Chapter 5
Friction

If a body rests on an incline plane, the friction force exerted on it by the surface prevents it from sliding down the incline. The question is, what is the steepest incline on which the body can rest?

A body is placed on a horizontal surface. The body is pushed with a small horizontal force F. If the force F is sufficiently small, the body does not move.

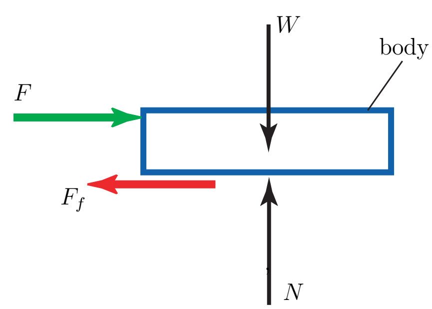

Fig. 5.1 Free-body diagram of the body

Figure 5.1 shows the free-body diagram of the body, where the force W is the weight force of the body, and N is the normal force exerted by the surface on the body. The force F is the horizontal force, and F_f is the friction force exerted by the surface. Friction force arises in part from the interactions of the roughness, or asperities, of the contacting surfaces. The body is in equilibrium and $F_f = F$.

The force F is slowly increased. As long as the body remains in equilibrium, the friction force F_f must increase correspondingly, since it equals the force F. The body slips on the surface. The friction force, after reaching the maximum value, cannot maintain the body in equilibrium. The force applied to keep the body moving on the surface is smaller than the force required to cause it to slip. Why more force is required to start the body sliding on a surface than to keep it sliding is explained in part by the necessity to break the asperities of the contacting surfaces before sliding can begin.

The theory of dry friction, or *Coulomb friction*, predicts:

• the maximum friction forces that can be exerted by dry, contacting surfaces that are stationary relative to each other;

• the friction forces exerted by the surfaces when they are in relative motion, or sliding.

Static Coefficient of Friction

The magnitude of the maximum friction force, F_f, that can be exerted between two plane dry surfaces in contact is

$$F_f = \mu_s N, \tag{5.1}$$

where μ_s is a constant, the *static coefficient of friction*, and N is the normal component of the contact force between the surfaces. The value of the static coefficient of friction, μ_s, depends on:

• the materials of the contacting surfaces;

• the conditions of the contacting surfaces namely smoothness and degree of contamination.

Typical values of μ_s for various materials are shown in Table 5.1.

Table 5.1. Typical values of the static coefficient of friction.

Materials	μ_s
metal on metal	0.15 - 0.20
metal on wood	0.20 - 0.60
metal on masonry	0.30 - 0.70
wood on wood	0.25 - 0.50
masonry on masonry	0.60 - 0.70
rubber on concrete	0.50 - 0.90

Equation (5.1) gives the maximum friction force that the two surfaces can exert without causing it to slip. If the static coefficient of friction μ_s between the body and the surface is known, the largest value of F one can apply to the body without causing it to slip is $F = F_f = \mu_s N$. Equation (5.1) determines the magnitude of the maximum friction force but not its direction. The friction force resists the impending motion.

Kinetic coefficient of friction

The magnitude of the friction force between two plane dry contacting surfaces that are in motion relative to each other is

$$F_f = \mu_k N, \tag{5.2}$$

where μ_k is the *kinetic coefficient of friction* and N is the normal force between the surfaces. The value of the kinetic coefficient of friction is generally smaller than the value of the static coefficient of friction, μ_s.

To keep the body in Fig. 5.1 in uniform motion (sliding on the surface) the force exerted must be $F = F_f = \mu_k N$. The friction force resists the relative motion, when two surfaces are sliding relative to each other.

The body RB shown in Fig. 5.2(a) is moving on the fixed surface 0.

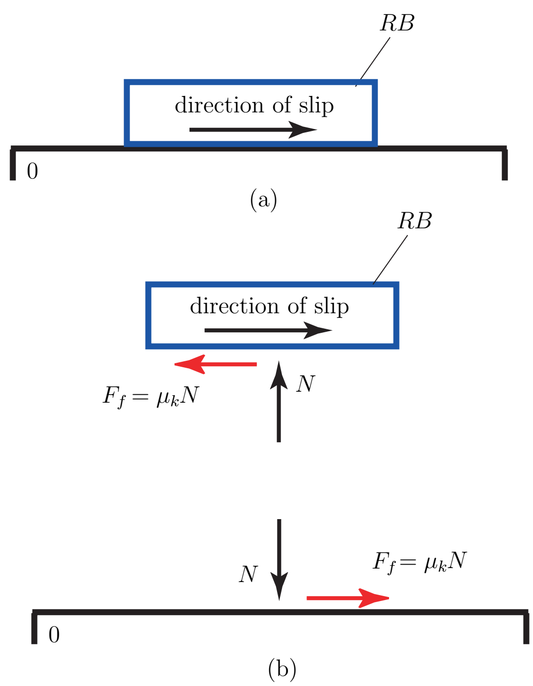

Fig. 5.2 Directions of the friction forces

The direction of motion of RB is the positive axis x. The friction force on the body RB acts in the direction opposite to its motion, and the friction force on the fixed surface is in the opposite direction as shown in Fig. 5.2(b).

Angles of friction

The *angle of friction*, θ, is the angle between the friction force, $F_f = |\mathbf{F}_f|$, and the normal force to the surface $N = |\mathbf{N}|$, as shown in Fig. 5.3.

The magnitudes of the normal force and friction force, and θ are related by

$$F_f = R \sin\theta,$$
$$N = R \cos\theta,$$

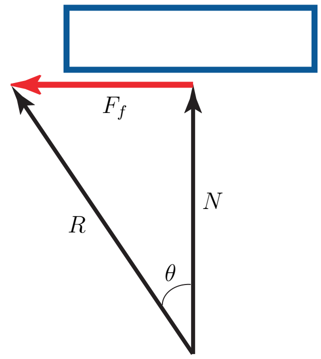

Fig. 5.3 Angle of friction, θ

where $R = |\mathbf{R}| = |\mathbf{N} + \mathbf{F}_f|$.

The value of the angle of friction when slip is impending is called the *static angle of friction*, θ_s,

$$\tan\theta_s = \mu_s.$$

The value of the angle of friction when the surfaces are sliding relative to each other is called the the *kinetic angle of friction*, θ_k,

$$\tan\theta_k = \mu_k.$$