Problem Set 2

Problem 2.1 Extremizing a Moment Due to a Force
Determine the direction θ of the force $F = 40$ lb. so that it produces
a) the maximum moment about point A, and
b) the minimum moment about point A.

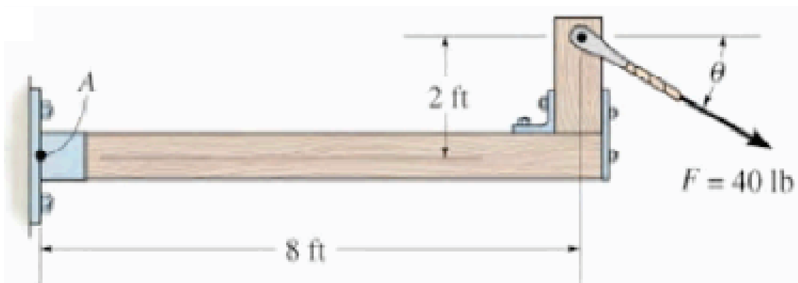

Figure P2.1: Problem 2.1

Problem 2.2 Moment of a Force-Vector Formulation

Determine the magnitude of the force **F** that should be applied at the end of the lever such that this force creates a clockwise moment about point O of 15 N m when $\theta = 30°$.

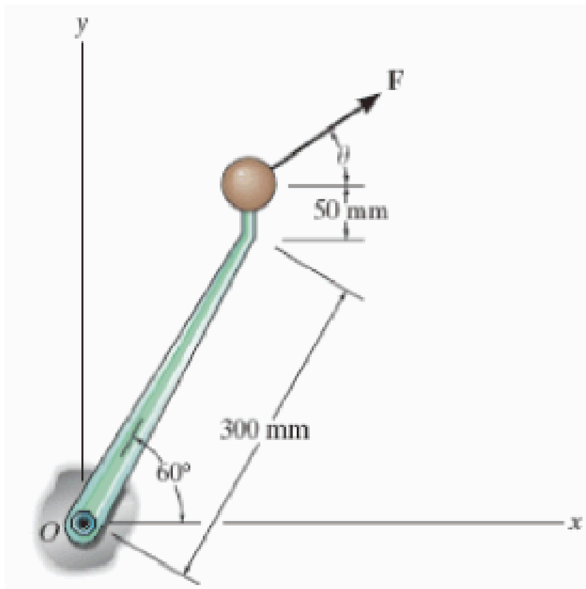

Figure P2.2: Problem 2.2

Problem 2.3

Moment Calculation Using Cross Products

The man pulls on the rope with a force of F = 20 N. Determine the moment that this force exerts about the base of the pole at O. Solve the problem two ways: (a) use a position vector from O-A, and (b) use a position vector from O-B.

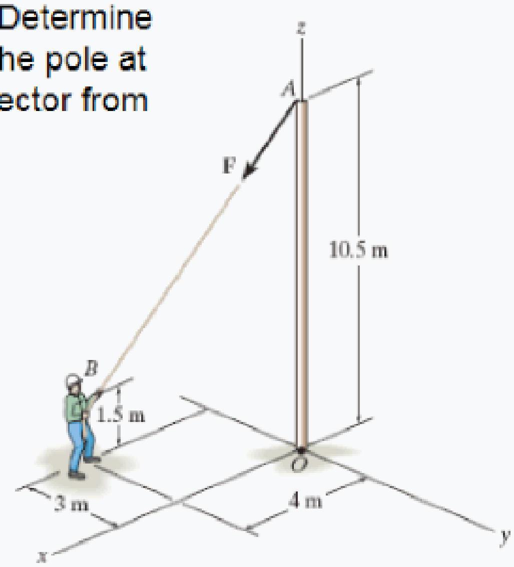

Figure P2.3: Problem 2.3

Problem 2.4

Torque to Loosen a Lug Nut

The lug nut on the automobile wheel is to be removed using the wrench and the vertical force of F = 30 N. If the torque required to loosen the lug nut is 14 N-m *around the x-axis* is this force sufficient? If the 30 N force can be applied anywhere on the wrench, will it be possible to loosen the lug nut?

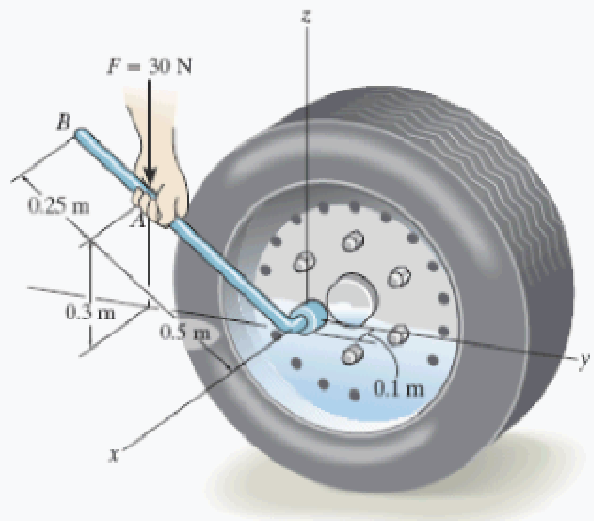

Figure P2.4: Problem 2.4

Problem 2.5

Equivalent Force and Couple Moment

Replace the loading system acting on the pole by an equivalent resultant force and couple moment at P.

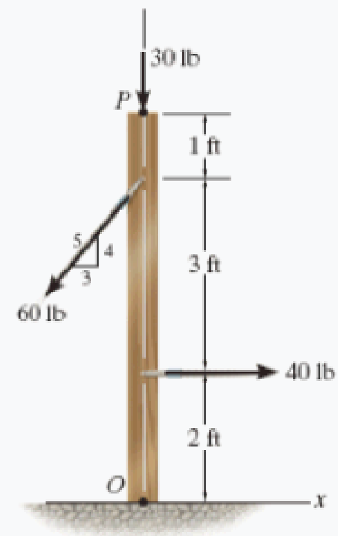

Figure P2.5: Problem 2.5

Problem 2.6

a) Determine the resultant of the forces $\mathbf{F}_1 = F_{1x}\,\imath + F_{1y}\,\jmath + F_{1z}\,\mathbf{k}$, $\mathbf{F}_2 = F_{2x}\,\imath + F_{2y}\,\jmath + F_{2z}\,\mathbf{k}$, and $\mathbf{F}_3 = F_{3x}\,\imath + F_{3y}\,\jmath + F_{3z}\,\mathbf{k}$, which are concurrent at the point $P(x_P,\, y_P,\, z_P)$, where $F_{1x} = 2$, $F_{1y} = 3.5$, $F_{1z} = -3$, $F_{2x} = -1.5$, $F_{2y} = 4.5$, $F_{2z} = -3$, $F_{3x} = 7$, $F_{3y} = -6$, $F_{3z} = 5$, $x_P = 1$, $y_P = 2$, and $z_P = 3$. b) Find the total moment of the given forces about the origin $O(0,\, 0,\, 0)$. The units for the forces are in Newtons and for the coordinates are given in meters.

Solution

```
% input data
F1x = 2; F1y = 3.5; F1z = -3; % N
F2x = -1.5; F2y = 4.5; F2z = -3; % N
F3x = 7; F3y = -6; F3z = 5; % N
xP = 1; yP = 2; zP = 3; % m

% a)
% vector forces F1, F2, F3
F1 = [F1x, F1y, F1z];
F2 = [F2x, F2y, F2z];
F3 = [F3x, F3y, F3z];
% resultant of the forces
R = F1 + F2 + F3;
% R = [7.5, 2, -1] N

% b)
% position vector of P
rP = [xP, yP, zP];
% moment of forces about the origin =
% moment resultant about the origin
% M_O = rP x F1 + rP x F2 + rP x F3 =
% M_O = rP x R
% M_O = [-8, 23.5, -13] N m
```

Problem 2.7

a) Determine the resultant of the three forces shown in Fig. P2.7. The force $\mathbf{F}_1$ acts along the x-axis, the force $\mathbf{F}_2$ acts along the z-axis, and the direction of the force $\mathbf{F}_3$ is given by the line O_3P_3, where $O_3 = O(x_{O_3}, y_{O_3}, z_{O_3})$ and $P_3 = P(x_{P_3}, y_{P_3}, z_{P_3})$. The application point of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ is the origin $O(0, 0, 0)$ of the reference frame as shown in Fig. P2.7. b) Find the total moment of the given forces about the point P_3. Numerical application: $|\mathbf{F}_1| = F_1 = 250$ N, $|\mathbf{F}_2| = F_2 = 300$ N, $|\mathbf{F}_3| = F_3 = 300$ N, $O_3 = O_3(1, 2, 3)$ and $P_3 = P_3(5, 7, 9)$. The coordinates are given in meters.

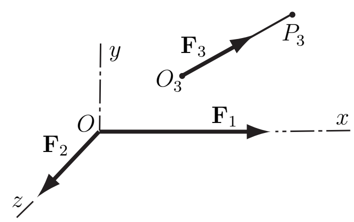

Figure P2.7: Problem 2.7

Solution

```
% input data
F1m = 250; % N
F2m = 300; % N
F3m = 300; % N
xO3 = 1; yO3 = 2; zO3 = 3; % m
xP3 = 5; yP3 = 7; zP3 = 9; % m

% vector force F1
F1 = [F1m, 0, 0];
% vector force F2
F2 = [0, 0, F2m];

% position vector of O3
rO3 = [xO3, yO3, zO3];
% position vector of P3
rP3 = [xP3, yP3, zP3];
% rP3 = [5, 7, 9] m
```

```
% vector O3P3
rO3P3 = rP3 - rO3;
% unit vector of  O3P3
u3 = rO3P3/norm(rO3P3);
% u3 =  [0.456, 0.57, 0.684]

% vector force F3
F3 = F3m*u3;
% F3 = [136.753, 170.941, 205.129] N

% a)
% resultant of the forces
R = F1+F2+F3;
% R = [386.753, 170.941, 505.129] N

% b)
% moment of forces about P3 =
% MP3 = rP3O x (F2 + F3)
% rP3O = -rP3
% MP3 = [-2100, -750, 1750] N m
```

Problem 2.8

Replace the three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, shown in Fig. P2.8, by a resultant force, $\mathbf{R}$, through O and a couple. The force $\mathbf{F}_2$ acts along the x-axis, the force $\mathbf{F}_1$ is parallel to the y-axis, and the force $\mathbf{F}_3$ is parallel to the z-axis. The application point of the forces $\mathbf{F}_2$ is O, the application point of the forces $\mathbf{F}_1$ is B, and the application points of the force $\mathbf{F}_3$ is A. The distance between O and A is d_1 and the distance between A and B is d_2 as shown in Fig. P2.8. Numerical application: $|\mathbf{F}_1| = F_1 = 250$ N, $|\mathbf{F}_2| = F_2 = 300$ N, $|\mathbf{F}_3| = F_3 = 400$ N, $d_1 = 1.5$ m and $d_2 = 2$ m.

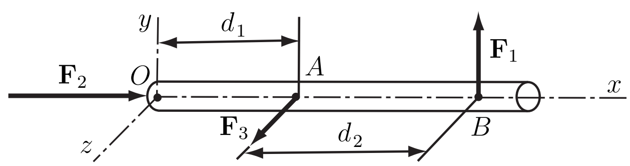

Figure P2.8: Problem P2.8

Solution

```
% input data
F1m = 250; % N
F2m = 300; % N
F3m = 400; % N
d1 = 1.5;  % m
d2 = 2;    % m

% vector force F1
F1 = [0, F1m, 0];
% vector force F2
F2 = [F2m, 0, 0];
% vector force F3
F3 = [0, 0, F3m];
% F1 = [0, 250, 0] N
% F2 = [300, 0, 0] N
% F3 = [0, 0, 400] N

% position vector of A
rA = [d1, 0, 0];
```

```
% position vector of B
rB = [d1+d2, 0, 0];

% resultant of the forces
R = F1+F2+F3;
% R = [300, 250, 400] N

% moment of forces about O
% MO = rA x F3 + rB x F1
% MO = [0, -600, 875] N m
```

Problem 2.9

Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem a) using a Cartesian vector approach and b) using a scalar approach.

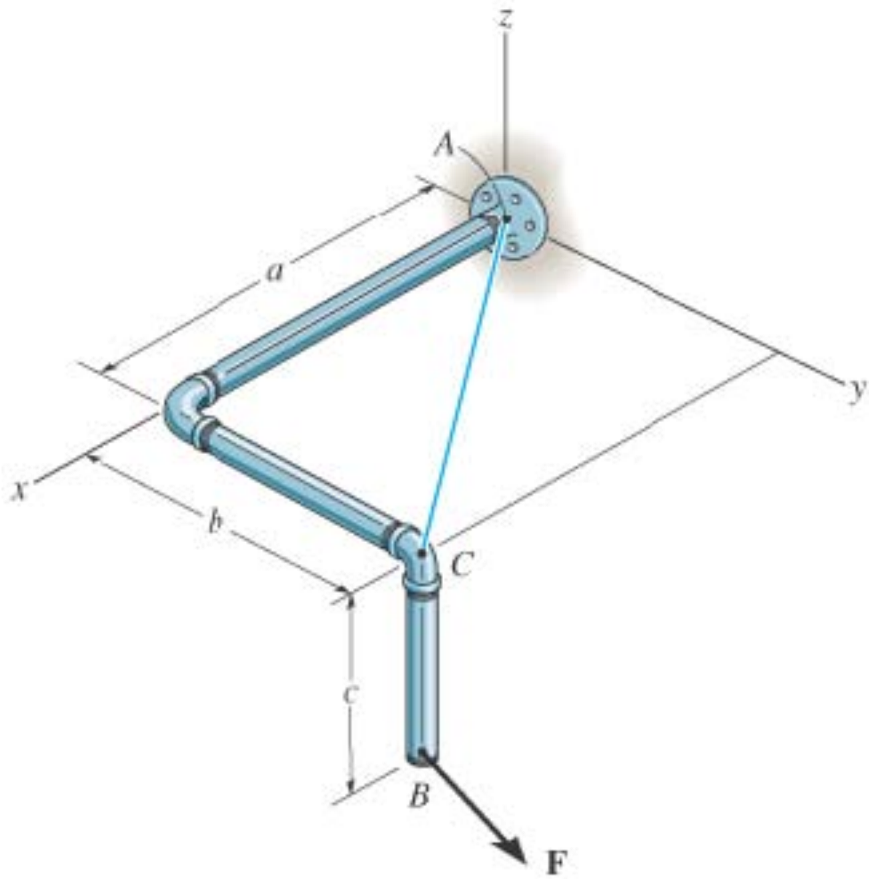

Figure P2.9: Problem 2.9

Solution

```
% input data
Fx = 4;  % lb
Fy = 12; % lb
Fz = -3; % lb
a = 4; % ft
```

```
b = 3; % ft
c = 2; % ft

% a) vector approach
% position vector AB
rAB = [a, b, -c];
% force F at B
F = [Fx, Fy, Fz];
% unit vectors
i = [1, 0, 0];
j = [0, 1, 0];
k = [0, 0, 1];

% moment of F about A: M = rAB x F
% moment of F about x-axis: Mx = (rAB x F).i
% moment of F about y-axis: Mx = (rAB x F).j
% moment of F about z-axis: Mx = (rAB x F).k

% Mx = 15 lb ft
% My = 4 lb ft
% Mz = 36 lb ft

% b) scalar approach
Mxs =  Fy*c + Fz*b;
Mys = -Fx*c - Fz*a;
Mzs = -Fx*b + Fy*a;

% Mxs = 15 lb ft
% Mys = 4 lb ft
% Mzs = 36 lb ft
```

Problem 2.10

Determine the moment of the force **F** about an axis extending between A and C. Express the result as a Cartesian vector.

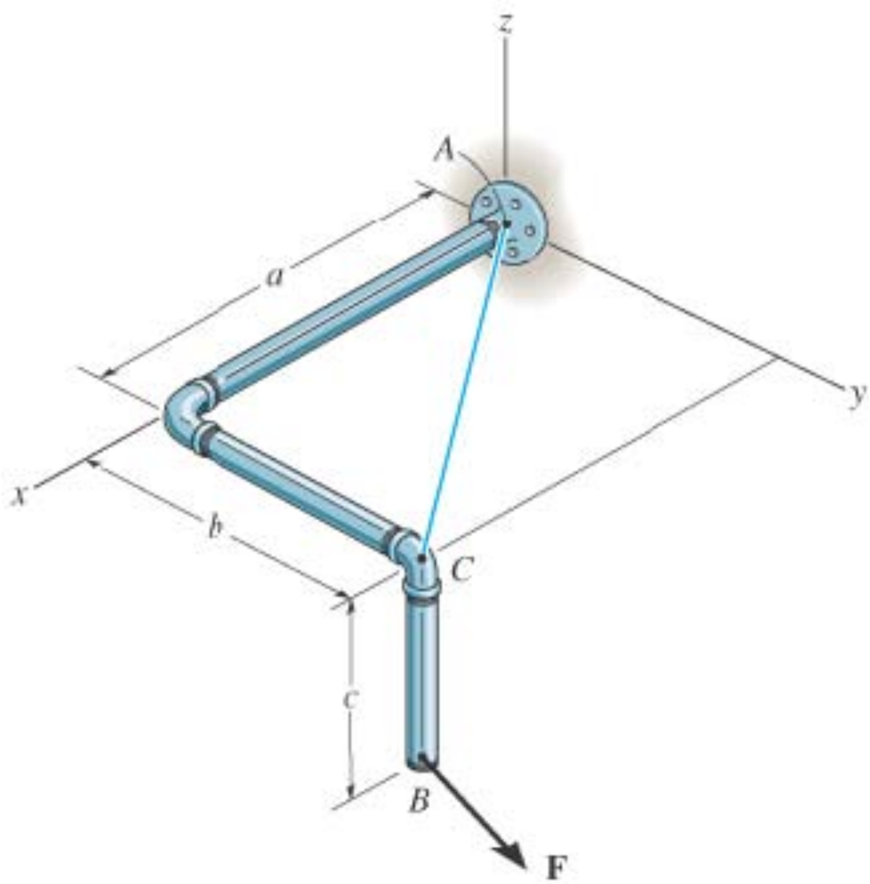

Figure P2.10: Problem 2.10

Solution

```
% input data
Fx = 4;  % lb
Fy = 12; % lb
Fz = -3; % lb
a = 4; % ft
b = 3; % ft
```

```
c = 2; % ft

% force F at B
F = [Fx, Fy, Fz];
% position vector AB
rAB = [a, b, -c];
% position vector AC
rAC = [a, b, 0];
% unit vector AC: uAC = rAC/|rAC|

% moment of F about AC-axis
% M_AC=[(rAB x F).uAC].uAC

% M_AC = [11.52, 8.64, 0] lb ft
```