{C_2} \boldsymbol{\alpha}_2.$$

The *Mathematica^{TM}* commands for the total force $\mathbf{F}_2$ and the moment $\mathbf{M}_2$ are

```
m2=rho hSlider wSlider d /.rule;
rC2=rB; vC2=vB; aC2=aB;
Fin2=-m2 aC2 /.rule;
G2={0,-m2*g,0} /.rule;
F2=(Fin2+G2) /.rule;
IC2=m2 (hSlider^2+wSlider^2)/12 /.rule;
M2=Min2=-IC2 alpha2 /.rule;
Print["F2 = ",F2];
Print["M2 = ",M2];
```

Link 3
The mass of link 3 is

$$m_3 = \rho \; FD \; h \; d.$$

The position, velocity, and acceleration for the center of mass C_3 are

$$x_{C_3} = x_C + (FD/2 - CD)\ \cos\phi_3,\ y_{C_3} = y_C + (FD/2 - CD)\ \sin\phi_3,$$
$$\mathbf{r}_{C_3} = x_{C_3}\mathbf{\imath} + y_{C_3}\mathbf{\jmath},\ \mathbf{v}_{C_3} = \dot{x}_{C_3}\mathbf{\imath} + \dot{y}_{C_3}\mathbf{\jmath},\ \text{and } \mathbf{a}_{C_3} = \ddot{x}_{C_3}\mathbf{\imath} + \ddot{y}_{C_3}\mathbf{\jmath}.$$

The inertia force is

$$\mathbf{F}_{in3} = -m_3\ \mathbf{a}_{C_3}.$$

The gravitational force is

$$\mathbf{G}_3 = -m_3\ g\ \mathbf{k}.$$

The total force at C_3 is

$$\mathbf{F}_3 = \mathbf{F}_{in3} + \mathbf{G}_3.$$

The mass moment of inertia is

$$I_{C_3} = m_3\ (FD^2 + h^2)/12.$$

The total moment on link 3 is

$$\mathbf{M}_3 = \mathbf{M}_{in3} = -I_{C_3}\boldsymbol{\alpha}_3.$$

The force $\mathbf{F}_3$ and the moment $\mathbf{M}_3$ with *Mathematica^{TM}* are

```
m3=rho FD h d /.rule;
xC3=xC+(FD/2-CD) Cos[phi3];
yC3=yC+(FD/2-CD) Sin[phi3];
rC3={xC3,yC3,0};
vC3=D[rC3,t];
aC3=D[D[rC3,t],t];
Fin3=-m3 aC3 /.rule;
G3={0,-m3*g,0} /.rule;
F3=(Fin3+G3) /.rule;
IC3=m3 (FD^2+h^2)/12 /.rule;
M3=Min3=-IC3 alpha3 /.rule;
Print["F3 = ",F3];
Print["M3 = ",M3];
```

Link 4

The mass of link 4 is

$$m_4 = \rho\ h_{Slider}\ w_{Slider}\ d.$$

The position, velocity, and acceleration for the center of mass C_4 are

$$\mathbf{r}_{C_4} = \mathbf{r}_D, \ \mathbf{v}_{C_4} = \mathbf{v}_D, \ \text{and } \mathbf{a}_{C_4} = \mathbf{a}_D.$$

The inertia force is

$$\mathbf{F}_{in4} = -m_4 \ \mathbf{a}_{C_4}.$$

The gravitational force is

$$\mathbf{G}_4 = -m_4 \ g \ \mathbf{k}.$$

The total force on slider 4 at D is

$$\mathbf{F}_4 = \mathbf{F}_{in4} + \mathbf{G}_4.$$

The mass moment of inertia is

$$I_{C_4} = m_4(h^2_{Slider} + w^2_{Slider})/12.$$

The moment of inertia is

$$\mathbf{M}_4 = \mathbf{M}_{in4} = -I_{C_4} \ \boldsymbol{\alpha}_4.$$

To calculate and print the numerical values of the total force $\mathbf{F}_4$ and the moment $\mathbf{M}_4$ the following *Mathematica*^{TM} commands are used

```
m4=rho hSlider wSlider d /.rule;
rC4=rD; vC4=vD; aC4=aD;
Fin4=-m4 aC4 /.rule;
G4={0,-m4*g,0} /.rule;
F4=(Fin4+G4) /.rule;
IC4=m4 (hSlider^2+wSlider^2)/12 /.rule;
M4=Min4=-IC4 alpha4 /.rule;
Print["F4 = ",F4];
Print["M4 = ",M4];
```

Link 5
The mass of link 5 is

$$m_5 = \rho \ EG \ h \ d.$$

The position, velocity, and acceleration for the center of mass C_5 are

$$x_{C_5} = (EG/2)\cos\phi_5, \ y_{C_5} = (EG/2)\sin\phi_5,$$
$$\mathbf{r}_{C_5} = x_{C_5}\mathbf{\imath} + y_{C_5}\mathbf{\jmath}, \ \mathbf{v}_{C_5} = \dot{x}_{C_5}\mathbf{\imath} + \dot{y}_{C_5}\mathbf{\jmath}, \ \text{and } \mathbf{a}_{C_5} = \ddot{x}_{C_5}\mathbf{\imath} + \ddot{y}_{C_5}\mathbf{\jmath}.$$

The inertia force is

$$\mathbf{F}_{in5} = -m_5 \; \mathbf{a}_{C_5}.$$

The gravitational force is

$$\mathbf{G}_5 = -m_5 \; g \; \mathbf{k}.$$

The total force on link 5 at C_5 is

$$\mathbf{F}_5 = \mathbf{F}_{in5} + \mathbf{G}_5.$$

The mass moment of inertia is

$$I_{C_5} = m_5 \; (EG^2 + h^2)/12.$$

The moment of inertia is

$$\mathbf{M}_5 = \mathbf{M}_{in5} = -I_{C_5} \; \boldsymbol{\alpha}_5.$$

The total force $\mathbf{F}_5$ and the moment $\mathbf{M}_5$ with $Mathematica^{TM}$ are

```
m5=rho EG h d /.rule;
xC5=EG/2 Cos[phi5];
yC5=EG/2 Sin[phi5];
rC5={xC5,yC5,0};
vC5=D[rC5,t];
aC5=D[D[rC5,t],t];
Fin5=-m5 aC5 /.rule;
G5={0,-m5*g,0} /.rule;
F5=(Fin5+G5) /.rule;
IC5=m5 (EG^2+h^2)/12 /.rule;
M5=Min5=-IC5 alpha5 /.rule;
M5e={0,0,Me} /.rule;
Print["F5 = ",F5];
Print["M5 = ",M5];
```

The numerical values are

$$\mathbf{F}_1 = 0.018\imath - 0.099\jmath \text{ N}, \; \mathbf{M}_1 = \mathbf{0} \text{ N} \cdot \text{m},$$
$$\mathbf{F}_2 = 0.026\imath - 0.063\jmath \text{ N}, \; \mathbf{M}_2 = -0.00002\mathbf{k} \text{ N} \cdot \text{m},$$
$$\mathbf{F}_3 = 0.049\imath - 0.333\jmath \text{ N}, \; \mathbf{M}_3 = -0.00621\mathbf{k} \text{ N} \cdot \text{m},$$
$$\mathbf{F}_4 = -0.036\imath - 0.063\jmath \text{ N}, \; \mathbf{M}_4 = 0.00001\mathbf{k} \text{ N} \cdot \text{m},$$
$$\mathbf{F}_5 = -0.055\imath - 0.410\jmath \text{ N}, \; \mathbf{M}_5 = 0.00481\mathbf{k} \text{ N} \cdot \text{m}.$$

Joint reaction forces

The diagram representing the mechanism is shown in Fig. 7.11(b) and it has two contours 0-1-2-3-0 and 0-3-4-5-0.

Reaction force $\mathbf{F}_{05}$

The rotation joint E_R between the links 0 and 5 is replaced with the unknown reaction force $\mathbf{F}_{05}$ (Fig. 7.12)

$$\mathbf{F}_{05} = F_{05x}\mathbf{\imath} + F_{05y}\mathbf{\jmath}.$$

With *Mathematica*^{TM}, the force $\mathbf{F}_{05}$ is written as

```
F05Sol={F05xSol,F05ySol,0};
```

Following the path I, as shown in Fig. 7.12, a force equation is written for the translation joint D_T. The projection of all forces, that act on the link 5, onto the sliding direction $\mathbf{r}_{DE}$ is zero

$$\sum \mathbf{F}^{(5)} \cdot \mathbf{r}_{DE} = (\mathbf{F}_5 + \mathbf{F}_{05}) \cdot \mathbf{r}_{DE} = 0, \tag{7.19}$$

where $\mathbf{r}_{DE} = \mathbf{r}_{AE} - \mathbf{r}_{AD}$.
Equation (7.19) with *Mathematica*^{TM} becomes

```
rDE=(rE-rD)/.rule;
eqER1=(F5+F05Sol).rDE==0;
```

where the command `a.b` gives the scalar product of the vectors **a** and **b**.
Continuing on the path I, a moment equation is written for the rotation joint D_R

$$\sum \mathbf{M}_D^{(4\&5)} = \mathbf{r}_{DE} \times \mathbf{F}_{05} + \mathbf{r}_{DC_5} \times \mathbf{F}_5 + \mathbf{M}_4 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}, \tag{7.20}$$

where $\mathbf{r}_{DC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AD}$.
Equation (7.20) with *Mathematica*^{TM} gives

```
rDC5=(rC5-rD)/.rule;
```

```
  eqER2=(Cross[rDE,F05Sol]+Cross[rDC5,F5]+
M4+M5+M5e)[[3]]==0;
```

The system of two equations is solved using *Mathematica*^{TM} command

```
  solF05=Solve[{eqER1,eqER2}, {F05xSol,F05ySol}];
```

The following numerical solution is obtained

$$\mathbf{F}_{05} = 268.127\mathbf{\imath} + 135.039\mathbf{\jmath} \text{ N}.$$

Reaction force $\mathbf{F}_{45}$

The translation joint D_T between the links 4 and 5 is replaced with the unknown reaction force $\mathbf{F}_{45}$ (Fig. 7.13)

$$\mathbf{F}_{45} = -\mathbf{F}_{54} = F_{45x}\mathbf{\imath} + F_{45y}\mathbf{\jmath}.$$

The position of the application point P of the force $\mathbf{F}_{45}$ is unknown

$$\mathbf{r}_{AP} = x_P\mathbf{\imath} + y_P\mathbf{\jmath},$$

where x_P and y_P are the plane coordinates of the point P.
The force $\mathbf{F}_{45}$ and its point of application P with *Mathematica*^{TM} is written as

```
  F45Sol={F45xSol,F45ySol,0};
  rPSol={xPSol,yPSol,0};
```

Following the path I, Fig. 7.13, a moment equation is written for the rotation joint E_R

$$\sum \mathbf{M}_E^{(5)} = \mathbf{r}_{EP} \times \mathbf{F}_{45} + \mathbf{r}_{EC_5} \times \mathbf{F}_5 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}, \tag{7.21}$$

where $\mathbf{r}_{EP} = \mathbf{r}_{AP} - \mathbf{r}_{AE}$, and $\mathbf{r}_{EC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AE}$.
One can write Eq. (7.21) using the *Mathematica*^{TM} commands

```
  rEP=(rPSol-rE)/.rule;
  rEC5=(rC5-rE)/.rule;
```

```
eqDT1=(Cross[rEP,F45Sol]+Cross[rEC5,F5]+
M5+M5e)[[3]]==0;
```

Following the path II, Fig. 7.13, a moment equation is written for the rotation joint D_R

$$\sum \mathbf{M}_D^{(4)} = \mathbf{r}_{DP} \times \mathbf{F}_{54} + \mathbf{M}_4 = \mathbf{0}, \tag{7.22}$$

where $\mathbf{r}_{DP} = \mathbf{r}_{AP} - \mathbf{r}_{AD}$ and $\mathbf{F}_{54} = -\mathbf{F}_{45}$.
Equation (7.22) with *Mathematica^{TM}* is

```
rDP=(rPSol-rD)/.rule;
eqDT2=(Cross[rDP,F54Sol]+M4)[[3]]==0;
```

The direction of the unknown joint force $\mathbf{F}_{45}$ is perpendicular to the sliding direction $\mathbf{r}_{DE}$

$$\mathbf{F}_{45} \cdot \mathbf{r}_{DE} = 0, \tag{7.23}$$

and using *Mathematica^{TM}* command

```
eqDT3=F45Sol.rDE==0;
```

The application point P of the force $\mathbf{F}_{45}$ is located on the direction DE, that is

$$\frac{y_D - y_E}{x_D - x_E} = \frac{y_P - y_E}{x_P - x_E}. \tag{7.24}$$

One can write Eq. (7.24) using the *Mathematica^{TM}* commands

```
eqDT4=((yD-yE)/(xD-xE)/.rule)==
((yPSol-yE)/(xPSol-xE)/.rule);
```

The system of four equations is solved using the *Mathematica^{TM}* command

```
solF45=Solve[{eqDT1,eqDT2,eqDT3,eqDT4},
{F45xSol,F45ySol,xPSol,yPSol}];
```

The following numerical solutions are obtained

$$\mathbf{F}_{45} = -268.072\boldsymbol{\imath} - 134.628\boldsymbol{\jmath} \text{ N} \quad \text{and} \quad \mathbf{r}_{AP} = -0.149\boldsymbol{\imath} + 0.047\boldsymbol{\jmath} \text{ m}.$$

Reaction force $\mathbf{F}_{34}$

The rotation joint D_R between the links 3 and 4 is replaced with the unknown reaction force $\mathbf{F}_{34}$ (Fig. 7.14)

$$\mathbf{F}_{34} = -\mathbf{F}_{34} = F_{34x}\mathbf{\imath} + F_{34y}\mathbf{\jmath},$$

and with *Mathematica*^{TM}

```
F34Sol={F34xSol,F34ySol,0};
```

Following the path I, a force equation can be written for the translation joint D_T. The projection of all forces, that act on the link 4, onto the sliding direction ED is zero

$$\sum \mathbf{F}^{(4)} \cdot \mathbf{ED} = (\mathbf{F}_4 + \mathbf{F}_{34}) \cdot \mathbf{r}_{ED} = 0, \tag{7.25}$$

where $\mathbf{r}_{ED} = \mathbf{r}_{AD} - \mathbf{r}_{AE}$.
Equation (7.25) using the *Mathematica*^{TM} gives

```
rED=(rD-rE)/.rule;
eqDR1=(F4+F34Sol).rED==0;
```

Continuing on the path I, Fig. 7.14, a moment equation is written for the rotation joint E_R

$$\sum \mathbf{M}_E^{(4\&5)} = \mathbf{r}_{ED} \times \mathbf{F}_{34} + \mathbf{r}_{EC_4} \times \mathbf{F}_4 + \mathbf{r}_{EC_5} \times \mathbf{F}_5 + \mathbf{M}_4 + \mathbf{M}_5 + \mathbf{M}_e = \mathbf{0}. \tag{7.26}$$

where $\mathbf{r}_{EC_5} = \mathbf{r}_{AC_5} - \mathbf{r}_{AE}$, and $\mathbf{r}_{EC_4} = \mathbf{r}_{AC_4} - \mathbf{r}_{AE}$.
Equation (7.26) with *Mathematica*^{TM} becomes

```
rEC5=(rC5-rE)/.rule;
rEC4=(rC4-rE)/.rule;
eqDR2=(Cross[rEC4,F4]+Cross[rEC5,F5]+
Cross[rED,F34Sol]+M4+M5+M5e)[[3]]==0;
```

The system of two equations is solved using the *Mathematica*^{TM} commands

```
solF34=Solve[{eqDR1,eqDR2}, {F34xSol,F34ySol}];
```

The following numerical solution is obtained

$$\mathbf{F}_{34} = -268.035\mathbf{\imath} - 134.564\mathbf{\jmath} \text{ N}.$$

Reaction force $\mathbf{F}_{03}$

The rotation joint C_R between the links 0 and 3 is replaced with the unknown reaction force $\mathbf{F}_{03}$ (Fig. 7.15)

$$\mathbf{F}_{03} = F_{03x}\mathbf{\imath} + F_{03y}\mathbf{\jmath},$$

and with *Mathematica*^{TM} the force $\mathbf{F}_{03}$ is written as

```
F03Sol={F03xSol,F03ySol,0};
```

Following the path I, Fig. 7.15, a force equation is written for the translation joint B_T. The projection of all forces, that act on the link 3, onto the sliding direction CD is zero

$$\sum \mathbf{F}^{(3)} \cdot \mathbf{r}_{CD} = (\mathbf{F}_{03} + \mathbf{F}_{43} + \mathbf{F}_3) \cdot \mathbf{r}_{CD} = 0, \quad (7.27)$$

where $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$. Equation (7.27) with *Mathematica*^{TM} commands is

```
rCD=(rD-rC)/.rule;
eqCR1=(F03Sol+F43+F3).rCD==0;
```

Continuing on the path II, Fig. 7.15, a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(3\&2)} = \mathbf{r}_{BC_3} \times \mathbf{F}_3 + \mathbf{r}_{BC} \times \mathbf{F}_{03} + \mathbf{r}_{BD} \times \mathbf{F}_{43} + \mathbf{M}_3 + \mathbf{M}_2 = \mathbf{0}, \quad (7.28)$$

where $\mathbf{r}_{BC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AB}$, $\mathbf{r}_{BC} = \mathbf{r}_{AC} - \mathbf{r}_{AB}$, and $\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$.
With *Mathematica*^{TM} Eq. (7.28) gives

```
rBC3=(rC3-rB)/.rule;
rBC=(rC-rB)/.rule;
rBD=(rD-rB)/.rule;
eqCR2=(Cross[rBC3,F3]+Cross[rBC,F03Sol]+
Cross[rBD,F43]+M2+M3)[[3]]==0;
```

To solve the system of two equations the a *Mathematica^{TM}* command is used

```
solF03=Solve[{eqCR1,eqCR2}, {F03xSol,F03ySol}];
```

The following numerical solution is obtained

$$\mathbf{F}_{03} = -256.71\mathbf{\imath} - 272.141\mathbf{\jmath}\ \text{N}.$$

Reaction force $\mathbf{F}_{23}$

The translation joint B_T between the links 2 and 3 is replaced with the unknown reaction force $\mathbf{F}_{23}$ (Fig. 7.16)

$$\mathbf{F}_{23} = -\mathbf{F}_{32} = F_{23x}\mathbf{\imath} + F_{23y}\mathbf{\jmath}.$$

The position of the application point Q of the force $\mathbf{F}_{23}$ is unknown

$$\mathbf{r}_{AQ} = x_Q\mathbf{\imath} + y_Q\mathbf{\jmath},$$

where x_Q and y_Q are the plane coordinates of the point Q.
The force $\mathbf{F}_{23}$ and its point of application Q are written in *Mathematica^{TM}* as

```
F34Sol={F34xSol,F34ySol,0};
rQSol={xQSol,yQSol,0};
```

Following the path I, Fig. 7.16, a moment equation is written for the rotation joint C_R

$$\sum \mathbf{M}_C^{(3)} = \mathbf{r}_{CQ} \times \mathbf{F}_{23} + \mathbf{r}_{CC_3} \times \mathbf{F}_3 + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_3 = \mathbf{0}, \qquad (7.29)$$

where $\mathbf{r}_{CQ} = \mathbf{r}_{AQ} - \mathbf{r}_{AC}$, $\mathbf{r}_{CC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AC}$, and $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$.
Using *Mathematica^{TM}*, Eq. (7.29) is written as

```
rCQ=(rQSol-rC)/.rule;
rCC3=(rC3-rC)/.rule;
rCD=(rD-rC)/.rule;
eqBT1=(Cross[rCQ,F23Sol]+Cross[rCC3,F3]+
Cross[rCD,F43]+M3)[[3]]==0;
```

Following the path II, Fig. 7.16, a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(2)} = \mathbf{r}_{BQ} \times \mathbf{F}_{32} + \mathbf{M}_2 = \mathbf{0}, \tag{7.30}$$

where $\mathbf{r}_{BQ} = \mathbf{r}_{AQ} - \mathbf{r}_{AB}$. Equation (7.30) with *Mathematica*^{TM} becomes

```
rBQ=(rQSol-rB)/.rule;
eqBT2=(Cross[rBQ,F32Sol]+M2)[[3]]==0;
```

The direction of the unknown joint force $\mathbf{F}_{23}$ is perpendicular to the sliding direction BC and the following relation is written

$$\mathbf{F}_{23} \cdot \mathbf{r}_{BC} = 0,$$

or with *Mathematica*^{TM}

```
eqBT3=F23Sol.rBC==0;
```

The application point Q of the force $\mathbf{F}_{23}$ is located on the direction BC, that is

$$\frac{y_C - y_B}{x_C - x_B} = \frac{y_C - y_Q}{x_C - x_Q}. \tag{7.31}$$

Equation (7.31) with *Mathematica*^{TM} gives

```
eqBT4=((yC-yB)/(xC-xB)/.rule)==
((yC-yQSol)/(xC-xQSol)/.rule);
```

The system of four equations is solved using the *Mathematica*^{TM} command

```
solF23=Solve[{eqBT1,eqBT2,eqBT3,eqBT4},
{F23xSol,F23ySol,xQSol,yQSol}];
```

and the following numerical solutions are obtained

$$\mathbf{F}_{23} = -11.374\imath + 137.91\jmath \text{ N} \quad \text{and} \quad \mathbf{r}_{AQ} = 0.121\imath + 0.070\jmath \text{ m}.$$

Reaction force $\mathbf{F}_{12}$

The rotation joint B_R between the links 1 and 2 is replaced with the unknown reaction force $\mathbf{F}_{12}$ (Fig. 7.17)

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = F_{12x}\mathbf{\imath} + F_{12y}\mathbf{\jmath},$$

or with *Mathematica*^{TM}

```
F12Sol={F12xSol,F12ySol,0};
```

Following the path I, Fig. 7.17, a force equation is written for the translation joint B_T. The projection of all forces, that act on the link 2, onto the sliding direction BC is zero

$$\sum \mathbf{F}^{(2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{12} + \mathbf{F}_2) \cdot \mathbf{r}_{BC} = 0, \tag{7.32}$$

or using *Mathematica*^{TM}

```
rBC=(rC-rB)/.rule;
eqBR1=(F12Sol+F2).rBC==0;
```

Continuing on the path I, a moment equation is written for the rotation joint C_R

$$\sum \mathbf{M}_C^{(2\&3)} = \mathbf{r}_{CB} \times \mathbf{F}_{12} + \mathbf{r}_{CC_2} \times \mathbf{F}_2 + \mathbf{r}_{CC_3} \times \mathbf{F}_3 + \mathbf{r}_{CD} \times \mathbf{F}_{43} + \mathbf{M}_2 + \mathbf{M}_3 = \mathbf{0}, \tag{7.33}$$

where $\mathbf{r}_{CB} = \mathbf{r}_{AB} - \mathbf{r}_{AC}$, $\mathbf{r}_{CC_2} = \mathbf{r}_{AC_2} - \mathbf{r}_{AC}$, $\mathbf{r}_{CC_3} = \mathbf{r}_{AC_3} - \mathbf{r}_{AC}$, and $\mathbf{r}_{CD} = \mathbf{r}_{AD} - \mathbf{r}_{AC}$.
Using the *Mathematica*^{TM} commands Eq. (7.33) gives

```
rCB=(rB-rC)/.rule;
rCC2=(rC2-rC)/.rule;
rCC3=(rC3-rC)/.rule;
rCD=(rD-rC)/.rule;
eqBR2=(Cross[rCB,F12Sol]+Cross[rCC2,F2]+
Cross[rCC3,F3]+Cross[rCD,F43]+M2+M3)[[3]]==0;
```

The system of two equations is solved using the *Mathematica*^{TM} command

```
solF12=Solve[{eqBR1,eqBR2},{F12xSol,F12ySol}];
```

and the following numerical solution is obtained

$$\mathbf{F}_{12} = -11.401\mathbf{\imath} + 137.974\mathbf{\jmath} \text{ N}.$$

The motor moment $\mathbf{M}_m$

The motor moment needed for the dynamic equilibrium of the mechanism is $\mathbf{M}_m = M_m\ \mathbf{k}$ (Fig. 7.18) and with *Mathematica^{TM}*

```
M1mSol={0,0,MmSol};
```

Following the path I, Fig. 7.18, a moment equation is written for the rotation joint A_R

$$\sum \mathbf{M}_A^{(1)} = \mathbf{r}_{AB} \times \mathbf{F}_{21} + \mathbf{r}_{AC_1} \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M}_m = \mathbf{0}. \tag{7.34}$$

Equation (7.34) is solved using *Mathematica^{TM}*

```
  eqMA=(Cross[rAB,F21]+Cross[rAC1,F1]+M1+
M1mSol)[[3]]==0;
  solMm=Solve[eqMA,MmSol];
  Mm=MmSol/.solMm[[1]];
  M1m={0,0,Mm};
  Print["Mm = ",Mm];
```

The numerical solution is

$$\mathbf{M}_m = 17.533\ \mathbf{k} \quad \text{N} \cdot \text{m}.$$

Reaction force $\mathbf{F}_{01}$

The rotation joint A_R between the links 0 and 1 is replaced with the unknown reaction force $\mathbf{F}_{01}$ (Fig. 7.19)

$$\mathbf{F}_{01} = -\mathbf{F}_{10} = F_{01x}\mathbf{\imath} + F_{01y}\mathbf{\jmath},$$

and with *Mathematica^{TM}*

```
F01Sol={F01xSol,F01ySol,0};
```

Following the path I, Fig. 7.19, a moment equation is written for the rotation joint B_R

$$\sum \mathbf{M}_B^{(1)} = \mathbf{r}_{BA} \times \mathbf{F}_{01} + \mathbf{r}_{BC_1} \times \mathbf{F}_1 + \mathbf{M}_1 + \mathbf{M}_m = \mathbf{0}, \qquad (7.35)$$

where $\mathbf{r}_{BA} = -\mathbf{r}_{AB}$, and $\mathbf{r}_{BC_1} = \mathbf{r}_{AC_1} - \mathbf{r}_{AB}$.
Equation (7.35) using the $Mathematica^{TM}$ gives

```
rBA=-rB/.rule;
rBC1=(rC1-rB)/.rule;
eqAR1=(Cross[rBA,F01Sol]+Cross[rBC1,F1]+M1+
M1m)[[3]]==0;
```

Continuing on the path I, Fig. 7.19, a force equation is written for the translation joint B_T. The projection of all forces, that act on the links 1 and 2, onto the sliding direction BC is zero

$$\sum \mathbf{F}^{(1\&2)} \cdot \mathbf{r}_{BC} = (\mathbf{F}_{01} + \mathbf{F}_1 + \mathbf{F}_2) \cdot \mathbf{r}_{BC} = 0, \qquad (7.36)$$

or with $Mathematica^{TM}$

```
eqAR2=(F01Sol+F1+F2).rBC==0;
```

The system of two equations is solved using the $Mathematica^{TM}$ command

```
solF01=Solve[{eqAR1,eqAR2},{F01xSol,F01ySol}];
```

The following numerical solution is obtained

$$\mathbf{F}_{01} = -11.419\,\mathbf{\imath} + 138.073\,\mathbf{\jmath} \quad \text{N}.$$

The $Mathematica^{TM}$ program for the dynamic force analysis is presented in Program 7.6.

7.5 Problems

7.1 Referring to Example 3.1 (Fig. 3.11). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.2 Referring to Example 3.2 (Fig. 3.12). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.3 Referring to Example 3.3 (Fig. 3.15). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.4 Referring to Problem 3.4 (Fig. 3.19). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.5 Referring to Problem 3.5 (Fig. 3.20). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.5 Referring to Problem 3.11 (Fig. 3.26). Write a *Mathematica*^{TM} program for the position analysis of the mechanism.

7.7 Referring to Example 4.1 (Fig. 4.7). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism.

7.8 Referring to Example 4.2 (Fig. 4.8). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism.

7.9 Referring to Problem 4.1 (Fig. 3.16(a)). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism.

7.10 Referring to Problem 5.1 (Fig. 3.16(a)). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism using the contour equations method.

7.11 Referring to Problem 4.3 (Fig. 4.10). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism.

7.12 Referring to Problem 5.3 (Fig. 3.10). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism using the contour equations method.

7.13 Referring to Problem 4.4 (Fig. 3.19). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism.

7.14 Referring to Problem 5.4 (Fig. 3.19). Write a *Mathematica*^{TM} program for the velocity and acceleration analysis of the mechanism using the contour equations method.

7.15 Referring to Problem 6.3 (Fig. 4.10). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

7.16 Referring to Problem 6.16 (Fig. 3.31). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

7.17 Referring to Problem 6.18 (Fig. 3.33). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

7.18 Referring to Problem 6.24 (Fig. 3.11). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

7.19 Referring to Problem 6.25 (Fig. 3.12). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

7.20 Referring to Problem 6.26 (Fig. 3.15). Write a *Mathematica*^{TM} program for the equilibrium moment and the joint forces of the mechanism.

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Figure captions

Figure 7.1 R-RTR-RTR mechanism

Figure 7.2 Solutions for the position of the joint D for $0 \leq \phi \leq 90°$ $(x_D \leq x_C)$

Figure 7.3 Solutions for the position of the joint D for $90° < \phi \leq 180°$ $(x_D \geq x_C)$

Figure 7.4 Solutions for the position of the joint D for $180° < \phi < 270°$ $(x_D \geq x_C)$

Figure 7.5 Solutions for the position of the joint D for $270° \leq \phi \leq 360°$ $(x_D \leq x_C)$

Figure 7.6 Graph of the mechanism for a complete rotation, $0 \leq \phi \leq 360°$

Figure 7.7 Distance condition for position analysis: $d_k^I < d_k^{II} \Rightarrow D_{k+1} = D_{k+1}^I$

Figure 7.8 (a) R-RTR-RTR mechanism (b) contour diagram

Figure 7.9 First independent contour

Figure 7.10 Second independent contour

Figure 7.11 Forces and moments for R-RTR-RTR mechanism

Figure 7.12 Rotation joint E_R and reaction force $\mathbf{F}_{05}$

Figure 7.13 Translation joint D_T and reaction force $\mathbf{F}_{45}$

Figure 7.14 Rotation joint D_R and reaction force $\mathbf{F}_{34}$

Figure 7.15 Rotation joint C_R and reaction force $\mathbf{F}_{03}$

Figure 7.16 Translation joint B_T and reaction force $\mathbf{F}_{23}$

Figure 7.17 Rotation joint B_R and reaction force $\mathbf{F}_{12}$

Figure 7.18 Dynamic equilibrium moment $\mathbf{M}_m$

Figure 7.19 Rotation joint A_R and reaction force $\mathbf{F}_{01}$