MECH 2110- STATICS AND DYNAMICS

LAB MANUAL

DEPARTMENT OF MECHANICAL ENGINEERING

AUBURN UNIVERSITY

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Index

1. Introduction to vectors - Lab Instructor’s Guide

The **objective of this lab** is to familiarize the students with the basics of vector algebra. There will be no hands-on laboratory requirements for this assignment. The lab instructor is expected to give an overview of the following concepts and then encourage students to work some problems based on the concepts below. Since this would be the first lab meeting, the instructors are encouraged to discuss the lab protocols that will be followed through the semester.

1. Triangle/parallelogram law of vectors

2. Resolving vectors into X and Y components

3. Expressing vectors in polar and Cartesian coordinates

4. Vector addition, subtraction and resultants

5. Vector dot product and cross product

6. The Laws of Sine and Cosine

Lab Instructor’s duties

1. Prepare a set of problems and solutions based on the concepts above.

2. Review the above concepts in class

3. Divide students into groups

4. Assign groups with different problems.

5. Encourage them to discuss among the group. Guide them when necessary but do not give the final answer right away.

6. Collect students’ work and grade them.

1. Introduction to vectors - Student's Guide

The objective of this lab is to familiarize you with the basics of vector algebra. There will be no hands-on laboratory requirements for this assignment.

1. Pay attention to the instructor's lecture and take notes. You need them to complete this lab assignment.
2. After the lecture, the instructor will assign some problems. Work these problems with your group.
3. Actively participate in a discussion with your team and try to give your inputs to the group as and when possible.
4. Return your work in the format prescribed by the lab instructor.

2. Concurrent Forces - Lab Instructor's Guide

This lab uses a force table to demonstrate equilibrium of concurrent forces contained in a single plane. Three strings are attached to a ring. The ring is positioned horizontally in the center of a table with a fixed vertical rod passing through the ring. The three strings pass over pulleys attached to the edge of the table. The strings are attached to freely hanging weights. By appropriately choosing the weights and positioning the pulleys, the ring can be made to assume an equilibrium position where it is not touching the table or the center rod.

The **purpose of the lab** is for the students to determine appropriate weights and positions subject to certain constraints (specification of some weights and or positions is required). Of particular importance is the use of appropriate procedures by the students. Students should use the 4steps procedure: define mechanical systems, draw free body diagrams, sum forces and solve the equations in order to determine appropriate weight values and pulley positions. Also of importance is the examination of assumptions made in the solution process.

The students will be divided into groups of two (alphabetically by first name, one group may contain three students if there are an odd number of students). Each group will be given the task of determining appropriate weight values and pulley positions subject to the specification of certain weight values and/or pulley positions. They must analytically determine the required values as well as experimentally. The equations the students should develop and solution procedures are provided below. The required values and supporting documentation will be submitted to the laboratory instructor. After a successful solution has been found, the lab instructor will discuss briefly with the group a number of additional issues (see below) and ask the students to revise their submission to address these issues. Upon submission of an approved report, the group members should be encouraged to work with groups that appear to be having difficulty.

Instructor Duties

1. Prepare lab. Set up force table with equal weights attached to each of the three pulleys and the three pulleys spaced evenly around the circumference of the table (one pulley at 0 degrees). Ensure that the force table is level. Make sure that appropriate scales are available and operating. Make sure that weights are available and document the number and size of the available weights.

2. Establish values for each of the lab groups (you may choose to modify the values for known weights and pulley positions from those given in the assignment below). The following data is given: m_1 = 100 gm and θ_1=0°. Determine the solution for each of the lab groups (ensuring that all problems have practical solutions!). Solutions for the assignment given below are as follows:
   G1: m_2=61 gm, m_3=121 gm.
   G2: θ_2=145°, θ_3=283°
   G3: m_2=198 gm, θ_3=273°
   G4: m_3=35 gm, θ_3=268°
   G5: m_2=2 gm, m_3=82 gm
   G6: θ_2=133°, θ_3=262°
   G7: m_2=233 gm, θ_3=304°

G8: m_3=5 gm, θ_3=270 °.

3. Assign students to groups of two (alphabetically by first name) and sequentially assign integers beginning at 1 to the groups.

4. Provide each group with a copy of the lab assignment (possibly modified for your session).

5. Ask for any questions on the assignment from the class. Discuss these questions with the entire class. Caution the students not to discuss the assignment with students outside of their lab section.

6. Instruct the groups to begin working. Assist as required. Instruct the students to use the four steps procedure and explain in detail the FBD. Assist if necessary with a strategy for solving the equations in order to get an expression for the desired unknown.

7. When a group has developed a proposed set of weight values and pulley positions assist them in testing their proposal. Note that at this point they need not have fully completed the documentation portion of the lab. To test their proposal, first remove the hangers from the pulleys and the weights from the hangers. Then position the pulleys as necessary. Shift the strings on the center ring as necessitated by the pulley positions. Add the required weights to the hangers. Attach the hangers to the pulleys. Further adjust the strings on the center ring if necessary. If the desired equilibrium configuration is obtained, instruct the group to refine their supporting documentation (and note their finish rank). If the desired equilibrium configuration is not obtained, instruct the group to reconsider their proposal (and deduct three points).

8. When a group has successfully tested their proposal and submitted supporting documentation, review their report. Ensure that they have appropriately defined mechanical systems, drawn free body diagrams, and summed forces. Many groups will consider only the ring and will show the tension in a string to be equal to the weight attached to that string. Ask them to justify this assumption (note this will involve analysis of the string and/or the pulley). You should ensure that eventually their report includes the following analyses (items a, b, and c below should use the four step procedure; define mechanical system, draw free body diagram, sum forces, set equal to zero and solve) :

    a. Weight and small string segment to show that the string tension below the pulley is equal to the weight (can be combined with b).

    b. Pulley and small string segment wrapped around the portion of the pulley to show (using moments) that the tension in the string is the same on either side of the pulley (note that this should bring out the assumption of a freely turning pulley).
    c. Ring and three short segments of string.

    d. A discussion of why forces in the string can be assumed to be tension along the string direction (no shear force or bending moment). This discussion should focus on the fact that while a string resists stretching, it provides virtually no resistance to compression, shear, or bending (optional).

e. Identification of assumptions (neglect weight of ring, neglect weight of string, frictionless pulley, and so on).

9. When a group has successfully completed their work, encourage them to assist other groups (see grading policy).

10. When all groups have completed their work, ensure that no weights are missing. Report any missing weights or damaged equipment to the supervising faculty member.

Equations and Solution Procedure (note that some of the students may need some assistance with some of the algebra).

In the following the tensions in the three strings are denoted T_1, T_2, and T_3. Each of these tensions is equal to the supported mass multiplied by the acceleration of gravity. Note that the supported mass includes the 50 gram hanger and any added mass. The string with tension T_1 is presumed to be positioned at the zero angles. The angle between this string and the string with tension T_2 will be denoted θ_2. The angle between the string with tension T_1 and the string with tension T_3 will be denoted θ_3 and will be measured in the same direction as θ_2. Thus θ_2 will be between 0° and 180° while θ_3 will be between 180° and 360°. As the three tensions are the dominant forces acting on the ring in the desired equilibrium position, they must satisfy the following equations:

$T_1 + T_2 \cos(\theta_2) + T_3 \cos(\theta_3) = 0$
$T_2 \sin(\theta_2) + T_3 \sin(\theta_3) = 0$

The approach to solving the above equations depends on what is known. Each of the possible cases will be discussed below.

1. T_2 and T_3 unknown.

Multiply the top equation by $\sin(\theta_3)$ and the bottom equation by $\cos(q_3)$ and subtract the two equations. This yields:

$T_1 \sin(\theta_3) + T_2 \{ \cos(\theta_2) \sin(\theta_3) – \sin(\theta_2) \cos(\theta_3) \} = 0$

Using the double angle formula, one obtains:

$T_1 \sin(\theta_3) + T_2 \sin(\theta_3 - \theta_2) = 0$ or $\mathbf{T_2 = -T_1 \sin(\theta_3) / \sin(\theta_3 - \theta_2)}$

To solve for T_3, multiply the top equation by $\sin(\theta_2)$ and the bottom equation by $\cos(\theta_2)$ and subtract.

$T_1 \sin(\theta_2) + T_3 \{ \cos(\theta_3) \sin(\theta_2) – \sin(\theta_3) \cos(\theta_2) \} = 0$

Using the double angle formula, one obtains:

$T_1 \sin(\theta_2) – T_3 \sin(\theta_3 - \theta_2) = 0$ or $\mathbf{T_3 = T_1 \sin(\theta_2) / \sin(\theta_3 - \theta_2)}$

2. T_3 and θ_3 unknown (also covers T_2 and θ_2 unknown).

Rewrite the equations as:

$T_3 \cos(\theta_3) = - T_1 – T_2 \cos(\theta_2)$
$T_3 \sin(\theta_3) = - T_2 \sin(\theta_2)$

Squaring the two equations and adding:

$T_3^2 = T_1^2 + T_2^2 + 2 T_1 T_2 \cos(\theta_2)$

$$\mathbf{T_3 = \{ T_1^2 + T_2^2 + 2 T_1 T_2 \cos(\theta_2) \}^{1/2}}$$

With T_3 known, we can determine both the sine and the cosine of θ_3, and can thus determine the angle 3:

$$\mathbf{\sin(\theta_3) = -T_2 \sin(\theta_2)/T_3}$$
$$\mathbf{\cos(\theta_3) = -\{ T_1 + T_2 \cos(\theta_2) \} / T_3}$$

3. T_2 and θ_3 unknown (also covers T_3 and θ_2 unknown).

Rewrite the equations as:

$T_3 \cos(\theta_3) = - T_1 – T_2 \cos(\theta_2)$
$T_3 \sin(\theta_3) = - T_2 \sin(\theta_2)$

Squaring the two equations and adding:

$T_3^2 = T_1^2 + T_2^2 + 2 T_1 T_2 \cos(\theta_2)$

$T_2^2 + 2 T_1 \cos(\theta_2) T_2 + T_1^2 – T_3^2 = 0$

This is a quadratic equation in T_2. Thus:

$$\mathbf{T_2 = -T_1 \cos(\theta_2) +\{ T_1^2 \cos^2(\theta_2) + T_3^2 – T_1^2 \}^{1/2}}$$

Knowing T_2, we can solve for the angle 3:

$$\mathbf{\sin(\theta_3) = -T_2 \sin(\theta_2)/T_3}$$
$$\mathbf{\cos(\theta_3) = -\{ T_1 + T_2 \cos(\theta_2) \} / T_3}$$

4. θ_2 and θ_3 unknown.

Rewrite the equations as:

$T_3 \cos(\theta_3) = - T_1 - T_2 \cos(\theta_2)$
$T_3 \sin(\theta_3) = - T_2 \sin(\theta_2)$

Squaring and adding yields:

$T_3^2 = T_1^2 + T_2^2 + 2\, T_1\, T_2 \cos(\theta_2)$

Thus

$$\mathbf{\cos(\theta_2) = \{ T_3^2 - T_1^2 - T_2^2 \} / \{ 2\, T_1\, T_2 \}}$$

Knowing θ_2, we can obtain θ_3 from:

$$\mathbf{\sin(\theta_3) = -T_2 \sin(\theta_2)/T_3}$$
$$\mathbf{\cos(\theta_3) = -\{ T_1 + T_2 \cos(\theta_2) \} / T_3}$$

2. **Concurrent Forces – Lab Assignment**

This lab uses pulleys to investigate equilibrium. You are to design a system of pulleys that will enable two prescribed masses to mutually support one another. The definition of mutually supporting includes the following elements:

1. Each mass is in equilibrium and is supported by only a pulley (or by a single string coming from a pulley).

2. Either mass should be capable of being displaced without significant resistance..

3. Displacing one mass should cause the other mass to be displaced.

4. After displacing the masses and releasing them, both masses should remain in equilibrium after release.

Two examples of pulley systems with mutually supporting masses should be available for your inspection. The instructor will provide you with the mass values that you should work with (+- 2 grams). The recommended process in attacking this lab is as follows:

1. Work individually, apply the four step procedure (perhaps several times) to reveal why the example pulley systems are mutually supporting.

2. Come together as a team and discuss your work from step 1. Verify that you both understand how to analyze a pulley system.

3. Show your work to the instructor for any comments on your approach.

4. Work together to come up with a pulley system concept that you believe might work to enable the two masses specified to be mutually supporting.

5. Individually apply the four step procedure (perhaps several times) to see if your concept will work.

6. Come together as a team and discuss your work from step 5. If you both agree that the proposed pulley system concept is workable, move onto step 7. If the proposed concept is not workable, return to step 4.

7. As a team, prepare a submission including your pulley system design concept and all necessary supporting analysis (probably several applications of the four step procedure). Discuss how you will construct your pulley system from the available supplies.

8. Give your submission to the instructor and ask permission to begin assembly of your pulley system (note that you may want to keep a copy of the submission to assist you in construction). Upon granting of permission, you will have 5 minutes to construct your

pulley system. Upon construction completion, the instructor will verify that your design enables the prescribed masses to provide mutual support.

Grading: 10 points maximum for your submitted analysis (includes clarity and quality of presentation, incorporating effectiveness of use of the four step procedure).
10 points maximum for pulley system performance (1 point deducted for each minute beyond 5 minutes, 2 points deducted for each constructed pulley system that does not enable the specified masses to provide mutual support, 3 points deducted for any substantial difference between your constructed pulley system and your submitted pulley system design).

3. Pulley Statics - Lab Instructor's Guide

During this lab, the students should be introduced to some knowledge about the pulleys (pulley types, usage) and the notion of mechanical advantage.

The **purpose of this lab** is dual: the students will have to find a mass in a given system with two masses and they will also have to design a system that provide a maximum mechanical advantage and one that have a minimum mechanical advantage.

Preparation: Make sure the two "demonstration" mutually supporting pulley systems are set up. Currently one "demo" system enables masses of 70 g and 176 g to be mutually supporting. The other "demo" system enables masses of 252 g and 145 g to be mutually supporting. Verify that you can apply the four step procedure to understand both "demos". (Four step procedure = define mechanical system, draw free body diagram, write equations (sum forces and moments), solve; in this case typical mechanical systems will include the hanging masses, sections of string, pulleys, or combinations of these elements. Moment sums will normally be done about pulley centers.

For each lab you should prepare 10 pairs of prescribed masses (ready to handle up to 20 people). To prepare these pairs please note the following. There are a number of individual pulleys with masses ranging from 32 to 38 g (you should measure these masses and mark each pulley accordingly). The pulleys can be connected in one of two configurations, either with the string wrapped around the top of the pulley, or with the string wrapped around the bottom of the pulley (both configurations exist in the "demo" setups). If the string is wrapped around the top of the pulley, then the force supporting the pulley is equal to the pulley weight plus twice the tension on either side of the pulley. If the string is wrapped around the bottom of the pulley, then twice the tension in the string holding the pulley up is equal to the sum of the pulley weight and any force supported by the lower hook on the pulley. By combining these two pulley configurations in various ways, you can obtain a variety of mutually supporting mass pairs. For example, consider using three pulleys. One pulley will be hanging from the rack and have a string over its top. One end of this string will be connected to a large mass, the other end to a pulley of mass 36 gm. Over the top of this second pulley will be a second string. One end of this string will be fixed (using one of the provided weights). The other end will be attached to a third pulley of mass 35 gm. Over the top of this third pulley will be a third string. One end of this string will be fixed (using one of the provided weights). The other end will be attached to hanger and mass combination of total mass 56 g. This enables us to determine that the tension in the third string is equal to the weight of 56 g. Thus the tension in the second string is equal to twice the weight of 56 g plus the weight of 35 g, which is the weight of 147 g. The tension in the first string must be twice the weight of 147 g plus the weight of 36 g, which is the weight of 230 g. The "large" mass must be 230 g for the two masses to be mutually supporting. You should generate 10 such combinations and make sure that all combinations are feasible based upon the available pulleys, the available hangers, and the available masses (suitably matched with hangers, see below). You should try to ensure that the 10 assignments require different pulley system concepts (in terms of number of pulleys on either side of the main pulley, and string position on the different pulleys).

Note that as you and the students construct pulley systems, there should be no need to "force" masses onto the support holders (note that when I went to prepare the lab for this week, I noticed that several of the masses had to be "forced" off of the support). There are an adequate number of masses that fit each support – find them! They will slip easily on and off the support.

During the lab: Distribute the lab handout to the students. Assign the students to teams of two, attempting to ensure they are working with someone that they have not previously worked with. Show the students the two "demo" systems. Indicate the values of the mutually supporting masses. Demonstrate how the masses can be easily displaced, how displacing one mass does indeed displace the other, and how they remain in equilibrium after displacement. Address any questions or concerns. Distribute prescribed mass values to each team (each team should get a different pair of prescribed masses). Circulate among the teams while you wait for them to complete their analysis of one of the "demo" systems.

When the students show you their work on the "demo" systems (they need analyze only one of the two), look for the following:

- Each free body diagram should be clearly labeled with the mechanical system of interest.
- The free body diagram should only include those entities listed in the mechanical system definition.
- All external forces should be shown on the free body diagram (even those not necessarily of interest such as an attachment of a pulley to the supporting rack).
- Make sure they sure weights, not masses, on their free body diagrams (W = M g).
- Verify that they have summed moments to verify that the tension on both of sides a pulley is the same.
- Verify that they have been able to make a good estimate of the mass of the relevant pulley from their analysis.
- Make any other recommendations you believe are appropriate.

Continue to circulate among the teams after all teams have completed their "demo" system analysis and are working on the pulley system design for their pair of prescribed masses.

After receiving a pulley system design submission from a team; verify that the "rack" is clear of any previous work, make sure that all of the pulleys are readily available (labeled with their masses) and that the support hangers are available and are accompanied by masses that fit). If all is set, indicate to the team that they can begin construction and begin timing. Upon completion of construction, inspect their system to verify that it meets the mutually supporting requirement for the prescribed masses. Review their submitted report for clarity, quality, and consistency with the constructed system. Assign their grade as indicated below. Upon receiving their grades, students may either leave or watch the performance of other groups. After giving the team their grade, you should ready the area for the next team.

After the lab: If you are not the last lab of the week, return everything to its original condition. If you are the last lab of the week, put all of the materials away under the plastic sheet at the rear of the room (note that when I went to prepare the lab for this week, the force table had not been put away).

3. Pulley Statics – Lab Assignment

This lab uses pulleys to investigate equilibrium. You are to design a system of pulleys that will enable two prescribed masses to mutually support one another. The definition of mutually supporting includes the following elements:

1. Each mass is in equilibrium and is supported by only a pulley (or by a single string coming from a pulley).
2. Either mass should be capable of being displaced without significant resistance.
3. Displacing one mass should cause the other mass to be displaced.
4. After displacing the masses and releasing them, both masses should remain in equilibrium after release.

Two examples of pulley systems with mutually supporting masses should be available for your inspection. The instructor will provide you with the mass values that you should work with (+- 2 grams). The recommended process in attacking this lab is as follows:

1. Working individually, apply the four step procedure (perhaps several times) to reveal why the example pulley systems are mutually supporting.
2. Come together as a team and discuss your work from step 1. Verify that you both understand how to analyze a pulley system.
3. Show your work to the instructor for any comments on your approach.
4. Work together to come up with a pulley system concept that you believe might work to enable the two masses specified to be mutually supporting.
5. Individually apply the four step procedure (perhaps several times) to see if your concept will work.
6. Come together as a team and discuss your work from step 5. If you both agree that the proposed pulley system concept is workable, move onto step 7. If the proposed concept is not workable, return to step 4.
7. As a team, prepare a submission including your pulley system design concept and all necessary supporting analysis (probably several applications of the four step procedure). Discuss how you will construct your pulley system from the available supplies.
8. Give your submission to the instructor and ask permission to begin assembly of your pulley system (note that you may want to keep a copy of the submission to assist you in construction). Upon granting of permission, you will have 5 minutes to construct your pulley system. Upon construction completion, the instructor will verify that your design enables the prescribed masses to provide mutual support.

Grading: 10 points maximum for your submitted analysis (includes clarity and quality of presentation, incorporating effectiveness of use of the four step procedure). 10 points maximum for pulley system performance (1 point deducted for each minute beyond 5 minutes, 2 points deducted for each constructed pulley system that does not enable the specified masses to provide mutual support, 3 points deducted for any substantial difference between your constructed pulley system and your submitted pulley system design).

4. Free Body Diagram - Lab Instructor’s Guide

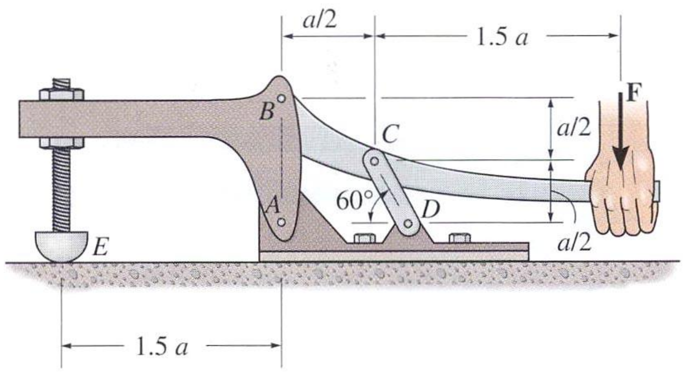
The **objective of this lab** is to familiarize the students with Free Body Diagrams. There will be no hands-on laboratory requirements for this assignment. The lab instructor is expected to give an overview of the concepts required to construct free body diagrams. He/she may start out with simple examples like a block on an inclined plane and trusses. Concepts of moments can also be explained in this class, this would give students a head start for the lab assignment on moments immediately following. Solving representative problems from the textbook to explain the concepts involved in free body diagram and moments, is strongly encouraged (bellow are a couple of example problems that can be used).

Lab Instructor’s duties

1. Prepare a set of problems and solutions based on the concepts above (refer to the next page for sample problems).
2. Review the above concepts in class
3. Divide students into groups
4. Assign groups with different problems and systems (vice-grips, cutting shears, etc.).
5. Encourage them to discuss among the group. Guide them when necessary but do not give the final answer right away.
6. Collect students’ work and grade them.

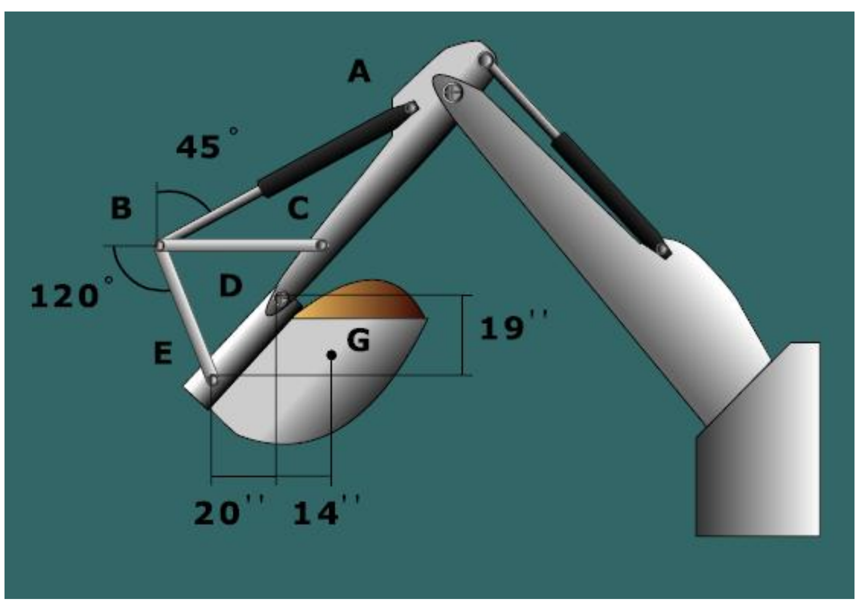
Problem 1:

The toggle clamp is subjected to a force F at the handle. Determine the vertical clamping force acting at E.



Problem 2:

The bucket of the backhoe with its load has a weight of 1200 lb and a center of gravity at G. Determine the forces of the hydraulic cylinder AB and in links BE and BC in order to hold the load in the position shown. The bucket is pinned at D.



4. Free Body Diagram - Lab Assignment

The objective of this lab is to familiarize you with the basics of free body diagram. Some concepts on moments will also be covered. There will be no hands-on laboratory requirements for this assignment.

1. Pay attention to the instructor's lecture and take notes. You need them to complete this lab assignment.
2. After the lecture, the instructor will assign some problems. Work these problems with your group.
3. Actively participate in a discussion with your team and try to give your inputs to the group as and when possible.
4. Return your work in the format prescribed by the lab instructor.

5: Determination of reaction forces using moments- Instructor's Guide

In this experiment, students will compute the reaction forces of a simply supported beam that is loaded by suspended weights. The apparatus consists of an aluminum base (1), on which two columns are mounted that serve as supports (see fig. 5.1). A rigid beam (3) is supported by the columns. The beam can be stressed through the use of weight mounts (4) and weights (5). The resulting bearing forces are displayed by the dynamometers (7) which are integrated in the columns. The columns can be adjusted by loosening the locking lever (8). It is also possible to shift the sites of weights on the beam by moving the slides (6) from which they are suspended. The position of the point where force is applied can be read on a millimeter scale (9).

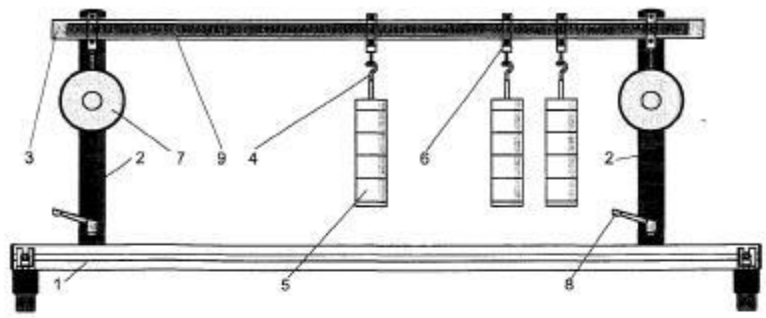


Figure 5.1

Fundamental Principles

When forces are applied to a beam as in Fig. 5.1, bearing/reaction forces occur in the support A and B. These bearing forces can be determined by "cutting away" the beam and setting up the equations for the equilibrium forces and moments. Equilibrium of forces must be present in all directions of the co-ordinate system. If the dynamometer readings are corrected to zero before loading weights by adjusting the dial, the mass of the simply supported beam can be ignored in the moments equations and equilibrium equations.

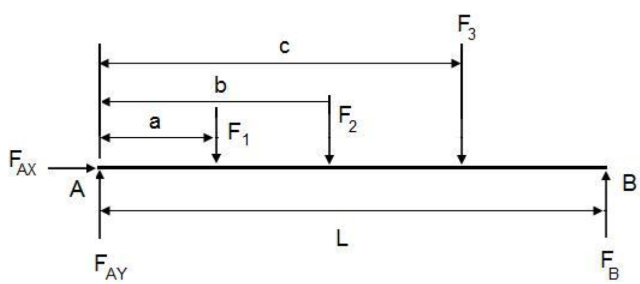


Figure 5.2

The following equation can be applied to the vertical forces, in accordance with Fig. 5.2:

$$F_{Ay} + F_B - F_1 - F_2 - F_3 = 0 \quad (1)$$

The following applies to the horizontal forces:

$$F_{Ax} = 0 \quad (2)$$

For the moments around the support A:

$$F_1.a + F_2.b + F_3.c - F_B.L = 0 \quad (3)$$

All unknown reaction forces can be determined with these equations:

$$F_B = \frac{F_1.a + F_2.b + F_3.c}{L} \quad (4)$$

$$F_{Ay} = F_1 + F_2 + F_3 - F_B \quad (5)$$

Instructor Duties:

1. Prepare lab. Ensure that hooks and weights are available and ready to use.

2. Go over definition of moment in a cross product form. Talk about different types of beams and loads and prepare simple examples of loaded beams that students can sum the moments for.

3. Prepare different sets of loading conditions and have solutions ready for each of these sets.

4. Assign students to groups of two. Make sure every group has at least one student who has completed the differential equations course.

5. Provide each group with a copy of the lab assignment. Ask for any questions on the assignment from the class. Discuss these questions with the entire class.

6. Assign each group with a setting of loading conditions by specifying the different forces acting on the beam and their distance from one of the ends of the beam.

7. Instruct the groups to begin working. Assist as required (do not provide them with the analysis, rather help them discover it on their own to the extent possible). Encourage them to write moments equation to find the reaction forces.

8. After checking their calculations, instruct the students to load the beam as assigned and measure the values in the dynamometer.

9. With the experimental data obtained, ask the class to determine the error in the dynamometer.

10. Collect students' lab reports.

11. Evaluate students' work.

5. Determination of reaction forces using moments – Lab Assignment

In this lab you will find the reaction forces acting a stress loaded simply supported beam mathematically and experimentally. You are provided with a simply supported beam. You will load the beam using weights at specified locations (magnitude of weight and their point of action will be specified by the instructor) and note the values of reaction forces from the dynamometers. You will also write moments and equilibrium equations and mathematically compute the reaction forces.

1. Closely observe the experimental set up and draw a free body diagram of the simply supported beam.
2. By resolving forces along X and Y axis and using a moments equation, derive expressions for the unknown reaction forces.
3. Your lab instructor will assign you a set of weights and their points of action on the beam. Using the above expressions find the unknown reaction forces.
4. Show your work to your instructor before starting your experiment.
5. Correct for zero error by rotating the dynamometer when no weights are added to the beam.
6. Load the beam as specified in your problem statement.
7. Note the values of reaction forces from the dynamometer reading.
8. Compare your experimental and mathematical results and find the percentage error in the dynamometers.
9. Discuss the sources of error.
10. Submit a report of your findings and discussions to the instructor in the prescribed format.

6. Determination of Coefficient of Friction – Instructor's Guide

The **objective of this lab** is to find the coefficient of static friction between 2 surfaces in contact. An inclined ramp and block are used as the surfaces in contact.

Go over some the definition of coefficient of friction, the difference between kinetic coefficient of friction and static coefficient of friction.

Make sure the inclined ramp and the two blocks (one with a glass surface) are available and clean. Make sure that the scales are available, clean, and operational. Make sure the pulley rack is available along with the required pulleys and string. Note that the normal setup of the pulley rack would use two pulleys (see part 2 of the lab assignment below). One pulley will be supported from the top bar. The string will over this pulley. One end of the string will be used to support the mass. The string will run over the top pulley and down to a pulley attached to the bottom of the rack. The string will make a quarter turn under this pulley and run to the object of interest. Make sure you can easily get this setup for the students.

You should do a little testing on your own to get rough ideas of the static coefficients of friction for the various surface combinations. You should try the pulley setup so that you have some idea just how predictable the behavior of the system will be.

Divide the students into teams of two and give them the assignment. Tell them they will need to figure out just how they can determine the static coefficient of friction using the inclined ramp. Indicate that for part 1 of the lab, they must determine coefficients of friction for the metal-ramp pair, the wood-ramp pair, and felt-ramp pair. Ask for questions. Indicate that they should begin work. Circulate and assist as needed. Grade their work as indicated below. They should meet with you after they have determined the coefficient of friction for each of the three surfaces. They should share their predictions with you for part 2 before performing the actual tests. You should monitor the testing process and assign grades as indicated below.

6. Determination of Coefficient of Static Friction – Lab Assignment

1. Use the inclined ramp to determine the static coefficient of friction between the ramp and the various surfaces available. Support your determination of the static coefficient of friction with an appropriate analysis. Investigate whether or not the static coefficient of friction depends upon the relative orientation of the two surfaces (rotation about axis normal to the contacting surfaces). Share your analysis and results with the instructor (5 points).
2. Consider a situation in which the ramp would be in a horizontal position, the available pulleys, pulley rack, and string, would be used so that a string tension could apply a horizontal force to a specified object resting on the ramp. Without any additional physical testing and based upon your results from step 1, predict the largest mass that can be supported by the pulley system without the block beginning to move. Based upon your results from step 1, predict the smallest mass that can be supported by the pulley system that will cause the block to begin moving. Share the analysis upon which your predictions were based with the instructor. Test your predictions (see grading procedures below). Note that you only get one chance!

   Grade: 5 points for the analysis supporting your predictions.
   2 points if your predicted "largest" mass does not move the object.
   2 points if your predicted "smallest" mass does move the object.
   Calculate the ratio: r = {largest-smallest}/largest
   6(1-r) points (round up to next integer).

7. Shape Properties – Instructor's guide

Review the lab assignment to be given to the students. Do not give these pages to the students. They need to work through this on their own. Verify that you understand each element of the solutions given below.

The **objective of this lab** is for the students to find the centroids of a hanging shape and to calculate the moments of inertia for a set shape.

During lab, first distribute the figure sheet to each student and assign hanging shape (semi-circle, con section or irregular shape) for each group of students. For the hanging shape, the students need to find the centroids in two ways: first by hanging the shape, and measure the centroid with respect to one corner of the shape, and second, by calculating it using the tabular method. They will have to compute the error between their two results. Answer any questions they may have. After resolving their questions, distribute the assignment sheet to each student. Score their work as they submit it to you. Make sure they have the limits indicated, and have correctly and precisely evaluated all integrals. Make sure they include the intermediate step that shows the integral evaluated over the interval of integration. Make them work hard, neatly, and do it properly. Their results should essentially be as follows:

$$A = \int dA$$

$$A = \int_0^h b \cdot dY = b \cdot Y \Big|_0^h = b \cdot h$$

$$A = \int_0^h b \cdot \frac{Y}{h} dY = \frac{b}{h} \frac{Y^2}{2} \Big|_0^h = \frac{b \cdot h}{2}$$

$$A = \int_{-r}^{r} 2\sqrt{r^2 - Y^2} \cdot dY$$

$$Y = r \cdot \sin(\theta)$$

$$dY = r \cdot \cos(\theta) \cdot d\theta$$

$$\sqrt{r^2 - Y^2} = r \cdot \cos(\theta)$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2 \cdot r^2 \cdot \cos^2(\theta) \cdot d\theta$$

$$\cos^2(\theta) = \frac{1}{2} + \frac{\cos(2\theta)}{2}$$

$$A = r^2 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + \cos(2\theta)) d\theta$$

$$A = r^2 \left[\theta + \frac{1}{2} \sin(2\theta) \right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$$

$$A = r^2 \cdot \pi$$

Note that the area integration for the circle is more challenging. Brush up on double angle relationships if necessary.

$$A = \int_0^h b \sqrt{\frac{Y}{h}} \cdot dY = \frac{b}{\sqrt{h}} \frac{2}{3} Y^{\frac{3}{2}} \Big|_0^h = \frac{2}{3} b \cdot h$$

End of first portion.

$$M_X = \int Y \cdot dA$$

$$M_X = \int_0^h Y \cdot b \cdot dY = b\frac{Y^2}{2}\Big|_0^h = b\frac{h^2}{2}$$

$$M_X = \int_0^h Y \cdot b \cdot \frac{Y}{h} dY = \frac{b}{h}\frac{Y^3}{3}\Big|_0^h = b\frac{h^2}{3}$$

$$M_X = \int_{-r}^{r} Y \cdot 2\sqrt{r^2 - Y^2} \cdot dY$$

$$Y = r \cdot \sin(\theta)$$

$$M_X = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} r \cdot \sin(\theta) \cdot 2 \cdot r \cdot \cos(\theta) \cdot r \cdot \cos(\theta) \cdot d\theta$$

$$M_X = 2 \cdot r^3 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2(\theta) \cdot \sin(\theta) \cdot d\theta$$

$$M_X = 2 \cdot r^3 \left[-\frac{1}{3}\cos^3(\theta) \right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = 0$$

$$M_X = \int_0^h Y \cdot b \cdot \sqrt{\frac{Y}{h}} \cdot dY = \frac{b}{\sqrt{h}}\frac{2}{5}Y^{\frac{5}{2}}\Big|_0^h = \frac{2}{5}b \cdot h^2$$

End of second portion.

$$Y_C = \frac{M_X}{A}$$

$$Y_C = \frac{\frac{1}{2}b \cdot h^2}{b \cdot h} = \frac{h}{2}$$

$$Y_C = \frac{\frac{1}{3}b \cdot h^2}{\frac{1}{2}b \cdot h} = \frac{2}{3}h$$

$$Y_C = \frac{0}{\pi \cdot r^2} = 0$$

$$Y_C = \frac{\frac{2}{5}b \cdot h^2}{\frac{2}{3}b \cdot h} = \frac{3}{5}h$$

End of first phase of third portion.

$$I_X = \int Y^2 \cdot dA$$

$$I_X = \int_0^h Y^2 \cdot b \cdot dY = b\frac{Y^3}{3}\Big|_0^h = \frac{1}{3}b \cdot h^3$$

$$I_X = \int_0^h Y^2 \cdot b \cdot \frac{Y}{h} dY = \frac{b}{h}\frac{Y^4}{4}\Big|_0^h = \frac{1}{4}b \cdot h^3$$

$$I_X = \int_{-r}^{r} Y^2 \cdot 2\sqrt{r^2 - Y^2}\, dY$$

$$I_X = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} r^2 \cdot \sin^2(\theta) \cdot 2 \cdot r \cdot \cos(\theta) \cdot r \cdot \cos(\theta) \cdot d\theta$$

$$I_X = \frac{1}{2}r^4 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^2(2\theta) \cdot d\theta = \frac{1}{4}r^4 \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} [1 - \cos(4\theta)] \cdot d\theta$$

$$I_X = \frac{1}{4}r^4\left[\theta - \frac{\sin(4\theta)}{4}\right]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{4}r^4 \cdot \pi$$

$$I_X = \int_0^h Y^2 \cdot b \cdot \sqrt{\frac{Y}{h}} dY = \frac{b}{\sqrt{h}}\frac{2}{7}Y^{\frac{7}{2}}\Big|_0^h = \frac{2}{7}b \cdot h^3$$

End of second phase of third portion. Their explanations of the differences in area should address the fact that the fourth shape has additional area between the boundaries of triangle and the rectangle. Their explanations of the differences in centroid should address the fact that more of the area of the triangle is nearer the top, while the fourth area is intermediate in terms of how much area is toward the top.

End of third portion.

$$\bar{I}_X = I_X - A \cdot {Y_C}^2$$

$$\bar{I}_X = \frac{1}{3}b \cdot h^3 - b \cdot h\left(\frac{h}{2}\right)^2 = \left(\frac{1}{3} - \frac{1}{4}\right)b \cdot h^3 = \frac{1}{12}b \cdot h^3$$

$$\bar{I}_X = \frac{1}{4}b \cdot h^3 - \frac{1}{2}b \cdot h\left(\frac{2h}{3}\right)^2 = \frac{1}{36}b \cdot h^3$$

$$\bar{I}_X = \frac{1}{4}\pi \cdot r^4 - 0 = \frac{1}{4}\pi \cdot r^4$$

$$\bar{I}_X = \frac{2}{7}b \cdot h^3 - \frac{2}{3}b \cdot h\left(\frac{3h}{5}\right)^2 = \frac{8}{175}b \cdot h^3$$

Completes the first phase of the fourth portion of the lab.

$$\bar{k}_X = \sqrt{\frac{\bar{I}_X}{A}}$$

$$\bar{k}_X = \sqrt{\frac{\frac{1}{12} b \cdot h^3}{b \cdot h}} = \frac{1}{2\sqrt{3}} h$$

$$\bar{k}_X = \sqrt{\frac{\frac{1}{36} b \cdot h^3}{\frac{1}{2} b \cdot h}} = \frac{1}{3\sqrt{2}} h$$

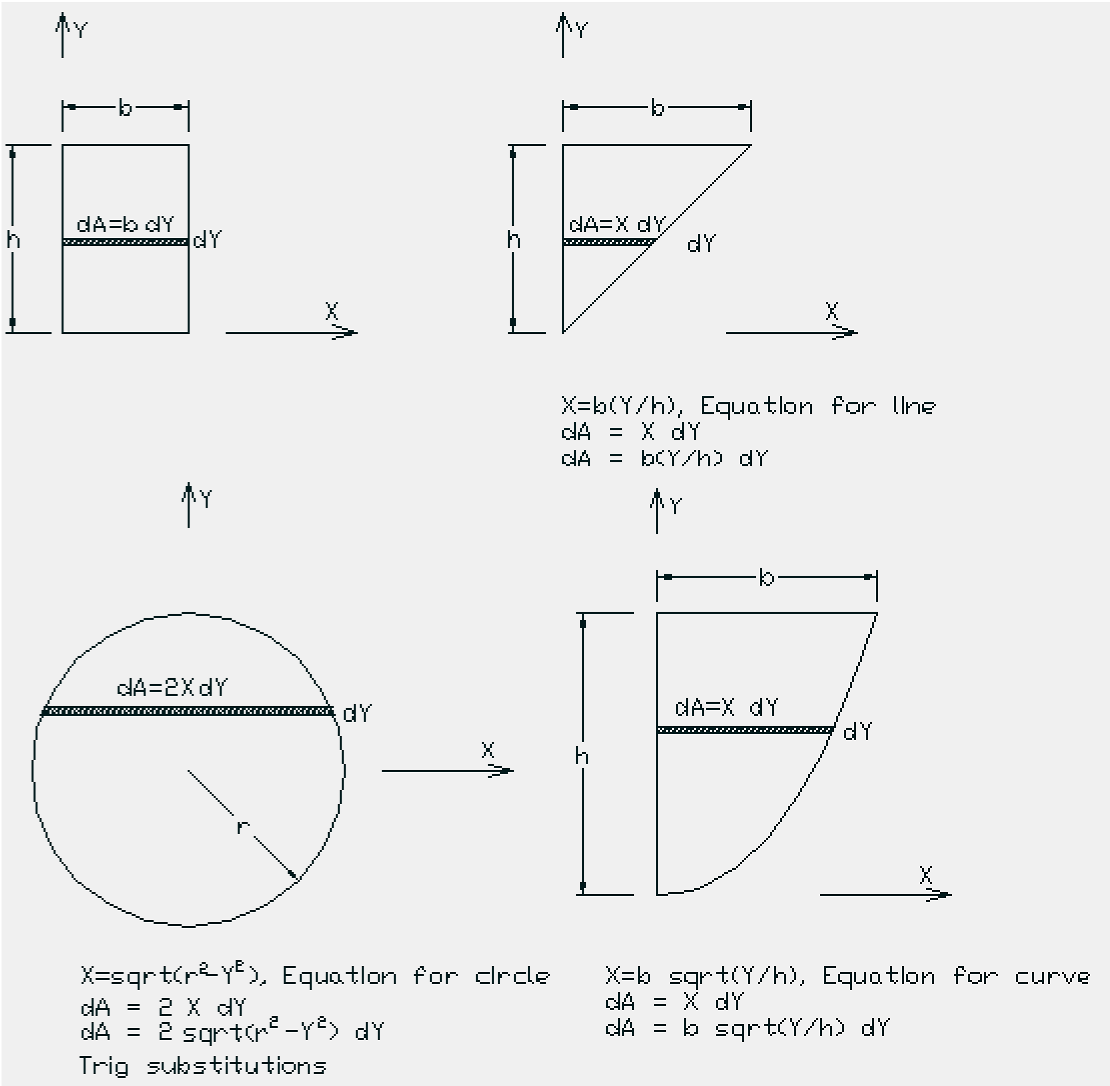
$$\bar{k}_X = \sqrt{\frac{\frac{1}{4} \pi \cdot r^4}{\pi \cdot r^2}} = \frac{r}{2}$$

$$\bar{k}_X = \sqrt{\frac{\frac{8}{175} b \cdot h^3}{\frac{2}{3} b \cdot h}} = \frac{2\sqrt{3}}{5\sqrt{7}} h$$

Completes the lab. Make them work!

7. Shape Properties – Lab Assignment

The lab investigates the properties of the following four shapes:



Review the figure. Ask your instructor for clarification on any aspect of the figure that you do not understand. When you understand the figure, you will be provided with the next part of the assignment.

You are to work alone on each of the following assignments. Address any questions to the instructor.

Evaluate the area of each shape using:

$$A = \int dA$$

Evaluate the first moment of area about the X-axis of each shape using:

$$M_X = \int y \cdot dA$$

Evaluate the Y-coordinate of the centroid of each shape using:

$$Y_C = \frac{M_X}{A}$$

Evaluate the area moment of inertia about the X-axis of each shape using:

$$I_X = \int y^2 \cdot dA$$

Write a brief explanation of why for the same values of b and h, the area of the fourth figure is between the areas of the rectangle and the triangle. Further explain why the centroid of the fourth figure is above that of the rectangle and below that of the triangle. Submit your explanations and your work to the instructor for evaluation (7 points).

For each shape, evaluate the area moment of inertia about an axis parallel to the X-axis but passing through the centroid of the shape using:

$$\bar{I}_X = I_X - A \cdot Y_C^2$$

For each shape, evaluate a radius of gyration for an axis parallel to the X-axis but passing through the centroid of the shape using:

$$\bar{k}_X = \sqrt{\frac{\bar{I}_X}{A}}$$

8. Gravity, One Dimensional Motion - Instructor's Guide

This lab uses a drop tower to investigate the motion of a sphere falling under the action of gravity. The students will be divided into groups of two (one group may contain three students if there are an odd number of students). Each group will be given the task of developing an equation relating distance to time as described below. They will be required to submit this equation and appropriate supporting documentation to you. You will then make measurements of the various times (as described below). The students will attempt to use their equations to predict a particular distance. They may modify their equations if they see fit but must submit modified supporting documentation as well. You will then measure the distance in question. You will rank the student predicted distances by accuracy and lead a discussion of the reasons for deviations between prediction and reality. This discussion should first focus on the "correct" equations describing the behavior of the ball considering only the gravitational forces. The discussion should then move onto deviations from this theoretical model. Such deviations would include air resistance, a lack of perfect alignment of the light beams, and so on.

The solutions submitted by the students should be evaluated against the following:

1. Mechanical System = Sphere

2. Free Body Diagram shows only the sphere and its weight (mg) acting downward (students should have a figure showing this). Neglecting air resistance!

3. A. Sum of the forces is mg downward.

   B. Mass times acceleration is m a downward where a is dV/dt (V is the speed) and/or a is given by V dV/ds (s = distance downward from drop point)

4. $m\, dV/dt = mg$
   $dV/dt = g$
   $V = g\,t + C$

$m\,V\;dV/ds = m\,g$
$V\,dV/ds = g$
$\frac{1}{2}V^2 = g\,s$

V at upper light (V_A) is D/t_A where D is the distance along the sphere that occludes the light (must be no more than the diameter of the ball).

V at lower light (V_B) is D/t_B. (Assuming ball position relative to second light is identical first – this may not be true – must have lights mounted exactly same on tower).

Change in V from time t_1 to $t_2 = g\,t_2 + C – (g\,t_1 + C) = g(t_2-t_1)$
Change in V from upper light to lower light, should be $g\,t_{AB}$.

Thus,

$D/t_B - D/t_A = g\, t_{AB}$ (Note that more precisely t_{AB} should be distance from center to center instead of time between initiation of interruptions, could improve this by modifying t_{AB} using t_A and t_B).

$$D\,(t_A - t_B)/(t_A\, t_B) = g\, t_{AB}$$

$D = g\, t_{AB}\, t_A\, t_B / (t_A - t_B)$ (Useful check at this point to make sure D calculated is near to and slightly less than diameter of particle).

Now to bring distance into problem we can use the $a = V\, dV/ds$ result (although many other approaches are just as good), note that drop between lights is the distance we are interested in. Denoting this drop as H.

$$H = s_B - s_A$$
$$g\, H = g\,(s_B - s_A) = ½\,(V_B^2 - V_A^2)$$

$$H = (V_B^2 - V_A^2) / (2\, g)$$
$$= \{ (D/t_B)^2 - (D/t_A)^2 \} / (2\, g)$$
$$= D^2\,(t_A^2 - t_B^2) / (2\, g\, t_A^2\, t_B^2)$$

Using result for D and noting cancellations:

$$H = (g/2)\, t_{AB}^2\, (t_A^2 - t_B^2) / (t_A - t_B)^2$$

Factoring numerator permits simplification:

$$H = (g/2)\, t_{AB}^2\, (t_A + t_B) / (t_A - t_B)$$

This is what they should get! Note that you should only accept equations that have an expression for H in terms of g and the three times. Any other variables must be eliminated. Force them to do the calculus and algebra to obtain an equation of the desired form.

Instructor Duties

1. Prepare lab. Set up drop tower and level base. Make sure the tower is on the ground to avoid vibrations. Attach timing lights to two locations on tower (make sure each time light is inserted so that it is flush against tower). Attach ball dropper and catcher to drop tower. Use plumb bob to ensure that catcher is directly below dropper. Drop several test balls and ensure that ball enters catcher. Note that you will get best performance if the ball is positioned so that it is just barely held by the jaws (at the lowest possible position in the jaws). This produces a much more consistent drop. Check the three times and be sure that they are consistent over all trials when the ball actually enters the catcher. This will take some practice.

2. Perform several pre-tests and ensure that the solution derived above yields reasonably accurate results. Adjust setup as necessary.

3. Assign students to groups of two as discussed above.

4. Provide each group with a copy of the lab assignment.

5. Ask for any questions on the assignment from the class. Discuss these questions with the entire class. Note that several students may ask for additional information. Inform them that they have sufficient information. Some of the things they might ask for are mass (roughly 28 g) and ball diameter (roughly 19 mm). You may give them this information if they are insistent. Caution them that the ball diameter is "dangerous" information as the "length" of the interruption of the light beam is known only to be less than or equal to the ball diameter (let them try to first recognize this on their own when told that the ball diameter information is "dangerous"). Less likely is that they ask for the drag coefficient of the sphere. You can instruct them to neglect air resistance, or if you want them to really get into it, give them a drag coefficient, the ball diameter, and the required air properties (density, viscosity).

6. Instruct the groups to begin working. Assist as required. Encourage them to use the four step procedure, to think about F=ma and where a = dv/dt or v dv/ds (note that they may well need to use both), and to check units at various stages in the process. Remind them that final equation is to express distance in terms of gravity and the three times (no other variables). Keep an eye on the clock to ensure that you can finish by the end of the allotted lab time.

7. Collect from each group an equation expressing the distance between light beams in terms of the three times and the gravitational acceleration. Collect from each group supporting documentation (should be similar to the four step procedure developed above). Instruct each group to keep a copy of their equation.

8. Conduct several drops and record the three times only for those drops in which the ball enters the catcher. Encourage the students to use values for the three times averaged over the successful trials. Discuss the reasons for averaging.

9. Ask each of the groups to use the data gathered to come up with a predicted distance using their equations. Note that if a group finds that their predicted distance is unreasonable, that group should reconsider their equation. They should reanalyze the problem and submit a new equation and predicted distance with supporting documentation.

10. After each group has submitted their predicted distance, measure the distance between light beams using the meter stick.

11. Rank the submitted distances based on how close they are to measured distance and lead a class discussion of deviations between theory and reality (as described above).

12. Evaluate the work of each group and record the scores of each of the groups.

8. Gravity, One Dimensional Motion – Lab Assignment

In this lab you will investigate the motion of a particle falling under the action of gravity. The particle will be released from rest and will fall vertically downward. As the spherical particle falls it will pass through two light beams. The continuity of these light beams is monitored electronically. Three time measurements will be made during the fall. The first time, t_A, will be the time during which the sphere interrupts the uppermost light beam. The second time, t_B, will be the time during which the sphere interrupts the lower light beam. The third time, t_{AB}, will be the time from the interruption of the upper light beam until the interruption of the lower light beam. Note that t_A and t_B are the measures of the brief time intervals during which the particle is occluding (blocking) a light beam.

Each group should independently develop an equation that relates the distance between light beams to the three times and the acceleration of gravity (you may take the acceleration of gravity to be 9.81 m/s^2). Don't forget the four step procedure! Don't forget that the acceleration of a particle in one dimensional motion can be expressed as dV/dt and/or as V dV/ds! Your final equation should express the distance between lights in terms of only the gravitational acceleration g, and the three times, t_A, t_B, and t_{AB}; no other symbols should appear in your equation.

When all groups have submitted their equations, the lab instructor will drop the ball several times and make several measurements of the three times. Each group should use the measured times and their equations to predict the distance between light beams. Note that if at this point you come to believe that your equations are in error, you may modify your equations and submit your modification and appropriate supporting documentation to your lab instructor (see grading policy below).

When all groups have submitted their predicted distance between light beams (and in some cases modified equations and supporting documentation), the lab instructor will measure the distance between the light beams. The predicted distances will then be shared with the class and any deviations between prediction and reality will be discussed.

Grading Policies: A maximum of 20 points may be earned. Three points will be deducted if the initially submitted equation is incorrect. An additional three points will be deducted if a modified equation is submitted and it is incorrect. An additional three points will be deducted if the predicted distance submitted is inconsistent with the equation submitted. The predicted distances will be ranked based on how close they are to the measured distance. 0.5 points will be deducted for each group ranked above your group. Up to ten points can be deducted based upon the quality of your development of the equation used. Up to five points can be deducted from **all** groups depending upon the lab instructor's assessment of the quality of the discussion of the results.

9. Projectile Motion – Instructor's Guide

In this lab projectile motion will be investigated. The projectile launcher has 4 spring loaded slots and when the launching marble is released from one of these slots, the x displacement can be measured. By changing the launching slot, the velocity of the marble can be varied. The angle of projection can also be varied along the angular scale. The initial velocity of the marble can be computed using the horizontal displacement measured. Since the experiment is carried out on a level surface, the y displacement is the height from which the marble is projected from the launcher.

$$x = \ vcos\theta \ t \ \Rightarrow t = \frac{x}{vcos\theta}$$

$$y = h = \ vsin\theta \ t - \frac{1}{2}gt^2$$

From the above equations substituting for *t* we have:

$$xtan\theta - \frac{1}{2}\frac{gx^2}{(vcos\theta)^2} = h$$

In the above equation the only unknown is the velocity *v* and can be easily calculated.

Instructor duties

1. Ensure that students have access to the projectile launcher, marble and a measuring tape.
2. Perform the experiments beforehand to have an idea of the values of *v* when launched from the 4 different slots.
3. Divide the class into groups of 2 and explain the problem statement. Assign the students a slot to launch the marble and an angle for the launcher. Initially the launcher will be set on the ground. And as a second experiment, put the launcher on a table (since *x* is in terms of *h*)
4. Encourage them to find the equation relating the *x* and *v*.
5. Monitor them while they work on the experiment and guide them as and when required.
6. Grade their work after they are done.

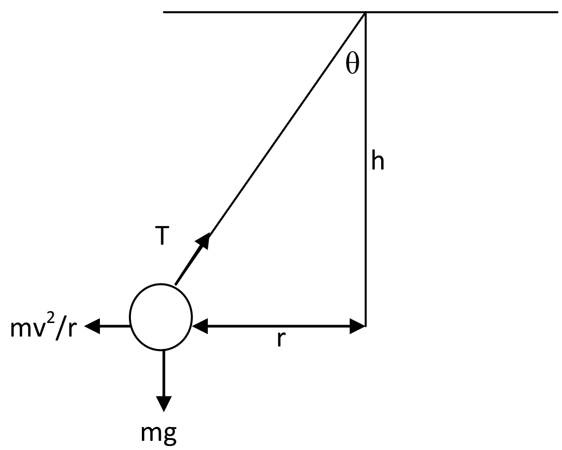
9. Projectile Motion - Lab Assignment

In this lab you will determine the initial velocity of a marble launched using a projectile launcher.

1. Take a look at the experimental setup and project the marble from the 4 different slots. Which slot would has the ability to displace the marble the highest?
2. Derive an expression to find the initial velocity of the marble using the measured x displacement and the height of launch, h. Note that your relationship must not have the time of flight ‘t’, since it is very difficult to accurately measure t.
3. Now project the marble from the slot and angle as advised by the lab instructor.
4. You may consider dipping the ball in water so that it leaves a trail as it lands on the floor.
5. Conduct the experiment 5 times and record the values of x. You may omit any outlier in your data.
6. Find the average of x values recorded and then the initial velocity of the marble.
7. Submit the lab report in the required format.

10(a) Determination of 'g' by swirling a mass – Instructor's Guide

In this lab acceleration due to gravity, 'g' will be determined by swirling a mass suspended using a string. A mass (100 grams) is tied to a string (about 8 inches in length) and swirled manually. Care must be taken to ensure that the point of pivot does not change a great deal while swirling. The free body diagram of the swirling mass is shown below:



Resolving the forces acting on the mass on X and y direction we have:

$$Fx = 0 \rightarrow T sin\theta = \frac{mv^2}{r}$$
$$Fy = 0 \rightarrow T cos\theta = mg$$

Dividing the above 2 equations we have

$$tan\theta = \frac{v^2}{rg}$$

Since $v=r\omega$ and $tan\theta = r/h$, the above equation becomes:

$$\frac{r}{h} = \frac{r^2\omega^2}{rg}$$

And hence $g=\omega^2/h$

The height h can be measured using a ruler, $\omega=2\pi/T$. The time period can be measured using a stop watch.

10(a) Determination of g by swirling a mass

In this lab you will investigate experimentally determine the value of ‘g’ by swirling a mass suspended by a thread.

1. Draw the free body diagram of a swirling mass suspended using a thread; clearly mention the forces acting on it.
2. Derive a relationship between g and other known parameters like length, height, and time period of oscillation.
3. Suspend a 100g using a thread about 6 inches long.
4. Swirl the mass and note the time taken for 5 complete rotations.
5. Repeat the previous step to get 5 readings.
6. From the above data, compute the time period. Time period is the time taken for one oscillation.
7. Using the experimental data and the derived equation, determine the value of g.
8. Compute the percentage error in the result obtained using the steps above.
9. Discuss the sources of error. Explain how a large displacement could affect your accuracy.
10. Submit a report of your findings and discussions to the instructor in the prescribed format.

10(b): Determination of 'g' using pendulum – Instructor's Guide

The period (duration of oscillation) of the pendulum depends on the length of the pendulum, the tilt angle β of the pendulum and acceleration due to gravity. Using the EM 025 'g' pendulum apparatus the influence of these parameters on the period can be investigated.

Evaluation of Experiments

The pendulum rod has a very low mass, this reason the pendulum can be considered a point mass of mass 'm'.

The acceleration due to gravity acting in the plane of oscillation is (Figure 9b.1)

$$G^* = m.g.sin\beta \quad (1)$$

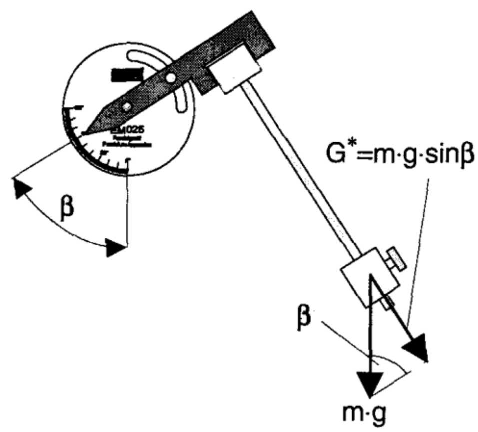


Figure 9b.1

The application of equilibrium of forces in the plane of oscillation along the X-axis (Figure 9b.2) produces the following equation:

$$m.l.\alpha \ + \ G^*.sin\alpha = 0 \quad (2)$$

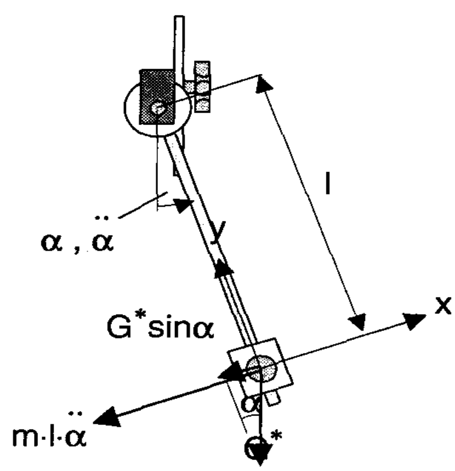


Figure 9b.2

The angular acceleration α is the acceleration along the path of the pendulum. For small pendulum length sinα ≈ α applies, that is

$$\alpha + \frac{g}{l}.cos\beta . \alpha = 0 \quad (3)$$

The middle term is the natural angular frequency ω_o^2:

$$\omega_o^2 = \frac{g}{l}.cos\beta \quad (4)$$

This is dependent on the pendulum length l and the acceleration due to gravity acting g.cosβ in the plane of the pendulum. This yield equation for undamped oscillation of a pendulum:

$$\alpha + \omega_o^2.\alpha = 0 \quad (5)$$

The natural angular frequency can also be represented by means of the measured period T:

$$\omega_o = \frac{2\pi}{T} \quad (6)$$

The drag on the pendulum is ignored. The natural frequency can be calculated and compared with the theoretical value.

Instructor Duties:

1. Prepare lab. Ensure that students have access to measuring tapes and stop watches.

2. Assign students to groups of two. Make sure every group has at least one student who has completed the differential equations course.

3. Provide each group with a copy of the lab assignment. Ask for any questions on the assignment from the class. Discuss these questions with the entire class.

4. Instruct the groups to begin working. Assist as required (do not provide them with the analysis, rather help them discover it on their own to the extent possible). Encourage them to derive the relationship between g, length of pendulum, time period of oscillation and tilt angle β.

5. With the assistance of the group, ask them to measure the time required for several cycles of oscillation of the mass. Note the length of the pendulum and the tilt angle.

6. With the experimental data obtained, ask the class determine the corresponding values of g.

7. Ask the class to find the error in the value of g, determined by the experiment. Encourage them to discuss the sources of error.

8. Collect students' lab reports.

9. Evaluate the work of each group and record the scores of each of the groups.

10(b): Determination of ‘g’ using pendulum - Lab Assignment

In this lab you will investigate the dependence of the time period (duration of oscillation) of the pendulum on the length of the pendulum, acceleration due to gravity and the tilt angle β.

1. Closely observe the pendulum apparatus and draw a free body diagram for of the system when $\beta=90°$ and when $\beta \neq 90°$
2. Derive mathematical equations to determine the relationship between the length of the pendulum (l), acceleration due to gravity (g), time period of oscillation (T) and tilt angle (β).
3. Level the pendulum apparatus using spirit level and leveling bolts. The length of pendulum is the distance of the centre of gravity of the pendulum from the axis of rotation of the pendulum. Set $\beta = 90°$, vary the length of the pendulum, measure (using stop watch) and tabulate the corresponding values for 5 complete oscillations. Note that the time period can be obtained by dividing the total taken by the number of oscillations.
4. Repeat the previous step for $\beta=10°$, 30° and 60°
5. **Note that the maximum displacement of the pendulum should not exceed from 10^o-15^o during the measurement**.
6. Using the experimental data and the derived equation, determine the value of g.
7. Compute the percentage error in the result obtained using the steps above.
8. Discuss the sources of error. Explain how a large displacement could affect your accuracy.
9. Submit a report of your findings and discussions to the instructor in the prescribed format.

11. Vibration – Assistant's Guide

The **purpose of this lab** is to investigate the free vibration of a spring-mass system. The system consists of a mass supported on an air bearing and attached at either end to springs. The students are to analyze the system, exploring the relationship between mass, stiffness, and natural frequency. They investigate the independence of natural frequency from amplitude. Finally they compare stiffnesses evaluated from the natural frequency with those measured based on extension produced by known forces. The analysis portion of the lab should involve the four step procedure and should include the following:

1. Mechanical system = Mass during vibration.

2. Free Body Diagram = Must include a horizontal spring force on each side of the mass. Both forces acting in the negative "X" direction with magnitudes k1 X and k2 X where k1 and k2 are the stiffnesses of the springs. The distance X should be measured from the equilibrium position of the mass. The free body diagram may also include the weight of the mass and the normal force from the air supporting the mass (both vertical). The free body diagram may also include the two equal and opposite forces exerted by the springs in the equilibrium position. Thus k1 X and k2 X actually represent the changes in spring force from the equilibrium values.

3. The sum of the forces should be (k1 + k2) X. Thus one can only determine the sum of the two stiffnesses from this experiment. The mass times the acceleration should be $M\ d^2X/dt^2$.

4. Setting up the differential equation should yield a natural frequency of:
   $\Omega = \{ (k1 + k2) / M \}^{1/2}$
   Measuring the period and evaluating the natural frequency in radians per second should permit the evaluation of k1 + k2 from the given mass.

Instructor Duties

12. Prepare lab. Make sure power is connected to air pump and timer. Ensure that mass vibrates freely when air pressure is applied.

13. Assign students to groups of two. Make sure every group has at least one student who has completed the differential equations course.

14. Provide each group with a copy of the lab assignment. Ask for any questions on the assignment from the class. Discuss these questions with the entire class.

15. Instruct the groups to begin working. Assist as required (do not provide them with the analysis, rather help them discover it on their own to the extent possible). Encourage them to use the four step procedure and to define variables in terms of changes from equilibrium. A difficult hurdle will be seeing that the two spring constants add to yield a single effective spring constant. Keep an eye on the clock to ensure that you can finish by the end of the allotted lab time. Try to arrange it so that all groups finish their analysis at roughly the same time.

16. With the assistance of the class, measure the time required for several cycles of oscillation of the mass. Note that you will need to manually fix one end of one spring. Ask the groups to use the measured time to evaluate the period and the natural frequency in radians per second. Run several experiments with different amplitudes of vibration. Compare the measured periods. Ask the groups to determine the effective stiffness of the two springs.

17. After all groups have submitted their analysis and predicted effective stiffness, ask the class to directly determine the stiffness of each spring. This can be done by supporting a known weight from the spring and measuring the position of the weight. If one then adds an additional weight and measures the change in deflection one can determine the relationship between extension and force (stiffness). Compare the sum of the two measured stiffnesses to the effective stiffness determined from the frequency calculation.

18. Evaluate the work of each group and record the scores of each of the groups.

11. Vibration - Lab Assignment

In this lab you will investigate the vibration of a spring-mass system. The mass will be supported by an air bearing to reduce friction. Two springs will be attached to the mass, one on either side. The ends of the springs not attached to the mass will be fixed. The lab includes the following elements:

11. Apply the four step procedure to determine a relationship between the stiffnesses of the spring, the mass of the slider, and the natural frequency of the system. In so doing, you will need to express acceleration as d^2x/dt^2 and solve a differential equation.
12. By measuring the time required for the system to make several oscillations, evaluate the natural frequency of the system in radians per second. Evaluate the natural frequency for a range of amplitudes of oscillation.
13. Use the natural frequency and the known mass of the slider to learn what you can about the stiffnesses of the springs.
14. Directly measure the stiffness of the springs by suspending weights from the springs and measuring the change in stretch in the spring as the weight supported by the spring is changed. Compare the information obtained by directly measuring the stiffnesses of the springs to that obtained from the natural frequency.

Grading Policies: A maximum of 20 points may be earned. The lab instructor will assign points based on your analysis and the accuracy of your predictions.

Auburn University

Department of Mechanical Engineering

MECH 2110 – Statics and Dynamics Laboratory

Grade _____________
(To be filled in by the instructor)

Name ______________________________

Experiment No. ___________
Date___________

Title:

Aim:

Apparatus used:

Description:

Figure(s)/FBD/Formulae/Derivations

(use additional sheets if required)

Calculations:

Use additional sheets if required

Results:

Conclusions and discussions:

Use additional sheets if required