Problem Set 3

Problem 3.1 Center of Gravity for a Bent Rod
Determine the distance $x_C$ and $y_C$ to the center of gravity of the bent rod.

Figure P9.1: Problem 9.1
Problem 3.2 Centroid of a Composite Section
Determine the location $y_C$ of the centroid of the beam’s cross sectional area. Neglect the size of the corner welds at $A$ and $B$ for the calculation.

Figure P9.2: Problem 9.2
**Problem 3.3** Centroid of a Tapered Cross Section

Locate the centroid \( y_C \) of the concrete beam with the tapered cross section shown.

Figure P9.3: Problem 9.3
Problem 3.4
Find the $x$-coordinate of the centroid of the indicated region where $A = 2 \text{ m}$ and $k = \pi/8 \text{ m}^{-1}$.

Figure 3.1: Problem 3.1
Problem 3.5
Determine the area and the centroid of the area.

\[ y(x) = x^{3/2} \]
where \( 0 < x < a \) and \( 0 < y < a \)

\( a = 1; \) \( \text{m} \)

\% differential element
\[ dA = y \, dx = x^{3/2} \, dx \]
\% centroid of the differential element is at
\% \( x \) and \( y/2 = (1/2) \, x^{3/2} \)

\% \( A = \int dA \) where \( 0 < x < a \)
\( A = \int (x^{3/2}, x, 0, a); \)
\% \( A = 0.400 \) \( (\text{m}^2) \)

\% \( My = \int (x \, dA) \) where \( 0 < x < a \)
\( My = \int (x \times x^{3/2}, x, 0, a); \)
\( xC = My/A; \)
\% \( xC = 0.714 \) \( (\text{m}) \)
% Mx = int(y/2 dA) where 0<x<a
Mx = int((1/2)*x^(-3/2)*x^((3/2)),x,0,a);
yC = Mx/A;
% yC = 0.312 (m)
Problem 3.6
Determine the area and the centroid of the area.
Problem 3.7
Determine the location of the centroid \( C \) of the area where \( a = 6 \) in, \( b = 6 \) in, \( c = 3 \) in, and \( d = 6 \) in.

Figure 3.7: Problem 3.7
Problem 3.8
Locate the center of mass of the homogeneous block assembly

Solution

% rectangular prism 1
V1 = 150*150*550; % mm^3
x1 = 75; % mm
y1 = 275; % mm
z1 = 75; % mm

% rectangular prism 2
V2 = 150*150*200; % mm^3
x2 = 225; % mm
y2 = 450; % mm
z2 = 75; % mm

% triangular prism 3
V3 = 150*150*100/2; % mm^3
x3 = 200; % mm
y3 = 50; % mm
z3 = 50; % mm

% total area
\[ V = V_1 + V_2 + V_3; \]

\[ x_C = \frac{x_1 V_1 + x_2 V_2 + x_3 V_3}{V}; \]
\[ y_C = \frac{y_1 V_1 + y_2 V_2 + y_3 V_3}{V}; \]
\[ z_C = \frac{z_1 V_1 + z_2 V_2 + z_3 V_3}{V}; \]

\% \ V = 1.8e+07 (\text{mm}^3)
\% \ x_C = 120.312 (\text{mm})
\% \ y_C = 304.688 (\text{mm})
\% \ z_C = 73.438 (\text{mm})
Problem 3.9
Locate the centroid of the paraboloid defined by the equation $x/a = (y/b)^2$ where $a = b = 4$ m. The material is homogeneous.

![Diagram of a paraboloid](image)

**Solution**

```matlab
a = 4; % m
b = 4; % m
y = b*sqrt(x/a);

% dV = pi*y^2 dy
V = int(pi*y^2, x, 0, a);
% V = 32 pi (m^3)

% xC = (1/V) int(x dV)
xC = int(x*pi*y^2, x, 0, a)/V;
% xC = 2.667 (m)
```
Problem 3.10

A circular V-belt has an inner radius \( r = 600 \text{ mm} \) and a cross-sectional area with the dimensions \( a = 25 \text{ mm}, b = 50 \text{ mm}, \) and \( c = 75 \text{ mm} \). Determine the volume of material required to make the belt.

Figure 3.10: Problem 3.10