Problem Set 6

Problem 6.1 Minimizing Tension in a Cable

The 200 lb. uniform tank is suspended by a 6 ft. cable which passes over the pulley at O. The cable can be attached at either points A and B, or at points C and D. Which attachment produces the smallest tension on the cable, and what is the tension?

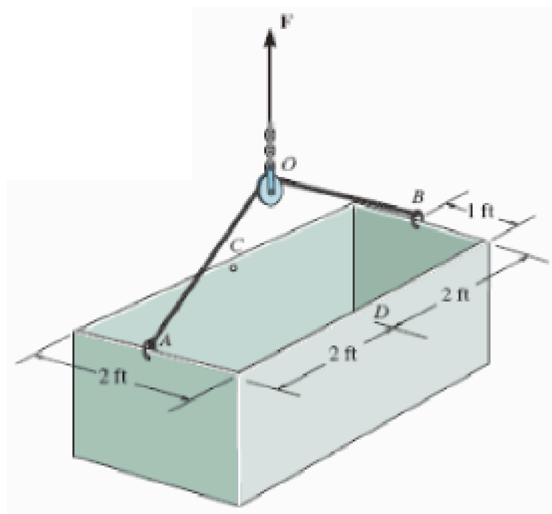

Figure P6.1: Problem 6.1

Problem 6.2 Tension in a Network of Cords

The 30 kg pipe is supported by a series of five cords. Determine the force in each cord for equilibrium.

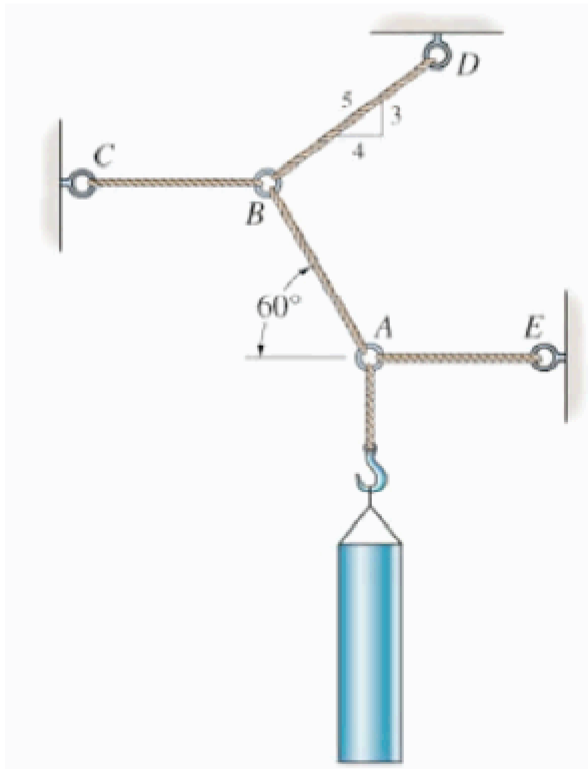

Figure P6.2: Problem 6.2

Problem 6.3 Tension in Three Cables

Determine the tension in cables AB, AC, and AD required to hold the 60 lb. crate in equilibrium.

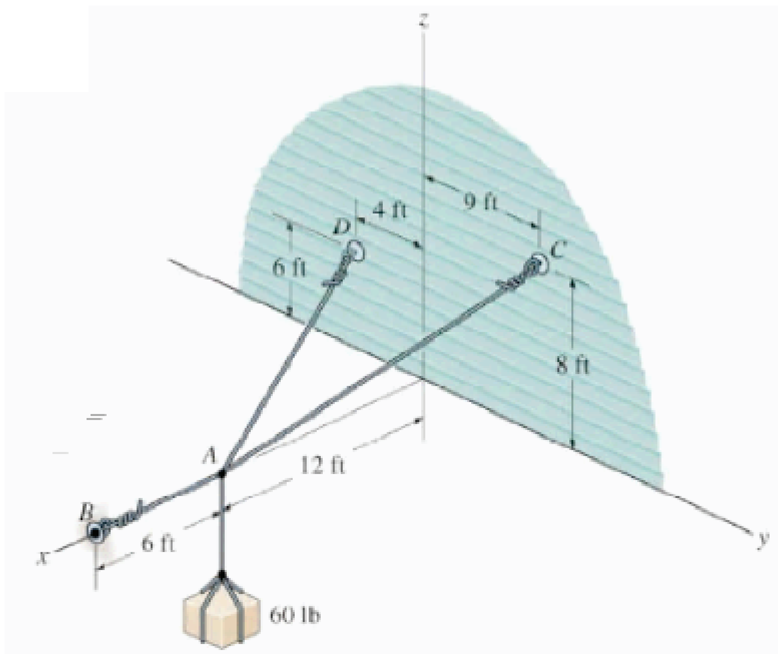

Figure P6.3: Problem 6.3

Problem 6.4

If cables BD and BC can withstand a maximum tensile force of 20 kN, determine the maximum mass of the girder that can be suspended from cable AB so that neither cable will fail. The center of mass of the girder is located at point G.

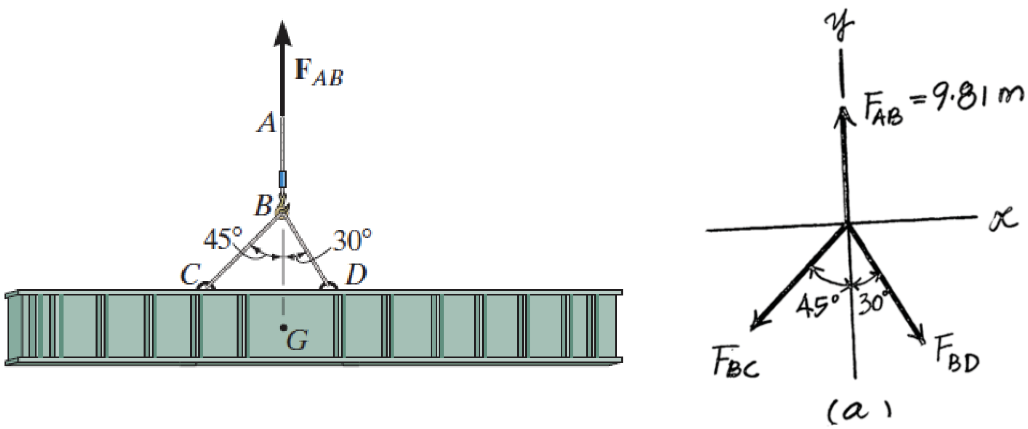

Figure P6.4: Problem 6.4

Solution

1. Mechanical System: girder, cables BD, BC, AB.
2. Free-Body Diagram (FBD): see Fig (a).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$.

```
% in order to meet the conditions of equilibrium
% the tensile force developed in cable AB
% must be equal to the weight of the girder
% F_AB = m*g where g = 9.8 m/s^2
% equation of equilibrium along the x and y-axes to the free-body diagram
thetaBC = 45*pi/180;
thetaBD = 30*pi/180;
% sum Fx = 0
% F_BD*sin(thetaBD)-F_BC*sin(thetaBC) = 0
% F_BD = F_BC*sin(thetaBC)/sin(thetaBD)
% F_BD = 1.414*F_BC
% since F_BD > F_BC => cable BD will break before cable BC
% substituting F_BD = 20 000 N =>
```

```
F_BD = 20000; % N
F_BC = F_BD*sin(thetaBD)/sin(thetaBC)
% F_BC = 1.4142e+04 N
% sum Fy = 0
% F_AB - F_BD*cos(thetaBD) - F_BC*cos(thetaBC) = 0
g = 9.8;
m = (F_BD*cos(thetaBD)+F_BC*cos(thetaBC))/g
```

4. Result: m = 2 787 kg.

Problem 6.5

Determine the unstretched length of spring AC if a force $P = 80$ lb causes the angle $\theta = 60°$ for equilibrium. Cord AB is 2 ft long. Take $k = 50$ lb/ft and $b = 2$ ft.

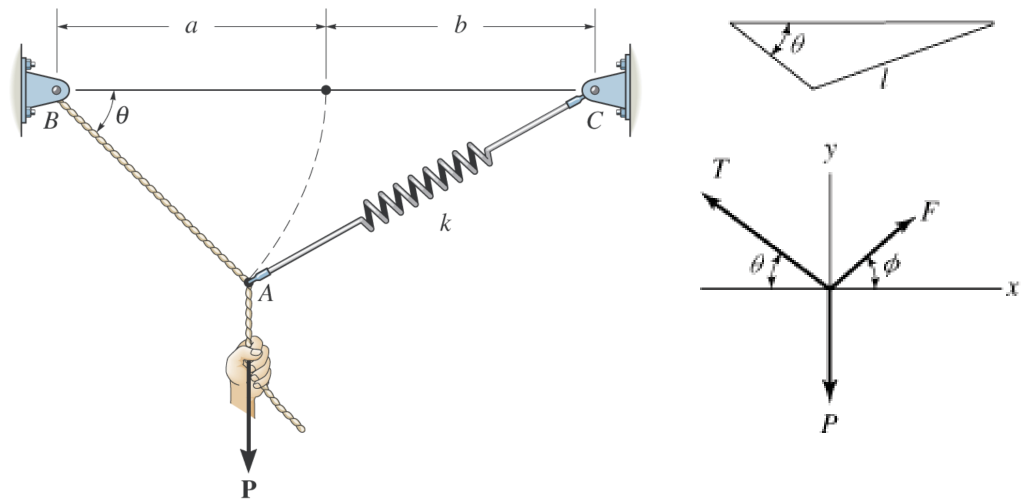

Figure P6.5: Problem 6.5

Solution

1. Mechanical System: spring AC and cord AB.
2. Free-Body Diagram (FBD): node A (see figure).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$.

```
P = 80; % lb
k = 50; % lb/ft
a = 2;  % ft
b = 2;  % ft
theta = 60*pi/180;
AB = a; BC = a + b;
% AC = l
l = sqrt(AB^2+BC^2-2*AB*BC*cos(theta));
%  AC = l = 3.4641 ft
% l/sin(theta) = a/sin(phi) =>
phi = asin(a*sin(theta)/l);
% phi = 0.5236 rad = 30 deg
% FBD of A
```

```
Fx = -T*cos(theta)+F*cos(phi);
Fy = T*sin(theta)+F*sin(phi)-P;
% solve Fx = 0 and Fy = 0;
sol = solve(Fx,Fy);
Fs = sol.F; Ts = sol.T;
% F = Fs = 40.000 (lb)
% Fs= k (l-lo) => lo = l - Fs/k
lo = l - Fs/k;
% lo =  2.664 (ft)
```

4. Result: unstretched length of the spring is 2.664 ft.

Problem 6.6

Two spheres A and B have an equal mass and are electrostatically charged such that the repulsive force acting between them has a magnitude of 20 mN and is directed along line AB. Determine the angle θ, the tension in cords AC and BC, and the mass m of each sphere.

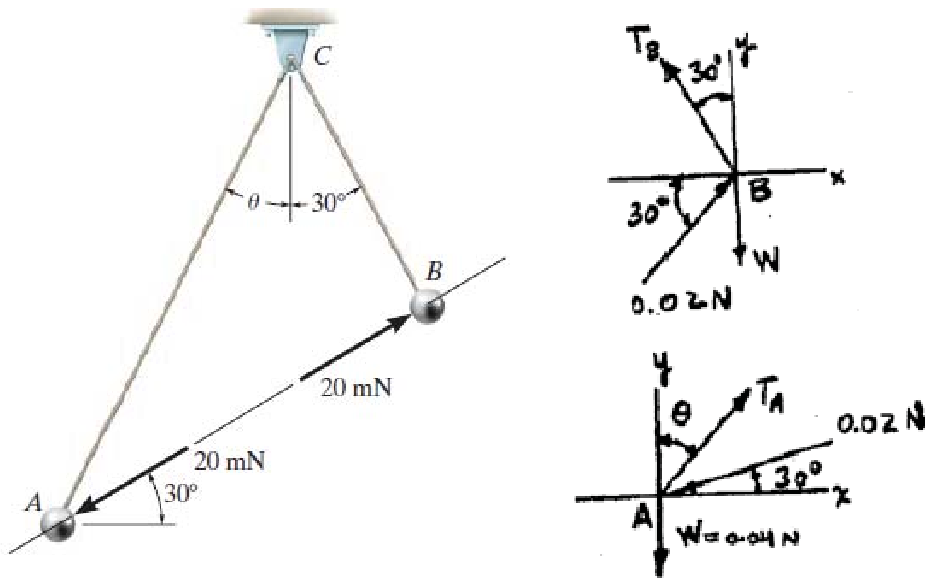

Figure P6.6: Problem 6.6

Solution

1. Mechanical System: sphere A, sphere B, cord AC, cord BC, and pin C.

2. Free-Body Diagram: FBD of sphere A and FBD of sphere B (see figures).

3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$.

```
% FBD of sphere B
FBx = 0.02*cosd(30) - TB*sind(30);
FBy = 0.02*sind(30) + TB*cosd(30) - W;
% solve FBx = 0 and FBy = 0;
solB = solve(FBx,FBy);
TBs = solB.TB; Ws = solB.W;
% TB = 0.03464 N and  W =  0.04 N
```

```
% FBD of sphere A
% FAx = -0.02*cosd(30) + TA*sin(theta) = 0
% FAy = -0.02*sind(30) + TA*cos(theta) - Ws = 0
% => TA*sin(theta) = 0.02*cosd(30)
% => TA*cos(theta) = 0.02*sind(30)+Ws
% tan(theta) = 0.02*cosd(30)/(0.02*sind(30)+Ws)
thetas = atand(0.02*cosd(30)/(0.02*sind(30)+Ws));
thetas = double(thetas);
% theta = 19.1066 deg
TA = 0.02*cosd(30)/sind(thetas);
% TA = 0.0529 N
g = 9.81; % m/s^2
m = Ws/g; % m = 0.004077 kg
```

4. Results: $\theta = 19.1066°$, $T_A = 0.0529$ N, $T_B = 0.03464$ N, and $m = 0.004077$ kg.

Problem 6.7

Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.

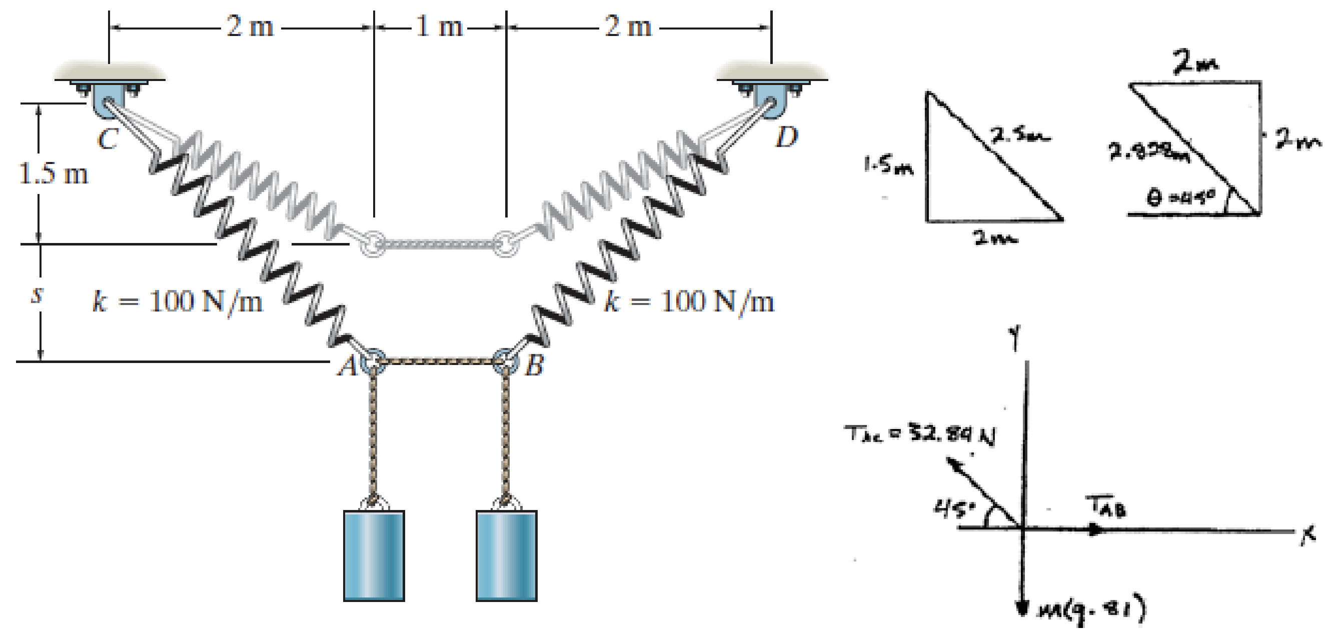

Figure P6.7: Problem 6.7

Solution

1. Mechanical System: cylinders, cord, springs, pins.
2. Free-Body Diagram: FBD of node A (see figure).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$.

```
k = 100.; % N/m
s = 0.5; % m
g = 9.81 % m/s^2
% unstretched length of spring
lo = sqrt(2^2+1.5^2)
% lo = 2.5000 m
% final length of spring
l = sqrt(2^2+(1.5+s)^2)
% l = 2.8284 m
```

```
TAC = k*(l-lo)
% TAC = 32.8427 N
% FBD for node A
% Fy = TAC sind(45) - m g = 0 =>
m = TAC*sind(45)/g
% m = 2.3673 kg
```

4. Result: $m = 2.3673$ kg.

Problem 6.8

Determine the force acting along the axis of each of the three struts needed to support the 500-kg block.

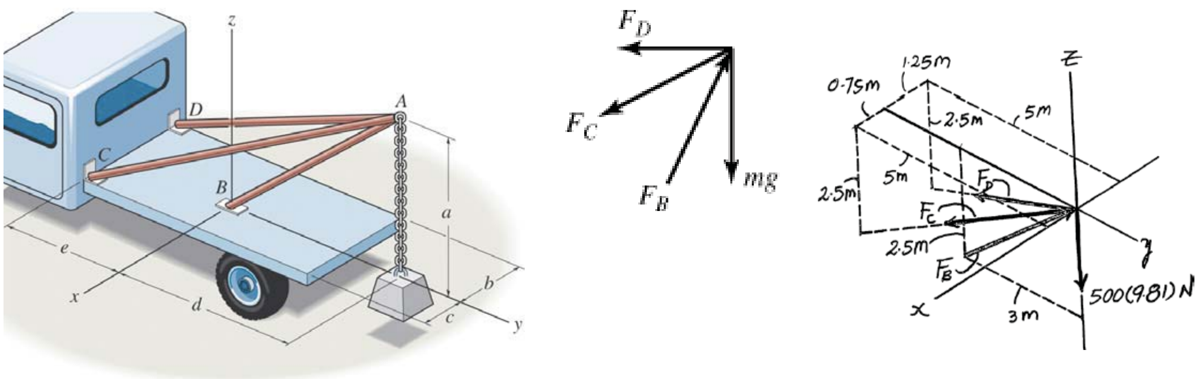

Figure P6.8: Problem 6.8

Solution

1. Mechanical System: three struts and 500-kg block.
2. Free-Body Diagram: FBD of node A (see figure).
3. Equations: $\sum F_x = 0$ & $\sum F_y = 0$.

```
mb = 500; % kg
a = 2.5; b = 1.25; c = 0.75; d = 3;  e = 2;  % m
g = 9.81; % m/s^2
% direction cosines of vector force F_B
cxB = 0; cyB = d/sqrt(d^2+a^2); czB = a/sqrt(d^2+a^2);
% unit vector of vector force F_B
uB = [cxB, cyB, czB] %  uB = [0    0.7682    0.6402]
F_B = FB*uB;
% direction cosines of vector force F_C
cxC = c/sqrt(c^2+(d+e)^2+a^2); cyC = -(d+e)/sqrt(c^2+(d+e)^2+a^2);
czC = -a/sqrt(c^2+(d+e)^2+a^2);
% unit vector of vector force F_C
uC = [cxC, cyC, czC] %  uC = [0.1330   -0.8865   -0.4432]
F_C = FC*uC;
% direction cosines of vector force F_D
cxD = -b/sqrt(b^2+(d+e)^2+a^2); cyD = -(d+e)/sqrt(b^2+(d+e)^2+a^2);
```

```
czD = -a/sqrt(b^2+(d+e)^2+a^2);
% unit vector of vector force F_D
uD = [cxD, cyD, czD] %  uD = [-0.2182   -0.8729   -0.4364]
F_D = FD*uD;
W = - mb*g*[0 0 1];
% FBD
F = F_B + F_C + F_D + W;
% Fx = 0.133*FC - 0.2182*FD = 0
% Fy = 0.7682*FB - 0.8865*FC - 0.8729*FD = 0
% Fz = 0.6402*FB - 0.4432*FC - 0.4364*FD - 4905.0 = 0
% FB = 19154.637 (N)  FC = 10374.548 (N) FD = 6321.806 (N)
```

4. Result: $F_B = 19\,154.637$ N, $F_C = 10\,374.548$ N, $F_D = 6\,321.806$ N.