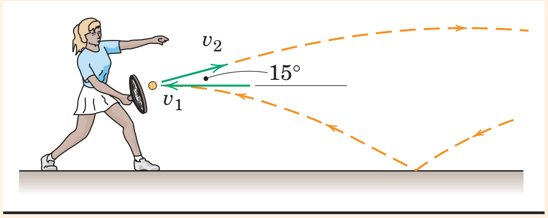
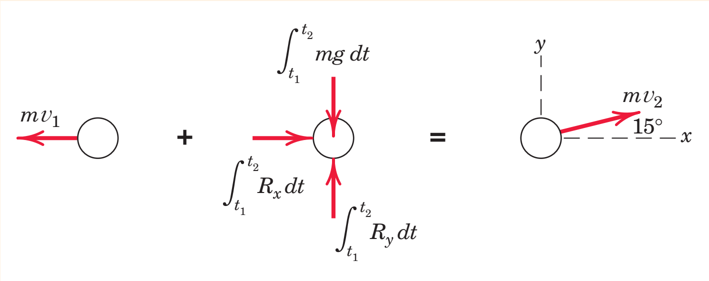
Sample Problem 3/19



A tennis player strikes the tennis ball with her racket when the ball is at the uppermost point of its trajectory as shown. The horizontal velocity of the ball just before impact with the racket is $v_1 = 50$ ft/sec and just after impact its velocity is $v_2 = 70$ ft/sec directed at the 15° angle as shown. If the 4-oz ball is in contact with the racket for 0.02 sec, determine the magnitude of the average force **R** exerted by the racket on the ball. Also determine the angle β made by **R** with the horizontal.

Solution. We construct the impulse-momentum diagrams for the ball as follows:

① 

② $\left[m(v_x)_1 + \int_{t_1}^{t_2} \Sigma F_x\, dt = m(v_x)_2\right]$ $\quad -\dfrac{4/16}{32.2}(50) + R_x(0.02) = \dfrac{4/16}{32.2}(70 \cos 15°)$

$\left[m(v_y)_1 + \int_{t_1}^{t_2} \Sigma F_y\, dt = m(v_y)_2\right]$

$$\frac{4/16}{32.2}(0) + R_y(0.02) - (4/16)(0.02) = \frac{4/16}{32.2}(70 \sin 15°)$$

We can now solve for the impact forces as

$$R_x = 45.7 \text{ lb}$$

$$R_y = 7.28 \text{ lb}$$

We note that the impact force $R_y = 7.28$ lb is considerably larger than the 0.25-lb weight of the ball. Thus, the weight mg, a nonimpulsive force, could have been neglected as small in comparison with R_y. Had we neglected the weight, the computed value of R_y would have been 7.03 lb.

We now determine the magnitude and direction of **R** as

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{45.7^2 + 7.28^2} = 46.2 \text{ lb} \qquad \textit{Ans.}$$

$$\beta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{7.28}{45.7} = 9.06° \qquad \textit{Ans.}$$

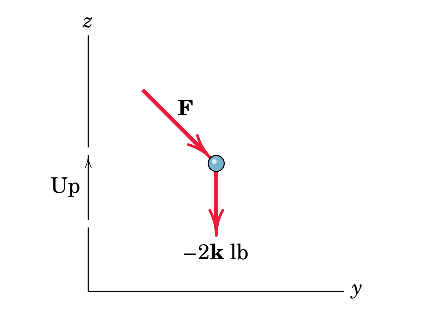
Helpful Hints

① Recall that for the impulse-momentum diagrams, initial linear momentum goes in the first diagram, all external linear impulses go in the second diagram, and final linear momentum goes in the third diagram.

② For the linear impulse $\int_{t_1}^{t_2} R_x\, dt$, the average impact force R_x is a constant, so that it can be brought outside the integral sign, resulting in $R_x \int_{t_1}^{t_2} dt = R_x(t_2 - t_1) = R_x \Delta t$. The linear impulse in the y-direction has been similarly treated.

Sample Problem 3/20

A 2-lb particle moves in the vertical y-z plane (z up, y horizontal) under the action of its weight and a force **F** which varies with time. The linear momentum of the particle in pound-seconds is given by the expression $\mathbf{G} = \frac{3}{2}(t^2 + 3)\mathbf{j} - \frac{2}{3}(t^3 - 4)\mathbf{k}$, where t is the time in seconds. Determine **F** and its magnitude for the instant when $t = 2$ sec.



Solution. The weight expressed as a vector is $-2\mathbf{k}$ lb. Thus, the force-momentum equation becomes

① $[\Sigma\mathbf{F} = \dot{\mathbf{G}}]$ $$\mathbf{F} - 2\mathbf{k} = \frac{d}{dt}[\tfrac{3}{2}(t^2 + 3)\mathbf{j} - \tfrac{2}{3}(t^3 - 4)\mathbf{k}]$$
$$= 3t\mathbf{j} - 2t^2\mathbf{k}$$

For $t = 2$ sec, $$\mathbf{F} = 2\mathbf{k} + 3(2)\mathbf{j} - 2(2^2)\mathbf{k} = 6\mathbf{j} - 6\mathbf{k} \text{ lb}$$ *Ans.*

Thus, $$F = \sqrt{6^2 + 6^2} = 6\sqrt{2} \text{ lb}$$ *Ans.*

Helpful Hint

① Don't forget that $\Sigma\mathbf{F}$ includes *all* external forces acting on the particle, including the weight.