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5 Contour Equations

This section aims at providing an algebraic method to compute the velocities and accelerations of any closed kinematic chain. The classical method for obtaining the velocities and accelerations involves the computation of the derivative with respect to time of the position vectors. The method of contour equations avoids this task and utilizes only algebraic equations [4, 55]. Using this approach, a numerical implementation is much more efficient. The method described here can be applied to planar and spatial mechanisms.

Two rigid links \((j)\) and \((k)\) are connected by a joint (kinematic pair) at \(A\), Fig. 5.1. The point \(A_j\) of the rigid body \((j)\) is guided along a path prescribed in the body \((k)\). The points \(A_j\) belonging to body \((j)\) and the \(A_k\) belonging to body \((k)\) are coincident at the instant of motion under consideration. The following relation exists between the velocity \(v_{A_j}\) of the point \(A_j\) and the velocity \(v_{A_k}\) of the point \(A_k\)

\[
v_{A_j} = v_{A_k} + v_{A_{jk}}^r, \tag{5.1}
\]

where \(v_{A_{jk}}^r = v_{A_jA_k}^r\) indicates the velocity of \(A_j\) as seen by an observer at \(A_k\) attached to body \(k\) or the relative velocity of \(A_j\) with respect to \(A_k\), allowed at the joint \(A\). The direction of \(v_{A_{jk}}^r\) is obviously tangent to the path prescribed in body \((k)\).

From Eq. (5.1) the accelerations of \(A_j\) and \(A_k\) are expressed as

\[
a_{A_j} = a_{A_k} + a_{A_{jk}}^r + a_{A_{jk}}^c, \tag{5.2}
\]

where \(a_{A_{jk}}^c = a_{A_jA_k}^c\) is known as the Coriolis acceleration and is given by

\[
a_{A_{jk}}^c = 2 \omega_k \times v_{A_{jk}}^r, \tag{5.3}
\]

where \(\omega_k\) is the angular velocity of the body \((k)\).

Equations (5.1) and (5.2) are useful even for coincident points belonging to two links that may not be directly connected. A graphical representation of Eq. (5.1) is shown in Fig. 5.1(b) for a rotating slider joint.

Figure 5.2 shows a monocontour closed kinematic chain with \(n\) rigid links. The joint \(A_i, i = 0, 1, 2, ..., n\) is the connection between the links \((i)\) and \((i - 1)\). The last link \(n\) is connected with the first link \(0\) of the chain. For the closed kinematic chain, a path is chosen from link 0 to link \(n\). At the joint \(A_i\) there are two instantaneously coincident points: 1) the point \(A_{i,i}\)
belonging to link \((i)\), \(A_{i,i} \in (i)\), and 2) the point \(A_{i,i-1}\) belonging to body \((i-1)\), \(A_{i,i-1} \in (i-1)\).

### 5.1 Contour Velocity Equations

The absolute angular velocity, \(\omega_i = \omega_{i,0}\), of the rigid body \((i)\), or the angular velocity of the rigid body \((i)\) with respect to the ‘fixed” reference frame \(Oxyz\) is

\[
\omega_i = \omega_{i-1} + \omega_{i,i-1},
\]

where \(\omega_{i-1} = \omega_{i-1,0}\) is the absolute angular velocity of the rigid body \((i-1)\) (or the angular velocity of the rigid body \((i-1)\) with respect to the ‘fixed” reference frame \(Oxyz\)) and \(\omega_{i,i-1}\) is the relative angular velocity of the rigid body \((i)\) with respect to the rigid body \((i-1)\).

For the \(n\) link closed kinematic chain the following expressions are obtained for the angular velocities

\[
\begin{align*}
\omega_1 &= \omega_0 + \omega_{1,0} \\
\omega_2 &= \omega_1 + \omega_{2,1} \\
&\cdots \\
\omega_i &= \omega_{i-1} + \omega_{i,i-1} \\
&\cdots \\
\omega_0 &= \omega_n + \omega_{0,n}.
\end{align*}
\]

(5.5)

Summing the expressions given in Eq. (5.5), the following relation is obtained

\[
\omega_{1,0} + \omega_{2,1} + \ldots + \omega_{0,n} = 0,
\]

(5.6)

which may be rewritten as

\[
\sum_{(i)} \omega_{i,i-1} = 0.
\]

(5.7)

Equation (5.7) represents the first vectorial equation for the angular velocities of a simple closed kinematic chain.

The following relation exists between the velocity \(v_{A_{i,i}}\) of the point \(A_{i,i}\) and the velocity \(v_{A_{i,i-1}}\) of the point \(A_{i,i-1}\)

\[
v_{A_{i,i}} = v_{A_{i,i-1}} + v_{A_{i,i-1}}^r,
\]

(5.8)
where $v_{A_{i+1},i}^{r} = v_{A_{i+1},i-1}^{r}$ is the relative velocity of $A_{i,i}$ on link $(i)$ with respect to $A_{i,i-1}$ on link $(i-1)$. Using the velocity relation for two particles on the rigid body $(i)$ the following relation exists

$$v_{A_{i+1},i} = v_{A_{i},i} + \omega_i \times r_{A_{i},A_{i+1}},$$  \hspace{1cm} (5.9)$$

where $\omega_i$ is the absolute angular velocity of the link $(i)$ in the reference frame $Oxyz$, and $r_{A_{i},A_{i+1}}$ is the distance vector from $A_i$ to $A_{i+1}$. Using Eqs. (5.8) and (5.9) the velocity of the point $A_{i+1,i} \in (i+1)$ is written as

$$v_{A_{i+1},i} = v_{A_{i},i-1} + \omega_i \times r_{A_{i},A_{i+1}} + v_{A_{i},i-1}^r.$$

For the $n$ link closed kinematic chain the following expressions are obtained

$$v_{A_{1,0}} = v_{A_{0,n}} + \omega_0 \times r_{A_{0,A_1}} + v_{A_{0,n}}^r,$$

$$v_{A_{2,1}} = v_{A_{1,0}} + \omega_1 \times r_{A_{1,A_2}} + v_{A_{1,0}}^r.$$

Summing the relations in Eq. (5.11)

$$[\omega_1 \times r_{A_{1,A_2}} + \omega_2 \times r_{A_{2,A_3}} + \ldots + \omega_i \times r_{A_{i,A_{i+1}}} + \ldots + \omega_0 \times r_{A_{0,A_1}}] +$$

$$[v_{A_{2,1}}^r + v_{A_{1,2}}^r + \ldots + v_{A_{i,i-1}}^r + \ldots + v_{A_{0,n}}^r + v_{A_{1,0}}^r] = 0.$$  \hspace{1cm} (5.12)$$

Because the reference system $Oxyz$ is considered “fixed”, the vector $r_{A_{i-1,A_i}}$ is written in terms of the position vectors of the points $A_{i-1}$ and $A_i$

$$r_{A_{i-1,A_i}} = r_{A_i} - r_{A_{i-1}},$$  \hspace{1cm} (5.13)$$

where $r_{A_i} = r_{O A_i}$ and $r_{A_{i-1}} = r_{O A_{i-1}}$. Equation (5.12) becomes

$$[r_{A_1} \times (\omega_1 - \omega_0) + r_{A_2} \times (\omega_2 - \omega_1) + \ldots + r_{A_0} \times (\omega_0 - \omega_n)] +$$

$$[v_{A_{1,0}}^r + v_{A_{2,1}}^r + \ldots + v_{A_{i,i-1}}^r + \ldots + v_{A_{0,n}}^r] = 0.$$  \hspace{1cm} (5.14)$$

Using Eq. (5.5), Eq. (5.14) becomes

$$[r_{A_1} \times \omega_{1,0} + r_{A_2} \times \omega_{2,1} + \ldots + r_{A_0} \times \omega_{0,n}] +$$

$$[v_{A_{1,0}}^r + v_{A_{2,1}}^r + \ldots + v_{A_{0,n}}^r] = 0.$$  \hspace{1cm} (5.15)$$
The previous equation is written as

\[
\sum_{i} r_{Ai} \times \omega_{i,i-1} + \sum_{i} v_{A,i-1}^r = 0. \tag{5.16}
\]

Equation (5.16) represents the second vectorial equation for the angular velocities of a simple closed kinematic chain.

Equations

\[
\sum_{i} \omega_{i,i-1} = 0 \quad \text{and} \quad \sum_{i} r_{Ai} \times \omega_{i,i-1} + \sum_{i} v_{A,i-1}^r = 0, \tag{5.17}
\]

represent the velocity equations for a simple closed kinematic chain.

### 5.2 Contour Acceleration Equations

The absolute angular acceleration, \( \alpha_i = \alpha_{i,0} \), of the rigid body \((i)\) (or the angular acceleration of the rigid body \((i)\) with respect to the ‘fixed” reference frame \(Oxyz\)) is

\[
\alpha_i = \alpha_{i-1} + \alpha_{i,i-1} + \omega_i \times \omega_{i,i-1}, \tag{5.18}
\]

where \( \alpha_{i-1} = \alpha_{i-1,0} \) is the absolute angular acceleration of the rigid body \((i-1)\) (or the angular acceleration of the rigid body \((i-1)\) with respect to the ‘fixed” reference frame \(Oxyz\)) and \( \alpha_{i,i-1} \) is the relative angular acceleration of the rigid body \((i)\) with respect to the rigid body \((i-1)\).

For the \(n\) link closed kinematic chain the following expressions are obtained for the angular accelerations

\[
\alpha_2 = \alpha_1 + \alpha_{2,1} + \omega_2 \times \omega_{2,1} \\
\alpha_3 = \alpha_2 + \alpha_{3,2} + \omega_3 \times \omega_{3,2} \\
\ldots \\
\alpha_i = \alpha_{i-1} + \alpha_{i,i-1} + \omega_i \times \omega_{i,i-1} \\
\ldots \\
\alpha_1 = \alpha_0 + \alpha_{1,0} + \omega_1 \times \omega_{1,0}. \tag{5.19}
\]

Summing all the expressions in Eq. (5.19)

\[
\alpha_{2,1} + \alpha_{3,2} + \ldots + \alpha_{1,0} + \omega_2 \times \omega_{2,1} + \ldots + \omega_1 \times \omega_{1,0} = 0. \tag{5.20}
\]
Equation (5.20) is rewritten as
\[ \sum_{(i)} \alpha_{i,i-1} + \sum_{(i)} \omega_i \times \omega_{i,i-1} = 0. \]  
(5.21)

Equation (5.21) represents the first vectorial equation for the angular accelerations of a simple closed kinematic chain.

Using the acceleration distributions of the relative motion of two rigid bodies \( (i) \) and \( (i-1) \)
\[ \mathbf{a}_{A_{i,i}} = \mathbf{a}_{A_{i,i-1}} + \mathbf{a}_{A_{i,i-1}} + \mathbf{a}_{A_{i,i-1}}. \]  
(5.22)

where \( \mathbf{a}_{A_{i,i}} \) and \( \mathbf{a}_{A_{i,i-1}} \) are the linear accelerations of the points \( A_{i,i} \) and \( A_{i,i-1} \),
\[ \mathbf{a}_{A_{i,i-1}} = \mathbf{a}_{A_{i,i-1}} \] is the relative acceleration between \( A_{i,i} \) on link \( (i) \) and \( A_{i,i-1} \) on link \( (i-1) \). Finally, \( \mathbf{a}_{A_{i,i-1}} \) is the Coriolis acceleration defined as
\[ \mathbf{a}_{A_{i,i-1}} = 2 \omega_{i-1} \times \mathbf{v}_{A_{i,i-1}}. \]  
(5.23)

Using the acceleration distribution relations for two particles on a rigid body
\[ \mathbf{a}_{A_{i,i+1}} = \mathbf{a}_{A_{i,i}} + \alpha_i \times \mathbf{r}_{A_{i,A_{i+1}}} + \omega_i \times (\omega_i \times \mathbf{r}_{A_{i,A_{i+1}}}), \]  
(5.24)
where \( \alpha_i \) is the angular acceleration of the link \( (i) \). From Eqs. (5.22) and (5.24)
\[ \mathbf{a}_{A_{i,i+1}} = \mathbf{a}_{A_{i,i-1}} + \mathbf{a}_{A_{i,i-1}} + \alpha_i \times \mathbf{r}_{A_{i,A_{i+1}}} + \omega_i \times (\omega_i \times \mathbf{r}_{A_{i,A_{i+1}}}). \]  
(5.25)

Writing similar equations for all the links of the kinematic chain, the following relations are obtained
\[ \mathbf{a}_{A_{i,2}} = \mathbf{a}_{A_{2,1}} + \mathbf{a}_{A_{2,1}} + \alpha_2 \times \mathbf{r}_{A_{2,A_{3}}} + \omega_2 \times (\omega_2 \times \mathbf{r}_{A_{2,A_{3}}}), \]
\[ \mathbf{a}_{A_{i,3}} = \mathbf{a}_{A_{3,2}} + \mathbf{a}_{A_{3,2}} + \alpha_3 \times \mathbf{r}_{A_{3,A_{4}}} + \omega_3 \times (\omega_3 \times \mathbf{r}_{A_{3,A_{4}}}), \]
\[ \vdots \]
\[ \mathbf{a}_{A_{i,0}} = \mathbf{a}_{A_{0,1}} + \mathbf{a}_{A_{0,1}} + \alpha_0 \times \mathbf{r}_{A_{0,A_{1}}} + \omega_0 \times (\omega_0 \times \mathbf{r}_{A_{0,A_{1}}}), \]
\[ \mathbf{a}_{A_{i,1}} = \mathbf{a}_{A_{1,0}} + \mathbf{a}_{A_{1,0}} + \alpha_1 \times \mathbf{r}_{A_{1,A_{2}}} + \omega_1 \times (\omega_1 \times \mathbf{r}_{A_{1,A_{2}}}). \]  
(5.26)

Summing the expressions in Eq. (5.26)
\[ \left[ \alpha_{A_{1,0}} + \mathbf{a}_{A_{2,1}} + \ldots + \mathbf{a}_{A_{0,1}} \right] + \left[ \alpha_{A_{1,0}} + \mathbf{a}_{A_{2,1}} + \ldots + \mathbf{a}_{A_{0,1}} \right] + \]
\[ \ldots \]
\[ \omega_1 \times (\omega_1 \times \mathbf{r}_{A_{1,A_{2}}}) + \omega_2 \times (\omega_2 \times \mathbf{r}_{A_{2,A_{3}}}) + \ldots + \]
\[ \omega_0 \times (\omega_0 \times \mathbf{r}_{A_{0,A_{1}}}) = 0. \]  
(5.27)
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Using the relation $r_{A_{i-1}A_i} = r_{A_i} - r_{A_{i-1}}$ in Eq. (5.27)

\[
\begin{align*}
\sum_{(i)} a_{A_{i-1},i}^r + \sum_{(i)} a_{A_{i-1},i}^e + \sum_{(i)} r_{A_i} \times (A_{i-1,0} + A_{i-1,1} + \ldots + A_{i-1,n}) + \\
r_{A_1} \times (A_{1,0} + A_{1,1} + \ldots + r_{A_0} \times (A_{0,0} + A_{0,1} + \ldots + A_{0,n})) + \\
\sum_{(i)} \omega_i \times (A_{i-1,0} + A_{i-1,1} + \ldots + \omega_i \times \omega_{i-1}) + \\
\sum_{(i)} a_{A_{i-1},i}^r = 0.
\end{align*}
\] (5.28)

Equation (5.28) is rewritten as

\[
\begin{align*}
\sum_{(i)} a_{A_{i-1},i}^r + \sum_{(i)} a_{A_{i-1},i}^e + \sum_{(i)} r_{A_i} \times (A_{i-1,0} + A_{i-1,1} + \ldots + A_{i-1,n}) + \\
\sum_{(i)} \omega_i \times (A_{i-1,0} + A_{i-1,1} + \ldots + \omega_i \times \omega_{i-1}) + \\
\sum_{(i)} a_{A_{i-1},i}^r = 0.
\end{align*}
\] (5.29)

Equation (5.29) represents the second vectorial equation for the angular accelerations of a simple closed kinematic chain. Equations

\[
\begin{align*}
\sum_{(i)} A_{i-1,i} + \sum_{(i)} a_{A_{i-1},i}^e + \sum_{(i)} r_{A_i} \times (A_{i-1,0} + A_{i-1,1} + \ldots + A_{i-1,n}) + \\
\sum_{(i)} \omega_i \times (A_{i-1,0} + A_{i-1,1} + \ldots + \omega_i \times \omega_{i-1}) + \\
\sum_{(i)} a_{A_{i-1},i}^e = 0.
\end{align*}
\] (5.30)

are the acceleration equations for the case of a simple closed kinematic chain.

Remarks
1. For a closed kinematic chain in planar motion, simplified relations are obtained because

\[
\omega_i \times (A_{i-1,0} + A_{i-1,1} + \ldots + \omega_i \times \omega_{i-1}) = -\omega_i^2 r_{A_{i-1,i}} + \omega_i \times \omega_{i-1} = 0.
\] (5.31)

Equations

\[
\begin{align*}
\sum_{(i)} A_{i-1,i} = 0 \quad \text{and} \\
\sum_{(i)} r_{A_i} \times A_{i-1,i} + \sum_{(i)} a_{A_{i-1},i}^e + \sum_{(i)} a_{A_{i-1},i}^e - \omega_i^2 r_{A_{i-1,i}} = 0.
\end{align*}
\] (5.32)
section

I.5 Contour Equations

represent the acceleration equations for a simple closed kinematic chain in planar motion.

2. The Coriolis acceleration, given by the expression

\[ a_{A,i,i-1}^{c} = 2\omega_{i-1} \times v_{A,i,i-1}^{r} \]  

(5.33)

vanishes when \( \omega_{i-1} = 0 \), or \( v_{A,i,i-1}^{r} = 0 \), or when \( \omega_{i-1} \) is parallel to \( v_{A,i,i-1}^{r} \).

5.3 Independent Contour Equations

A diagram is used to represent a mechanism in the following way: the numbered links are the nodes of the diagram and are represented by circles, and the joints are represented by lines which connect the nodes.

Figure 5.3 shows the diagram which represents a planar mechanism. The maximum number of independent contours is given by

\[ N = c - n \quad \text{or} \quad n_{c} = N = c - p + 1, \]  

(5.34)

where \( c \) is the number of joints, \( n \) is the number of moving links, and \( p \) is the number of links.

The equations for velocities and accelerations is written for any closed contour of the mechanism. However, it is best to write the contour equations only for the independent loops of the diagram representing the mechanism.

Step 1. Determine the position analysis of the mechanism.

Step 2. Draw a diagram representing the mechanism and select the independent contours. Determine a path for each contour.

Step 3. For each closed loop write the contour velocity relations, Eq. (5.17), and contour acceleration relations, Eq. (5.30). For a closed kinematic chain in planar motion the following equations will be used

\[ \sum_{(i)} \omega_{i,i-1} = 0, \]

\[ \sum_{(i)} r_{A,i} \times \omega_{i,i-1} + \sum_{(i)} v_{A,i,i-1}^{r} = 0. \]  

(5.35)

\[ \sum_{(i)} \alpha_{i,i-1} = 0, \]

\[ \sum_{(i)} r_{A,i} \times \alpha_{i,i-1} + \sum_{(i)} a_{A,i,i-1}^{r} + \sum_{(i)} a_{A,i,i-1}^{c} - \omega_{i}^{2} r_{A,i} r_{A,i+1} = 0. \]  

(5.36)
Step 4. Project on a cartesian reference system the velocity and acceleration equations. Linear algebraic equations are obtained where the unknowns are

- the components of the relative angular velocities \( \omega_{j,j-1} \);
- the components of the relative angular accelerations \( \alpha_{j,j-1} \);
- the components of the relative linear velocities \( v^r_{A_{j,j-1}} \);
- the components of the relative linear accelerations \( a^r_{A_{j,j-1}} \).

Solve the algebraic system of equations and determine the unknown kinematic parameters.

Step 5. Determine the absolute angular velocities \( \omega_j \) and the absolute angular accelerations \( \alpha_j \). Compute the velocities and accelerations of the characteristic points and joints.

In the following examples, the contour method is applied to determine the velocities and accelerations distribution for several planar mechanisms. The following notation will be used:

\( \omega_{ij} \) is the relative angular velocity vector of the link \( i \) with respect to the link \( j \). When the link \( j \) is the ground (denoted as link 0), then \( \omega_i = \omega_{i0} \) also denotes the absolute angular velocity vector of the link \( i \). The magnitude of \( \omega_{ij} \) is \( \omega_{ij} \) i.e. \( |\omega_{ij}| = \omega_{ij} \).

\( v^r_{A_{ij}} \) is the relative linear velocity of the point \( A_i \) on link \( i \) with respect to the point \( A_j \) on link \( j \). The point \( A_i \) belonging to link \( j \) and the point \( A_j \) belonging to link \( j \) are coincident at the instant of motion under consideration.

\( \alpha_{ij} \) is the relative angular acceleration vector of the link \( i \) with respect to the rigid body \( j \). When the link \( j \) is the ground, then \( \alpha_i = \alpha_{i0} \) also denotes the absolute angular acceleration vector of the rigid body \( i \).

\( a^r_{A_{ij}} \) is the relative linear acceleration vector of \( A_i \) on link \( i \) with respect to \( A_j \) on link \( j \).

\( a^c_{A_{ij}} \) is the Coriolis acceleration of \( A_i \) with respect to \( A_j \).

\( r_{BC} \) denotes a vector from the joint \( B \) to the joint \( C \).

\( x_B, y_B, z_B \) denote the coordinates of the point \( B \) with respect to the fixed reference frame.

\( v_B \) denotes the linear velocity vector of the point \( B \) with respect to the fixed reference frame.
\( a_B \) denotes the linear acceleration vector of the point \( B \) with respect to the fixed reference frame.
5.4 Example

The planar mechanism considered in this example is depicted in Fig. 5.4(a). The following data are given: \( AC = 0.100 \text{ m}, \ BC = 0.300 \text{ m}, \ BD = 0.900 \text{ m}, \) and \( L_a = 0.100 \text{ m}. \) The angle of the driver element (link \( AB \)) with the horizontal axis is \( \phi = 45^\circ. \) A Cartesian reference frame with the origin at \( A (x_A = y_A = 0) \) is selected. The coordinates of joint \( C \) are \( x_C = AC, \ y_C = 0. \) The coordinates of joint \( B \) are \( x_B = 0.256 \text{ m}, \ y_B = 0.256 \text{ m}. \) The coordinates of joint \( D \) are \( x_D = 1.142 \text{ m}, \ y_D = 0.100 \text{ m}. \) The position vectors \( \mathbf{r}_{AB}, \mathbf{r}_{AC} \) and \( \mathbf{r}_{AD} \) are defined as follows

\[
\mathbf{r}_{AB} = x_B \mathbf{i} + y_B \mathbf{j} = 0.256 \mathbf{i} + 0.256 \mathbf{j},
\]
\[
\mathbf{r}_{AC} = x_C \mathbf{i} + y_C \mathbf{j} = 0.100 \mathbf{i},
\]
\[
\mathbf{r}_{AD} = x_D \mathbf{i} + y_D \mathbf{j} = 1.142 \mathbf{i} + 0.100 \mathbf{j}.
\]

The angular velocity of the driver link is \( n_1 = 100 \text{ rpm}, \) or

\[
\omega_{10} = \omega_1 = \frac{n_1 \pi}{30} = \frac{100 \pi}{30} \text{ rad/s} = 10.472 \text{ rad/s}.
\]

The mechanism has six links and seven full joints. Using Eq. (5.34), the number of independent loops is given by

\[
n_c = l - p + 1 = 7 - 6 + 1 = 2.
\]

This mechanism has two independent contours. The first contour \( I \) contains the links 0, 1, 2 and 3, while the second contour \( II \) contains the links 0, 3, 4 and 5. The diagram representing the mechanism is given in Fig.5.4(b). Clockwise paths are chosen for each closed loop \( I \) and \( II. \)

**First contour**

According to Fig. 5.5, the first contour has

- rotational joint \( R \) between the links 0 and 1 (joint \( A \));
- translational joint \( T \) between the links 1 and 2 (joint \( B_T \));
- rotational joint \( R \) between the links 2 and 3 (joint \( B_R \));
- rotational joint \( R \) between the links 3 and 0 (joint \( C \)).

For the velocity analysis, the following equations are written using Eq. (5.35)

\[
\omega_{10} + \omega_{32} + \omega_{03} = 0,
\]
\[
\mathbf{r}_{AB} \times \omega_{32} + \mathbf{r}_{AC} \times \omega_{03} + \mathbf{v}_{B21}^r = 0,
\] (5.37)
where \( \omega_{10} = \omega_{10} \mathbf{k} = 10.47 \mathbf{k} \text{ rad/s}, \ \omega_{32} = \omega_{32} \mathbf{k}, \) and \( \omega_{03} = \omega_{03} \mathbf{k}. \)

The relative velocity of \( B_2 \) on link 2 with respect to \( B_1 \) on link 1, \( v_{B_{21}}^r \), has \( \mathbf{i} \) and \( \mathbf{j} \) components

\[
v_{B_{21}}^r = v_{B_{21}}^r \mathbf{i} + v_{B_{21}}^r \mathbf{j} = v_{B_{21}}^r \cos \phi \mathbf{i} + v_{B_{21}}^r \sin \phi \mathbf{j},
\]

where \( v_{B_{21}}^r \) is the magnitude of the vector \( v_{B_{21}}^r \) i.e. \( |v_{B_{21}}^r| = v_{B_{21}}^r \). The sign of the unknown relative velocities is selected as positive as shown in Figs. 5.4(a) and 5.5(b). Then the numerical computation will give the correct orientation of the unknown vectors. The unknowns in Eq. (5.37) are \( \omega_{32}, \omega_{03}, \) and \( v_{B_{21}}^r \).

Equation (5.37) becomes

\[
\omega_{10} \mathbf{k} + \omega_{32} \mathbf{k} + \omega_{03} \mathbf{k} = 0,
\]

where \( \omega_{32} \) is the unknown angular velocity of the link 3. Solving the algebraic equations, the following numerical values are obtained \( \omega_{32} = 2.539 \text{ rad/s}, \ \omega_{03} = -13.011 \text{ rad/s}, \) and \( v_{B_{21}}^r = -0.920 \text{ m/s}. \)

The absolute angular velocity of the link 3 is

\[
\omega_{30} = -\omega_{03} = 13.011 \mathbf{k} \text{ rad/s}.
\]
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The velocity of the point $B_2 = B_3$ is computed with the expression of velocity field of two points ($B_3$ and $C$) on the same rigid body (link 3)

$$v_{B_2} = v_{B_3} = v_C + \omega_{30} \times r_{CB} = \begin{vmatrix} 1 & J & k \\ 0 & 0 & \omega_{30} \\ x_B - x_C & y_B - y_C & 0 \end{vmatrix} = \begin{vmatrix} 1 & J & k \\ 0 & 0 & 13.011 \\ 0.256 - 0.100 & 0.256 & 0 \end{vmatrix} = -3.333 \ i + 2.032 \ j \ m/s,$$

where $v_C = 0$ because the joint $C$ is grounded.

The link 2 and the driver link 1 have the same angular velocity

$$\omega_{10} = \omega_{20} = \omega_{30} + \omega_{23} = 13.011 \ k - 2.539 \ k = 10.472 \ k \ rad/s.$$

The velocity of the point $B_1$ on link 1 is calculated with the expression of velocity field of two points ($B_1$ and $A$) on the same rigid body (link 1)

$$v_{B_1} = v_A + \omega_{10} \times r_{AB} = \omega_{10} \times r_{AB} = \begin{vmatrix} 1 & J & k \\ 0 & 0 & \omega_{10} \\ x_B & y_B & 0 \end{vmatrix} = \begin{vmatrix} 1 & J & k \\ 0 & 0 & 10.472 \\ 0.256 & 0.256 & 0 \end{vmatrix} = -2.682 \ i + 2.682 \ j \ m/s.$$

Another way of calculating the velocity of the point $B_2 = B_3$ is with the help of velocity field of two points ($B_1$ and $B_2$) not situated on the same rigid body ($B_1$ is on link 1 and $B_2$ is on link 2)

$$v_{B_2} = v_{B_1} + v'_{B_{21}},$$

where $v'_{B_{21}} = v'_{B_{21}} \ cos \phi \ i + v'_{B_{21}} \ sin \phi \ j = -0.651 \ i - 0.651 \ j \ m/s.$

For the acceleration analysis, the following equations are written using Eq. (5.36)

$$\alpha_{10} + \alpha_{32} + \alpha_{03} = 0,$$

$$r_{AB} \times \alpha_{32} + r_{AC} \times \alpha_{03} + a_{B_{21}} + a'_{B_{21}} - \omega_{10}^2 r_{AB} - \omega_{30}^2 r_{BC} = 0(5.42)$$
I.5 Contour Equations

where \( \alpha_{10} = \dot{\omega}_{10} \mathbf{k} = 0 \), \( \alpha_{32} = \alpha_{32} \mathbf{k} \), and \( \alpha_{03} = \alpha_{03} \mathbf{k} \).

The relative acceleration of \( B_2 \) on link 2 with respect to \( B_1 \) on link 1, \( a_{B_{21}}^r \), has \( \mathbf{i} \) and \( \mathbf{j} \) components

\[
a_{B_{21}}^r = a_{B_{21}x}^r \mathbf{i} + a_{B_{21}y}^r \mathbf{j} = a_{B_{21}x}^r \cos \phi \mathbf{i} + a_{B_{21}y}^r \sin \phi \mathbf{j}.
\]

The sign of the unknown relative accelerations is selected positive and then the numerical computation will give the correct orientation of the unknown acceleration vectors. The expression for the Coriolis acceleration is

\[
a_{B_{21}}^c = 2\omega_{10} \times v_{B_{21}}^r = 2\omega_{20} \times v_{B_{21}}^r = \begin{bmatrix} 0 & 0 & \omega_{10} \\ v_{B_{21}}^r \cos \phi & v_{B_{21}}^r \sin \phi & 0 \end{bmatrix} = -2v_{B_{21}}^r \omega_{10} \sin \phi \mathbf{i} + 2v_{B_{21}}^r \omega_{10} \cos \phi \mathbf{j} = -2(-0.920)(10.472) \sin 45^\circ \mathbf{i} + 2(-0.920)(10.472) \cos 45^\circ \mathbf{j} = 13.629 \mathbf{i} - 13.629 \mathbf{j} \text{ m/s}^2.
\]

The unknowns in Eq. (5.42) are \( \alpha_{32}, \alpha_{03}, \) and \( a_{B_{21}}^r \). Equation (5.42) becomes

\[
\alpha_{32} \mathbf{k} + \alpha_{03} \mathbf{k} = 0,
\]

\[
a_{B_{21}}^r - \omega_{10}^2 (x_B \mathbf{i} + y_B \mathbf{j}) - \omega_{30}^2 [(x_C - x_B) \mathbf{i} + (y_C - y_B) \mathbf{j}] = 0.
\]

The previous equations are projected onto the “fixed” reference frame \( Oxyz \)

\[
\alpha_{32} + \alpha_{03} = 0,
\]

\[
y_B \alpha_{32} + y_C \alpha_{03} + a_{B_{21}}^r \cos \phi - 2v_{B_{21}}^r \omega_{10} \sin \phi - \omega_{10}^2 x_B - \omega_{30}^2 (x_C - x_B) = 0,
\]

\[
x_B \alpha_{32} - x_C \alpha_{03} + a_{B_{21}}^r \sin \phi + 2v_{B_{21}}^r \omega_{10} \cos \phi - \omega_{10}^2 y_B - \omega_{30}^2 (y_C - y_B) = 0,
\]

or numerically

\[
\alpha_{32} + \alpha_{03} = 0,
\]

\[
0.256\alpha_{32} + a_{B_{21}}^r \cos 45^\circ + 13.626 - (10.472)^2(0.256) - (13.011)^2(0.100 - 0.256) = 0,
\]

\[
-0.256\alpha_{32} - 0.100\alpha_{03} + a_{B_{21}}^r \sin 45^\circ - 13.626 - (10.472)^2(0.256) - (13.011)^2(0 - 0.256) = 0.
\]
Equation (5.43) represents a system of three equations with three unknowns: \( \alpha_{32} \), \( \alpha_{03} \), and \( a_{B_{21}}^r \). Solving the algebraic equations, the following numerical values are obtained \( \alpha_{32} = -25.032 \text{ rad/s}^2 \), \( \alpha_{03} = 25.032 \text{ rad/s}^2 \), and \( a_{B_{21}}^r = -7.865 \text{ m/s}^2 \).

The absolute angular acceleration of the link 3 is

\[
\alpha_{30} = -\alpha_{03} = -25.032 \text{ k rad/s}^2.
\]

The velocity of the point \( B_2 = B_3 \) is computed with the expression of velocity field of two points (\( B_3 \) and \( C \)) on the same rigid body (link 3)

\[
v_{B_2} = v_{B_3} = v_C + \omega_{30} \times r_{CB} = \begin{vmatrix}
1 & j & k \\
0 & 0 & \omega_{30} \\
x_B - x_C & y_B - y_C & 0
\end{vmatrix} = -3.333 \text{ i} + 2.032 \text{ j m/s},
\]

where \( v_C = 0 \) because the joint \( C \) is grounded.

The link 2 and the driver link 1 have the same angular velocity

\[
\omega_{10} = \omega_{20} = \omega_{30} + \omega_{23} = 13.011 \text{ k} - 2.539 \text{ k} = 10.472 \text{ k rad/s}.
\]

The velocity of the point \( B_1 \) on link 1 is calculated with the expression of velocity field of two points (\( B_1 \) and \( A \)) on the same rigid body (link 1)

\[
v_{B_1} = v_A + \omega_{10} \times r_{AB} = \omega_{10} \times r_{AB} = \begin{vmatrix}
1 & j & k \\
0 & 0 & \omega_{10} \\
x_B & y_B & 0
\end{vmatrix} = -2.682 \text{ i} + 2.682 \text{ j m/s}.
\]

Another way of calculating the velocity of the point \( B_2 = B_3 \) is with the help of velocity field of two points (\( B_1 \) and \( B_2 \)) not situated on the same rigid body (\( B_1 \) is on link 1 and \( B_2 \) is on link 2)

\[
v_{B_2} = v_{B_1} + v_{B_{21}}^r,
\]

where \( v_{B_{21}}^r = v_{B_{21}}^r \cos \phi \text{ i} + v_{B_{21}}^r \sin \phi \text{ j} = -0.651 \text{ i} - 0.651 \text{ j m/s} \).

The angular acceleration of the link 3 is

\[
\alpha_{30} = -\alpha_{03} = \alpha_{32} = -25.032 \text{ k rad/s}^2.
\]
The absolute linear acceleration of the point $B_3$ is computed as follows
\[ \mathbf{a}_{B_3} = \mathbf{a}_C + \alpha_{30} \times \mathbf{r}_{CB} - \omega_{30}^2 \mathbf{r}_{CB} = -20.026 \ 1 - 47.277 \text{ m/s}^2. \]

**Second contour analysis**

According to Fig. 5.6, the second contour is described as
- rotational joint $R$ between the links 0 and 3 (joint $C$);
- rotational joint $R$ between the links 3 and 4 (joint $B$);
- rotational joint $R$ between the links 4 and 5 (joint $D_R$);
- translational joint $T$ between the links 5 and 0 (joint $D_T$).

For the velocity analysis, the following equations are written
\[
\begin{align*}
\omega_{30} + \omega_{43} + \omega_{54} &= 0, \\
r_{AC} \times \omega_{30} + r_{AB} \times \omega_{43} + r_{AD} \times \omega_{54} + v_{D_{05}}^r &= 0. \tag{5.44}
\end{align*}
\]

The relative linear velocity $v_{D_{05}}^r$ has only one component, along the $x$ axis
\[ v_{D_{05}}^r = v_{D_{05}}^r \mathbf{i}. \]

The unknown parameters in Eq. (5.44) are $\omega_{43}$, $\omega_{54}$ and $v_{D_{05}}^r$. The following numerical values are obtained $\omega_{43} = -15.304 \text{ rad/s}$, $\omega_{54} = 2.292 \text{ rad/s}$, and $v_{D_{05}}^r = 3.691 \text{ m/s}$.

The angular velocity of the link $BD$ is
\[ \omega_{40} = \omega_{30} + \omega_{43} = -\omega_{54} = -2.292 \mathbf{k} \text{ rad/s}. \]

The absolute linear velocity of the point $D_4 = D_5$ is computed as follows
\[ \mathbf{v}_{D_4} = \mathbf{v}_{D_5} = \mathbf{v}_{B_4} + \omega_{40} \times \mathbf{r}_{BD} = -v_{D_{05}}^r = -3.691 \ 1 \text{ m/s}, \]

where $\mathbf{v}_{B_4} = \mathbf{v}_{B_3}$.

For the acceleration analysis the following equations exist
\[
\begin{align*}
\alpha_{30} + \alpha_{43} + \alpha_{54} &= 0, \\
\mathbf{a}_{D_{05}}^r + \mathbf{a}_{D_{05}}^c + r_{AC} \times \alpha_{30} + r_{AB} \times \alpha_{43} + r_{AD} \times \alpha_{54} - \\
\omega_{30}^2 \mathbf{r}_{CB} - \omega_{40}^2 \mathbf{r}_{BD} &= 0. \tag{5.45}
\end{align*}
\]

Because the slider 5 does not rotate ($\omega_{50} = 0$), the Coriolis acceleration is
\[ \mathbf{a}_{D_{05}}^c = 2\omega_{50} \times v_{D_{05}}^r = 0. \]
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The unknowns in Eq. (5.45) are $\alpha_{43}$, $\alpha_{54}$ and $a_{D_{05}}^r$. The following numerical results are obtained $\alpha_{43} = 77.446 \text{ rad/s}^2$, $\alpha_{54} = -52.414 \text{ rad/s}^2$, and $a_{D_{05}}^r = 16.499 \text{ m/s}^2$.

The absolute angular acceleration of the link $BD$ is

$$\alpha_{40} = \alpha_{30} + \alpha_{43} = \alpha_{45} = 52.414 \text{ k rad/s}^2,$$

and the linear acceleration of the point $D_4 = D_5$ is

$$a_{D_4} = a_{D_5} = a_{B_4} + \alpha_{40} \times r_{BD} - \omega_{40}^2 r_{BD} = -16.499 \text{ m/s}^2,$$

where $a_{B_4} = a_{B_3}$. 

5.5 Problems

5.1 The four-bar mechanism shown in Fig. 3.10(a) has the dimensions: $AB = 80$ mm, $BC = 210$ mm, $CD = 120$ mm, and $AD = 190$ mm. The driver link $AB$ rotates with a constant angular speed of 200 rpm. Find the velocities and the accelerations of the four-bar mechanism using the contour equations method for the case when the angle of the driver link $AB$ with the horizontal is $\phi = 60^\circ$.

5.2 The angular speed of the driver link 1, of the mechanism shown in Fig. 4.9, is $\omega = \omega_1 = 20$ rad/s. The distance from the link 3 to the horizontal axis $Ax$ is $a = 55$ mm. Using the contour equations find the velocity and the acceleration of the point $C$ on the link 3 for $\phi = 30^\circ$.

5.3 The slider crank mechanism shown in Fig. 4.10 has the dimensions $AB = 0.4$ m and $BC = 1$ m. The driver link 1 rotates with a constant angular speed of $n = 160$ rpm. Find the velocity and acceleration of the slider 3 using the contour equations when the angle of the driver link with the horizontal is $\phi = 30^\circ$.

5.4 The planar mechanism considered is shown in Fig. 3.19. The following data are given: $AB=0.150$ m, $BC=0.400$ m, $CD=0.370$ m, $CE=0.230$ m, $EF=CE$, $L_a=0.300$ m, $L_b=0.450$ m, and $L_c=CD$. The angular speed of the driver link 1 is constant and has the value 180 rpm. Using the contour equations method find the velocities and the accelerations of the mechanism for $\phi = \phi_1 = 30^\circ$.

5.5 The R-RRR-RTT mechanism is shown in Fig. 3.20. The following data are given: $AB=0.080$ m, $BC=0.350$ m, $CE=0.200$ m, $CD=0.150$ m, $L_a=0.200$ m, $L_b=0.350$ m, $L_c=0.040$ m. The driver link 1 rotates with a constant angular speed of $n = 1200$ rpm. For $\phi = 145^\circ$ find the velocities and the accelerations of the mechanism with the contour equations.

5.6 The mechanism shown in Fig. 3.21 has the following dimensions: $AB = 80$ mm, $AD = 250$ mm, $BC = 180$ mm, $CE = 60$ mm, $EF = 200$ mm, and $a = 170$ mm. The constant angular speed of the driver link 1 is $n = 400$ rpm. Find the velocities and the accelerations of the mechanism using the contour equations when the angle of the driver link 1 with the horizontal is $\phi = \phi_1 = 300^\circ$.
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5.7 The dimensions for the mechanism shown in Fig. 3.22 are: \( AB = 150 \text{ mm}, \ BD = 400 \text{ mm}, \ BC = 140 \text{ mm}, \ CD = 400 \text{ mm}, \ DE = 250 \text{ mm}, \ CF = 500 \text{ mm}, \ AE = 380 \text{ mm}, \) and \( b = 100 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 40 \text{ rpm} \). Find the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 210^\circ \). Use the contour equations method.

5.8 The mechanism in Fig. 3.23 has the dimensions: \( AB = 200 \text{ mm}, \ AC = 100 \text{ mm}, \ BD = 400 \text{ mm}, \ DE = 550 \text{ mm}, \ EF = 300 \text{ mm}, \ L_a = 500 \text{ mm}, \) and \( L_b = 100 \text{ mm} \). Using the contour equations method find the velocities and the accelerations of the mechanism if the constant angular speed of the driver link 1 is \( n = 70 \text{ rpm} \) and for \( \phi = \phi_1 = 210^\circ \).

5.9 The dimensions for the mechanism shown in Fig. 3.24 are: \( AB = 150 \text{ mm}, \ BC = 400 \text{ mm}, \ AD = 360 \text{ mm}, \ CD = 210 \text{ mm}, \ DE = 130 \text{ mm}, \ EF = 400 \text{ mm}, \) and \( L_a = 40 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 250 \text{ rpm} \). Utilizing the contour equations find the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 30^\circ \).

5.10 The mechanism in Fig. 3.25 has the dimensions: \( AB = 250 \text{ mm}, \ AC = 800 \text{ mm}, \ BD = 1200 \text{ mm}, \ L_a = 180 \text{ mm}, \) and \( L_b = 300 \text{ mm} \). The driver link 1 rotates with a constant angular speed of \( n = 50 \text{ rpm} \). Find the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 210^\circ \). Use the contour equations method.

5.11 Figure 3.26 shows a mechanism with the following dimensions: \( AB = 120 \text{ mm}, \ BD = 400 \text{ mm}, \) and \( L_a = 150 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 600 \text{ rpm} \). Find the velocities and the accelerations of the mechanism, using the contour equations method, when the angle of the driver link 1 with the horizontal is \( \phi = 210^\circ \).

5.12 The mechanism in Fig. 3.27 has the dimensions: \( AB = 200 \text{ mm}, \ AC = 500 \text{ mm}, \ BD = 800 \text{ mm}, \ DE = 400 \text{ mm}, \ EF = 270 \text{ mm}, \ L_a = 70 \text{ mm}, \) and \( L_b = 300 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 40 \text{ rpm} \). Utilizing the contour equations find the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 300^\circ \).

5.13 Figure 3.28 shows a mechanism with the following dimensions: \( AB = 200 \text{ mm}, \ BC = 750 \text{ mm}, \ CD = DE = 300 \text{ mm}, \ EF = 500 \text{ mm}, \ L_a = 750 \text{ mm}, \) and \( L_b = L_c = 250 \text{ mm} \). Find the velocities and the
accelerations of the mechanism, using the contour equations method, if the constant angular speed of the driver link 1 is $n = 1100 \text{ rpm}$ and for $\phi = \phi_1 = 120^\circ$.

5.14 Figure 3.29 shows a mechanism with the following dimensions: $AB = 120 \text{ mm}$, $BC = 550 \text{ mm}$, $CE = 180 \text{ mm}$, $CD = 350 \text{ mm}$, $EF = 300 \text{ mm}$, $L_a = 320 \text{ mm}$, $L_b = 480 \text{ mm}$, and $L_c = 600 \text{ mm}$. The constant angular speed of the driver link 1 is $n = 100 \text{ rpm}$. Find the velocities and the accelerations of the mechanism, using the contour equations, for $\phi = \phi_1 = 30^\circ$.

5.15 Figure 3.30 shows a mechanism with the following dimensions: $AB = 180 \text{ mm}$, $BC = 520 \text{ mm}$, $CF = 470 \text{ mm}$, $CD = 165 \text{ mm}$, $DE = 540 \text{ mm}$, $L_a = 630 \text{ mm}$, $L_b = 360 \text{ mm}$, and $L_c = 210 \text{ mm}$. The constant angular speed of the driver link 1 is $n = 70 \text{ rpm}$. Use the contour equations to calculate the velocities and the accelerations of the mechanism when the angle of the driver link 1 with the horizontal is $\phi = 210^\circ$.

5.16 Figure 3.31 shows a mechanism with the following dimensions: $AB = 60 \text{ mm}$, $BC = 150 \text{ mm}$, $AD = 70 \text{ mm}$, and $BE = 170 \text{ mm}$. The constant angular speed of the driver link 1 is $n = 300 \text{ rpm}$. Find the velocities and the accelerations of the mechanism, using the contour equations, if the angle of the driver link 1 with the horizontal is $\phi = 210^\circ$.

5.17 The dimensions of the mechanism shown in Fig. 3.32 are: $AB = 90 \text{ mm}$, $BC = 240 \text{ mm}$, $BE = 400 \text{ mm}$, $CE = 600 \text{ mm}$, $CD = 220 \text{ mm}$, $EF = 900 \text{ mm}$, $L_a = 250 \text{ mm}$, $L_b = 150 \text{ mm}$, and $L_c = 100 \text{ mm}$. The constant angular speed of the driver link 1 is $n = 50 \text{ rpm}$. Employing the contour equations find the velocities and the accelerations of the mechanism for $\phi = \phi_1 = 210^\circ$.

5.18 The dimensions of the mechanism shown in Fig. 3.33 are: $AB = 180 \text{ mm}$, $AC = 300 \text{ mm}$, $CD = 400 \text{ mm}$, $DE = 200 \text{ mm}$, and $L_a = 360 \text{ mm}$. The constant angular speed of the driver link 1 is $n = 90 \text{ rpm}$. Use the contour equations to calculate the velocities and the accelerations of the mechanism when the angle of the driver link 1 with the horizontal is $\phi = 30^\circ$. 
5.19 The dimensions of the mechanism shown in Fig. 3.34 are: \( AB = 80 \text{ mm} \), \( AC = 40 \text{ mm} \), \( CD = 100 \text{ mm} \), and \( DE = 300 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 60 \text{ rpm} \). Use the contour equations to calculate the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 210^\circ \).

5.20 The dimensions of the mechanism shown in Fig. 3.35 are: \( AB = 200 \text{ mm} \), \( AC = 350 \text{ mm} \), and \( CD = 600 \text{ mm} \). For the distance \( b \) select a suitable value. The constant angular speed of the driver link 1 is \( n = 90 \text{ rpm} \). Find the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 120^\circ \).

5.21 The dimensions of the mechanism shown in Fig. 3.36 are: \( AB = 80 \text{ mm} \), \( AC = 60 \text{ mm} \), and \( CD = 70 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 220 \text{ rpm} \). Find the velocities and the accelerations of the mechanism, using the contour equations, for \( \phi = \phi_1 = 240^\circ \).

5.22 The dimensions of the mechanism shown in Fig. 3.37 are: \( AB = 150 \text{ mm} \), \( AC = 420 \text{ mm} \), \( BD = L_a = 650 \text{ mm} \), and \( DE = 350 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = n_1 = 650 \text{ rpm} \). Find the velocities and the accelerations of the mechanism, using the contour equations, for \( \phi = \phi_1 = 240^\circ \).

5.23 The dimensions of the mechanism shown in Fig. 3.38 are: \( AB = 200 \text{ mm} \), \( AD = 500 \text{ mm} \), and \( BC = 250 \text{ mm} \). The constant angular speed of the driver link 1 is \( n = 160 \text{ rpm} \). Use the contour equations to calculate the velocities and the accelerations of the mechanism for \( \phi = \phi_1 = 240^\circ \). Select a suitable value for the distance \( a \).

5.24 The mechanism in Fig. 3.11(a) has the dimensions: \( AB = 0.20 \text{ m} \), \( AD = 0.40 \text{ m} \), \( CD = 0.70 \text{ m} \), \( CE = 0.30 \text{ m} \), and \( y_E = 0.35 \text{ m} \). The constant angular speed of the driver link 1 is \( n = 2600 \text{ rpm} \). Using the contour equations find the velocities and the accelerations of the mechanism for the given input angle \( \phi = \phi_1 = 210^\circ \).

5.25 The mechanism in Fig. 3.12 has the dimensions: \( AB = 0.03 \text{ m} \), \( BC = 0.05 \text{ m} \), \( CD = 0.08 \text{ m} \), \( AE = 0.07 \text{ m} \), and \( L_a = 0.025 \text{ m} \). The constant angular speed of the driver link 1 is \( n = 90 \text{ rpm} \). Employing the contour equations find the velocities and the accelerations of the mechanism for the given input angle \( \phi = \phi_1 = \pi/3 \).
5.26 The mechanism in Fig. 3.15 has the dimensions: \( AC = 0.200 \, \text{m} \), \( BC = 0.300 \, \text{m} \), \( BD = 1.000 \, \text{m} \), and \( L_a = 0.050 \, \text{m} \). The constant angular speed of the driver link 1 is \( n = 1500 \, \text{rpm} \). Use the contour equations to calculate the velocities and the accelerations of the mechanism for \( \phi = 330^\circ \).
References


[60] * * *, *The theory of mechanisms and machines (Teoria mehanizmov i masin)*, Vassaia scola, Minsc, 1970.

**Figure captions**

Figure 5.1 Two rigid links \((j)\) and \((k)\) connected by a joint at \(A\): (a) general case, (b) slider joint in general motion

Figure 5.2 Monocontour closed kinematic chain with \(n\) rigid links

Figure 5.3 Planar mechanism and the diagram which represents the mechanism

Figure 5.4 (a) Example mechanism (R-TRR-RRT), and (b) diagram which represents the mechanism

Figure 5.5 First contour RTRR: (a) diagram, and (b) mechanism

Figure 5.6 Second contour RRRT: (a) diagram, and (b) mechanism