4/8 Chapter Review

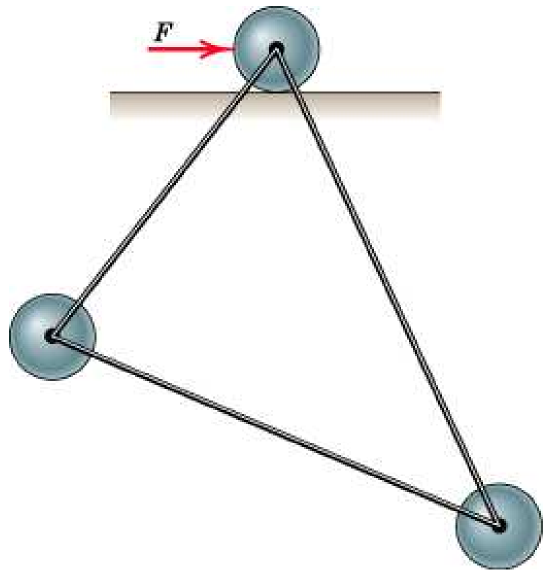
In this chapter we have extended the principles of dynamics for the motion of a single mass particle to the motion of a general system of particles. Such a system can form a rigid body, a nonrigid (elastic) solid body, or a group of separate and unconnected particles, such as those in a defined mass of liquid or gaseous particles. The following summarizes the principal results of Chapter 4.

1. We derived the generalized form of Newton's second law, which is expressed as the *principle of motion of the mass center*, Eq. 4/1 in Art. 4/2. This principle states that the vector sum of the external forces acting on any system of mass particles equals the total system mass times the acceleration of the center of mass.
2. In Art. 4/3, we established a *work-energy principle* for a system of particles, Eq. 4/3a, and showed that the total kinetic energy of the system equals the energy of the mass-center translation plus the energy due to motion of the particles relative to the mass center.
3. The resultant of the external forces acting on any system equals the time rate of change of the linear momentum of the system, Eq. 4/6 in Art. 4/4.
4. For a fixed point *O* and the mass center *G*, the resultant vector moment of all external forces about the point equals the time rate of change of angular momentum about the point, Eq. 4/7 and Eq. 4/9 in Art. 4/4. The principle for an arbitrary point *P*, Eqs. 4/11 and 4/13, has an additional term and thus does not follow the form of the equations for *O* and *G*.
5. In Art. 4/5 we developed the *law of conservation of dynamical energy*, which applies to a system in which the internal kinetic friction is negligible.
6. *Conservation of linear momentum* applies to a system in the absence of an external linear impulse. Similarly, *conservation of angular momentum* applies when there is no external angular impulse.
7. For applications involving steady mass flow, we developed a relation, Eq. 4/18 in Art. 4/6, between the resultant force on a system, the corresponding mass flow rate, and the change in fluid velocity from entrance to exit.
8. Analysis of angular momentum in steady mass flow resulted in Eq. 4/19a in Art. 4/6, which is a relation between the resultant moment of all external forces about a fixed point *O* on or off the system, the mass flow rate, and the incoming and outgoing velocities.
9. Finally, in Art. 4/7 we developed the equation of linear motion for variable-mass systems, Eq. 4/20. Common examples of such systems are rockets and flexible chains and ropes.

The principles developed in this chapter enable us to treat the motion of both rigid and nonrigid bodies in a unified manner. In addition, the developments in Arts. 4/2, 4/3 and 4/4–4/5 will serve to place on a rigorous basis the treatment of rigid-body kinetics in Chapters 6 and 7.

REVIEW PROBLEMS

4/93. Each of the identical steel balls weighs 4 lb and is fastened to the other two by connecting bars of negligible weight and unequal length. In the absence of friction at the supporting horizontal surface, determine the initial acceleration $\bar{a}$ of the mass center of the assembly when it is subjected to the horizontal force $F = 20$ lb applied to the supporting ball. The assembly is initially at rest in the vertical plane. Can you show that $\bar{a}$ is initially horizontal?

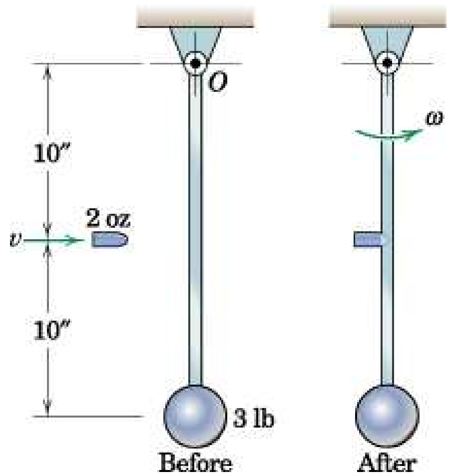


Problem 4/93

Answer:

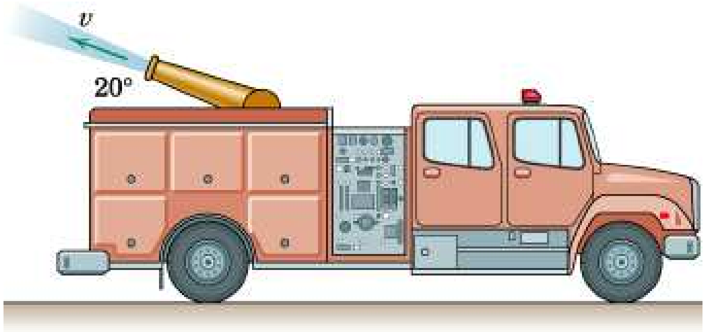
$\bar{a} = 53.7 \text{ ft/sec}^2$

4/94. A 2-oz bullet is fired horizontally with a velocity $v = 1000 \text{ ft/sec}$ into the slender bar of a 3-lb pendulum initially at rest. If the bullet embeds itself in the bar, compute the resulting angular velocity of the pendulum immediately after the impact. Treat the sphere as a particle and neglect the mass of the rod. Why is the linear momentum of the system not conserved?



Problem 4/94

4/95. In an operational design test of the equipment of the fire truck, the water cannon is delivering fresh water through its 2-in.-diameter nozzle at the rate of 1400 gal/min at the 20° angle. Calculate the total friction force F exerted by the pavement on the tires of the truck, which remains in a fixed position with its brakes locked. (There are 231 in.^3 in 1 gal.)



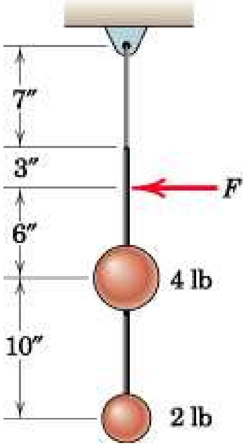
Problem 4/95

Answer:

$F = 812 \text{ lb}$

4/96. A small rocket of initial mass m_0 is fired vertically up near the surface of the earth (g constant), and the mass rate of exhaust m' and the relative exhaust velocity u are constant. Determine the velocity v as a function of the time t of flight if the air resistance is neglected and if the mass of the rocket case and machinery is negligible compared with the mass of the fuel carried.

4/97. The two balls are attached to the light rigid rod, which is suspended by a cord from the support above it. If the balls and rod, initially at rest, are struck with the force $F = 12 \text{ lb}$, calculate the corresponding acceleration $\bar{a}$ of the mass center and the rate $\ddot{\theta}$ at which the angular velocity of the bar is changing.



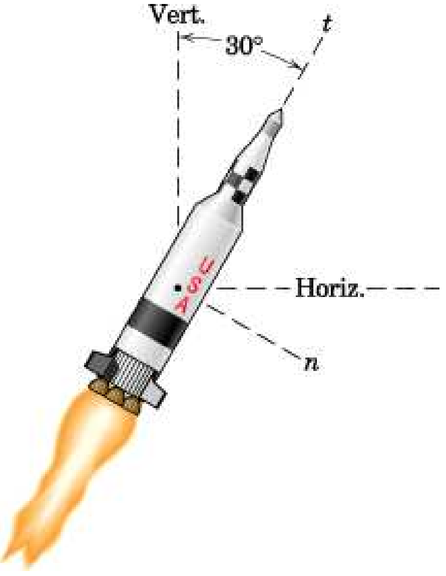
Problem 4/97

Answer:

$\bar{a} = 64.4 \text{ ft/sec}^2, \ddot{\theta} = 325 \text{ rad/sec}^2$

4/98. The rocket shown is designed to test the operation of a new guidance system. When it has reached a certain altitude beyond the effective influence of the earth's atmosphere, its mass has decreased to 2.80 Mg, and its trajectory is 30° from the vertical. Rocket fuel is being consumed at the rate of 120 kg/s with an exhaust velocity of 640 m/s relative to the nozzle. Gravitational acceleration is

$9.34\ \mathrm{m/s^2}$ at its altitude. Calculate the *n*- and *t*-components of the acceleration of the rocket.



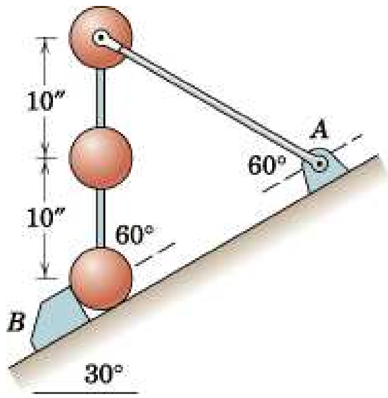
Problem 4/98

4/99. A two-stage rocket is fired vertically up and is above the atmosphere when the first stage burns out and the second stage separates and ignites. The second stage carries 1200 kg of fuel and has an empty mass of 200 kg. Upon ignition the second stage burns fuel at the rate of 5.2 kg/s and has a constant exhaust velocity of 3000 m/s relative to its nozzle. Determine the acceleration of the second stage 60 seconds after ignition and find the maximum acceleration and the time *t* after ignition at which it occurs. Neglect the variation of *g* and take it to be $8.70\ \mathrm{m/s^2}$ for the range of altitude averaging about 400 km.

Answer:

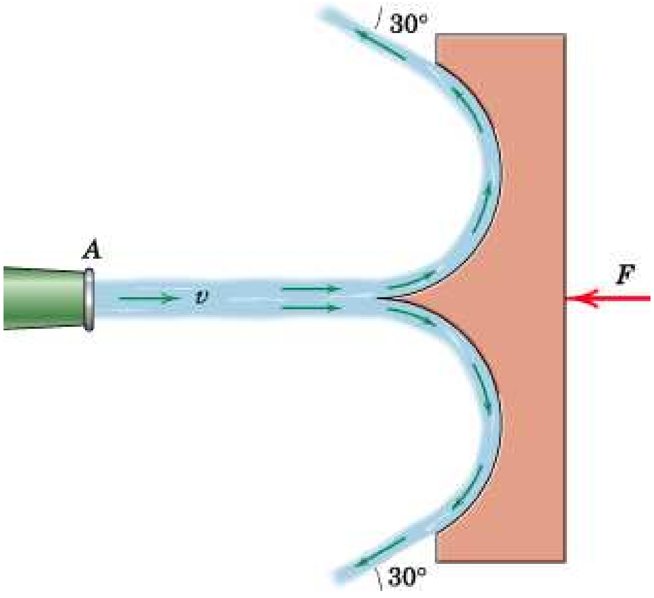
$$a = 5.64\ \mathrm{m/s^2}\ \text{at}\ t = 60\ \mathrm{s}$$
$$a_{max} = 69.3\ \mathrm{m/s^2}\ \text{at}\ t = 231\ \mathrm{s}$$

4/100. The three identical spheres, each of mass *m*, are supported in the vertical plane on the 30° incline. The spheres are welded to the two connecting rods of negligible mass. The upper rod, also of negligible mass, is pivoted freely to the upper sphere and to the bracket at *A*. If the stop at *B* is suddenly removed, determine the velocity *v* with which the upper sphere hits the incline. (Note that the corresponding velocity of the middle sphere is *v*/2.) Explain the loss of energy which has occurred after all motion has ceased.



Problem 4/100

4/101. A jet of fresh water under pressure issues from the 3/4-in.-diameter fixed nozzle with a velocity $v = 120\ \text{ft/sec}$ and is diverted into the two equal streams. Neglect any energy loss in the streams and compute the force F required to hold the vane in place.

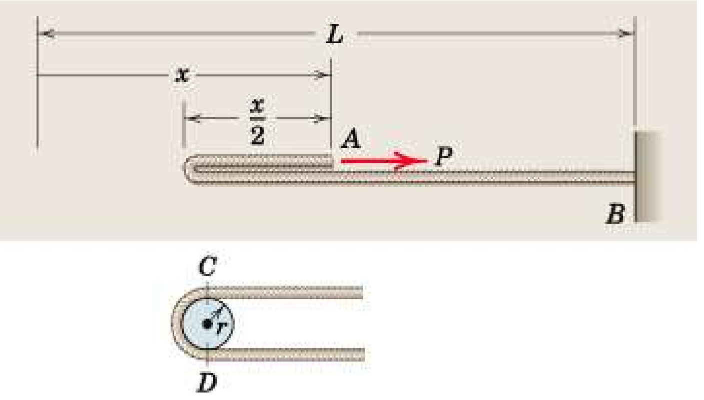


Problem 4/101

Answer:

$F = 159.8\ \text{lb}$

4/102. An ideal rope or bicycle-type chain of length *L* and mass ρ per unit length is resting on a smooth horizontal surface when end *A* is doubled back on itself by a force *P* applied to end *A*. End *B* of the rope is secured to a fixed support. Determine the force *P* required to give *A* a constant velocity *v*. (*Hint:* The action of the loop can be modeled by inserting a circular disk of negligible mass as shown in the separate sketch and then taking the disk radius as zero. It is easily shown that the tensions in the rope at *C*, *D*, and *B* are all equal to *P* under the ideal conditions imposed and with constant velocity.)

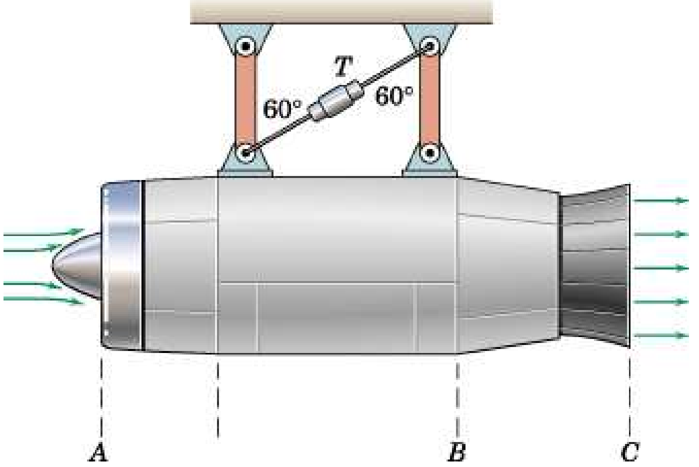


Problem 4/102

4/103. In the static test of a jet engine and exhaust nozzle assembly, air is sucked into the engine at the rate of 30 kg/s and fuel is burned at the rate of 1.6 kg/s. The flow area, static pressure, and axial-flow velocity for the three sections shown are as follows:

	Sec. *A*	Sec. *B*	Sec. *C*
Flow area, m^2	0.15	0.16	0.06
Static pressure, kPa	-14	140	14
Axial-flow velocity, m/s	120	315	600

Determine the tension *T* in the diagonal member of the supporting test stand and calculate the force *F* exerted on the nozzle flange at *B* by the bolts and gasket to hold the nozzle to the engine housing.

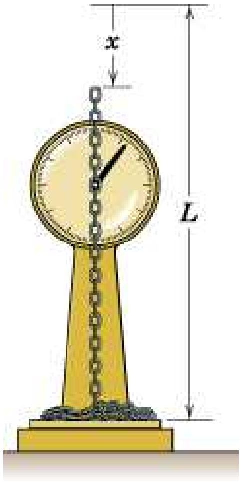


Problem 4/103

Answer:

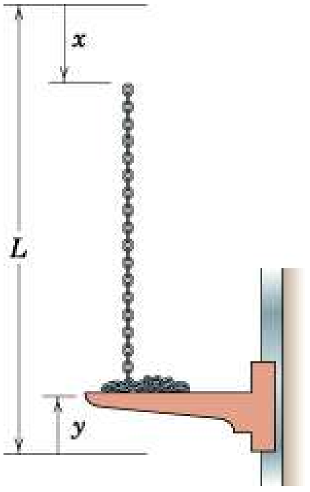
$T = 21.1\ \text{kN}, F = 12.55\ \text{kN}$

4/104. The upper end of the open-link chain of length *L* and mass ρ per unit length is released from rest with the lower end just touching the platform of the scale. Determine the expression for the force *F* read on the scale as a function of the distance x through which the upper end has fallen. (*Comment:* The chain acquires a free-fall velocity of $\sqrt{2gx}$ because the links on the scale exert no force on those above, which are still falling freely. Work the problem in two ways: first, by evaluating the time rate of change of momentum for the entire chain and second, by considering the force F to be composed of the weight of the links at rest on the scale plus the force necessary to divert an equivalent stream of fluid.)



Problem 4/104

4/105. The open-link chain of total length L and of mass ρ per unit length is released from rest at $x = 0$ at the same instant that the platform starts from rest at $y = 0$ and moves vertically up with a constant acceleration a. Determine the expression for the total force R exerted on the platform by the chain t seconds after the motion starts.

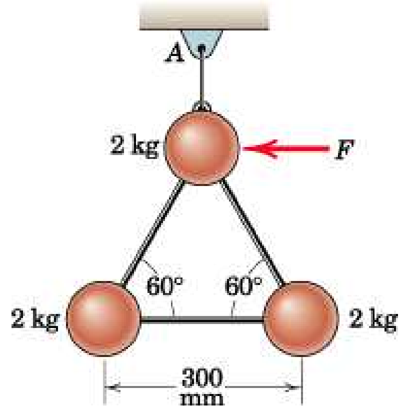


Problem 4/105

Answer:

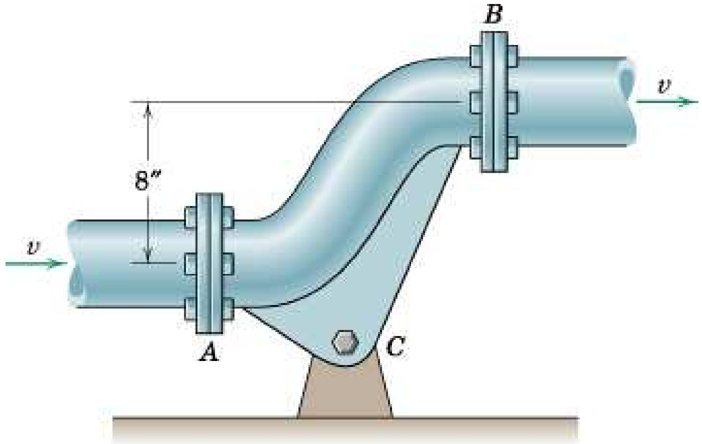
$$R = \frac{3}{2}\rho(a+g)^2t^2$$

4/106. The three identical 2-kg spheres are welded to the connecting rods of negligible mass and are hanging by a cord from point *A*. The spheres are initially at rest when a horizontal force $F = 16\ \text{N}$ is applied to the upper sphere. Calculate the initial acceleration $\bar{a}$ of the mass center of the spheres, the rate $\ddot{\theta}$ at which the angular velocity is increasing, and the initial acceleration *a* of the top sphere.



Problem 4/106

4/107. The diverter section of pipe between *A* and *B* is designed to allow the parallel pipes to clear an obstruction. The flange of the diverter is secured at *C* by a heavy bolt. The pipe carries fresh water at the steady rate of 5000 gal/min under a static pressure of $130\ \text{lb/in.}^2$ entering the diverter. The inside diameter of the pipe at *A* and at *B* is 4 in. The tensions in the pipe at *A* and *B* are balanced by the pressure in the pipe acting over the flow area. There is no shear or bending of the pipes at *A* or *B*. Calculate the moment *M* supported by the bolt at *C*. (Recall that 1 gallon contains $231\ \text{in.}^3$)

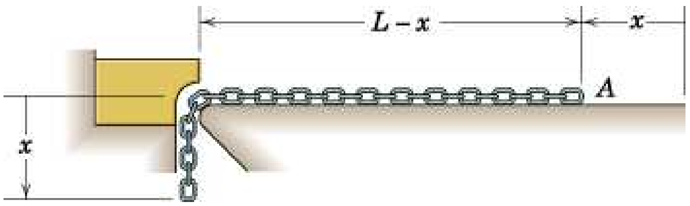


Problem 4/107

Answer:

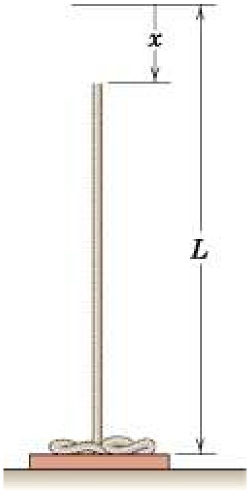
$M = 1837\ \text{lb-ft}$

4/108. The chain of length *L* and mass ρ per unit length is released from rest on the smooth horizontal surface with a negligibly small overhang *x* to initiate motion. Determine (*a*) the acceleration *a* as a function of *x*, (*b*) the tension *T* in the chain at the smooth corner as a function of *x*, and (*c*) the velocity *v* of the last link *A* as it reaches the corner.



Problem 4/108

4/109. ▸ A rope or hinged-link bicycle-type chain of length L and mass ρ per unit length is released from rest with $x = 0$. Determine the expression for the total force R exerted on the fixed platform by the chain as a function of x. Note that the hinged-link chain is a conservative system during all but the last increment of motion. Compare the result with that of Prob. 4/105 if the upward motion of the platform in that problem is taken to be zero.

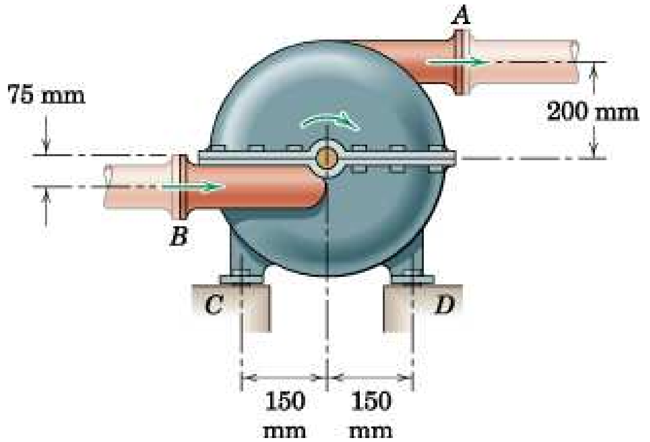


Problem 4/109

Answer:

$$R = \rho g x \frac{4L - 3x}{2(L - x)}$$

4/110. ▸ The centrifugal pump handles $20\ \mathrm{m}^3$ of fresh water per minute with inlet and outlet velocities of 18 m/s. The impeller is turned clockwise through the shaft at O by a motor which delivers 40 kW at a pump speed of 900 rev/min. With the pump filled but not turning, the vertical reactions at C and D are each 250 N. Calculate the forces exerted by the foundation on the pump at C and D while the pump is running. The tensions in the connecting pipes at A and B are exactly balanced by the respective forces due to the static pressure in the water. (*Suggestion:* Isolate the entire pump and water within it between sections A and B and apply the momentum principle to the entire system.)

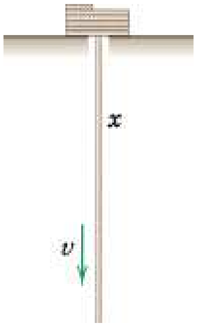


Problem 4/110

Answer:

$C = 4340$ N up, $D = 3840$ N down

4/111. ▸ Replace the pile of chain in Prob. 4/92 by a coil of rope of mass ρ per unit length and total length L as shown and determine the velocity of the falling section in terms of x if it starts from rest at $x = 0$. Show that the acceleration is constant at $g/2$. The rope is considered to be perfectly flexible in bending but inextensible and constitutes a conservative system (no energy loss). Rope elements acquire their velocity in a continuous manner from zero to v in a small transition section of the rope at the top of the coil. For comparison with the chain of Prob. 4/92, this transition section may be considered to have negligible length without violating the requirement that there be no energy loss in the present problem. Also determine the force R exerted by the platform on the coil in terms of x and explain why R becomes zero when $x = 2L/3$. Neglect the dimensions of the coil compared with x.

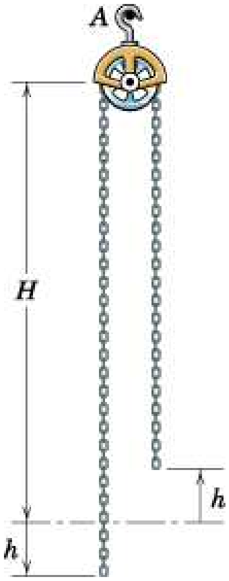


Problem 4/111

Answer:

$v = \sqrt{gx}$, $R = \rho g\left(L - \frac{3}{2}x\right)$

4/112. ▸ The chain of mass ρ per unit length passes over the small freely turning pulley and is released from rest with only a small imbalance h to initiate motion. Determine the acceleration a and velocity v of the chain and the force R supported by the hook at A, all in terms of h as it varies from essentially zero to H. Neglect the weight of the pulley and its supporting frame and the weight of the small amount of chain in contact with the pulley. (*Hint:* The force R does not equal two times the equal tensions T in the chain tangent to the pulley.)



Problem 4/112

Answer:

$a = (h / H)g,\ v = h\sqrt{g / H}$

$R = 2\rho g\left[H - \left(2h^2 / H\right)\right]$