Sample Problem 3/19

A tennis player strikes the tennis ball with her racket when the ball is at the uppermost point of its trajectory as shown. The horizontal velocity of the ball just before impact with the racket is \( v_1 = 50 \text{ ft/sec} \) and just after impact its velocity is \( v_2 = 70 \text{ ft/sec} \) directed at the 15° angle as shown. If the 4-oz ball is in contact with the racket for 0.02 sec, determine the magnitude of the average force \( \mathbf{R} \) exerted by the racket on the ball. Also determine the angle \( \beta \) made by \( \mathbf{R} \) with the horizontal.

Solution. We construct the impulse-momentum diagrams for the ball as follows:

1. \( [m(v_x)_1 + \int_{t_1}^{t_2} \Sigma F_x \, dt = m(v_x)_2] \)
   \[ \int_{t_1}^{t_2} \Sigma F_x \, dt = \frac{4}{16} \times 32.2 \times 50 = \frac{4}{16} \times 32.2 \times 70 \cos 15° \]

2. \( [m(v_y)_1 + \int_{t_1}^{t_2} \Sigma F_y \, dt = m(v_y)_2] \)
   \[ \int_{t_1}^{t_2} \Sigma F_y \, dt = \frac{4}{16} \times 32.2 \times 0 = \frac{4}{16} \times 32.2 \times 70 \sin 15° \]

We can now solve for the impact forces as

\[ R_x = 45.7 \text{ lb} \]
\[ R_y = 7.28 \text{ lb} \]

We note that the impact force \( R_y = 7.28 \text{ lb} \) is considerably larger than the 0.25-lb weight of the ball. Thus, the weight \( mg \), a nonimpulsive force, could have been neglected as small in comparison with \( R_y \). Had we neglected the weight, the computed value of \( R_y \) would have been 7.03 lb.

We now determine the magnitude and direction of \( \mathbf{R} \) as

\[ R = \sqrt{R_x^2 + R_y^2} = \sqrt{45.7^2 + 7.28^2} = 46.2 \text{ lb} \]
\[ \beta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{7.28}{45.7} = 9.06° \]
Sample Problem 3/20

A 2-lb particle moves in the vertical y-z plane (z up, y horizontal) under the action of its weight and a force \( \mathbf{F} \) which varies with time. The linear momentum of the particle in pound-seconds is given by the expression \( \mathbf{G} = \frac{\vec{3}}{2}(t^2 + 3\hat{j} - \frac{2}{3}(t^3 - 4)\hat{k}) \), where \( t \) is the time in seconds. Determine \( \mathbf{F} \) and its magnitude for the instant when \( t = 2 \) sec.

**Solution.** The weight expressed as a vector is \(-2\mathbf{k}\) lb. Thus, the force-momentum equation becomes

\[
[\Sigma \mathbf{F} = \dot{\mathbf{G}}] \quad \mathbf{F} - 2\mathbf{k} = \frac{d}{dt}\left[ \frac{3}{2}(t^2 + 3\hat{j} - \frac{2}{3}(t^3 - 4)\hat{k}) \right]
\]

For \( t = 2 \) sec,

\[
\mathbf{F} = 2\mathbf{k} + 3(2)\hat{j} - 2(2^2)\mathbf{k} = 6\hat{j} - 6\mathbf{k}
\]

Thus,

\[
F = \sqrt{6^2 + 6^2} = 6\sqrt{2} \text{ lb}
\]

**Sample Problem 3/21**

A particle with a mass of 0.5 kg has a velocity of 10 m/s in the x-direction at time \( t = 0 \). Forces \( \mathbf{F}_1 \) and \( \mathbf{F}_2 \) act on the particle, and their magnitudes change with time according to the graphical schedule shown. Determine the velocity \( \mathbf{v}_2 \) of the particle at the end of the 3-s interval. The motion occurs in the horizontal x-y plane.

**Solution.** First, we construct the impulse-momentum diagrams as shown.

Then the impulse-momentum equations follow as

\[
[\mathbf{m}(v_1)_z = 0]
\]

\[
\mathbf{m}(v_1)_y = 0.5(10) \text{ kg m/s}
\]

\[
\int_{t_i}^{t_f} \mathbf{F}_1 dt = \mathbf{m}(v_2)_y = \int_{t_i}^{t_f} \mathbf{F}_2 dt
\]

\[
\mathbf{m}(v_2)_z
\]

\[
\mathbf{m}(v_2)_x
\]

Thus,

\[
\mathbf{v}_2 = -6\hat{i} + 8\hat{j} \text{ m/s and } v_2 = \sqrt{6^2 + 8^2} = 10 \text{ m/s}
\]

\[
\theta_x = \tan^{-1} \frac{8}{-6} = 126.9^\circ
\]

**Helpful Hint**

\( \Delta \) The impulse in each direction is the corresponding area under the force-time graph. Note that \( F_1 \) is in the negative x-direction, so its impulse is negative.
Sample Problem 3/22

The loaded 150-kg skip is rolling down the incline at 4 m/s when a force \( P \) is applied to the cable as shown at time \( t = 0 \). The force \( P \) is increased uniformly with the time until it reaches 600 N at \( t = 4 \) s, after which time it remains constant at this value. Calculate (a) the time at which the skip reverses its direction and (b) the velocity \( v \) of the skip at \( t = 8 \) s. Treat the skip as a particle.

Solution. The stated variation of \( P \) with the time is plotted, and the impulse-momentum diagrams of the skip are drawn.

Part (a). The skip reverses direction when its velocity becomes zero. We will assume that this condition occurs at \( t = 4 + \Delta t \) s. The impulse-momentum equation applied consistently in the positive \( x \)-direction gives

\[
m(v_1)_x + \int \Sigma F_x \, dt = m(v_2)_x
\]

\[
150(-4) + \frac{1}{2}(4)(2)(600) + 2(600)(\Delta t) - 150(9.81)\sin 30^\circ(4 + \Delta t) = 150(0)
\]

\[
\Delta t = 2.46 \text{ s} \quad t' = 4 + 2.46 = 6.46 \text{ s} \quad \text{Ans.}
\]

Part (b). Applying the momentum equation to the entire 8-s interval gives

\[
m(v_1)_x + \int \Sigma F_x \, dt = m(v_2)_x
\]

\[
150(-4) + \frac{1}{2}(4)(2)(600) + 4(2)(600) - 150(9.81)\sin 30^\circ(8) = 150(v_2)_x
\]

\[
(v_2)_x = 4.76 \text{ m/s} \quad \text{Ans.}
\]

The same result is obtained by analyzing the interval from \( t' \) to 8 s.

Sample Problem 3/23

The 50-g bullet traveling at 600 m/s strikes the 4-kg block centrally and is embedded within it. If the block slides on a smooth horizontal plane with a velocity of 12 m/s in the direction shown prior to impact, determine the velocity \( v_2 \) of the block and embedded bullet immediately after impact.

Solution. Since the force of impact is internal to the system composed of the block and bullet and since there are no other external forces acting on the system in the plane of motion, it follows that the linear momentum of the system is conserved. Thus,

\[
[G_1 = G_2] \quad 0.050(600j) + 4(12)(\cos 30^\circ i + \sin 30^\circ j) = (4 + 0.050)v_2
\]

\[
v_2 = 10.26i + 13.33j \text{ m/s} \quad \text{Ans.}
\]

The final velocity and its direction are given by

\[
v = \sqrt{v_x^2 + v_y^2} \quad v_2 = \sqrt{(10.26)^2 + (13.33)^2} = 16.83 \text{ m/s} \quad \text{Ans.}
\]

\[
[\tan \theta = v_y/v_x] \quad \tan \theta = \frac{13.33}{10.26} = 1.299 \quad \theta = 52.4^\circ \quad \text{Ans.}
\]

Helpful Hint

1. The impulse-momentum diagram keeps us from making the error of using the impulse of \( P \) rather than \( 2P \) or of forgetting the impulse of the component of the weight. The first term in the linear impulse is the triangular area of the \( P-t \) relation for the first 4 s, doubled for the force of \( 2P \).
Sample Problem 3/24

A small sphere has the position and velocity indicated in the figure and is acted upon by the force \( F \). Determine the angular momentum \( \mathbf{H}_O \) about point \( O \) and the time derivative \( \dot{\mathbf{H}}_O \).

**Solution.** We begin with the definition of angular momentum and write

\[
\mathbf{H}_O = \mathbf{r} \times m \mathbf{v}
\]

\[
= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 2(5\mathbf{j})
\]

\[
= -40\mathbf{i} + 30\mathbf{k} \text{ N} \cdot \text{m/s}
\]

From Eq. 3/31,

\[
\dot{\mathbf{H}}_O = \mathbf{M}_O
\]

\[
= \mathbf{r} \times \mathbf{F}
\]

\[
= (3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}) \times 10\mathbf{k}
\]

\[
= 60\mathbf{i} - 30\mathbf{j} \text{ N} \cdot \text{m}
\]

Ans.

As with moments of forces, the position vector must run from the reference point (\( O \) in this case) to the line of action of the linear momentum \( m \mathbf{v} \). Here \( \mathbf{r} \) runs directly to the particle.

Sample Problem 3/25

A comet is in the highly eccentric orbit shown in the figure. Its speed at the most distant point \( A \), which is at the outer edge of the solar system, is \( v_A = 740 \text{ m/s} \). Determine its speed at the point \( B \) of closest approach to the sun.

**Solution.** Because the only significant force acting on the comet, the gravitational force exerted on it by the sun, is central (points to the sun center \( O \)), angular momentum about \( O \) is conserved.

\[
(H_O)_A = (H_O)_B
\]

\[
mr_Av_A = mr_Bv_B
\]

\[
v_B = \frac{r_Av_A}{r_B} = \frac{6(10^8)740}{75(10^6)}
\]

\[
v_B = 59200 \text{ m/s}
\]

Ans.
Sample Problem 3/26

The assembly of the light rod and two end masses is at rest when it is struck by the falling wad of putty traveling with speed \( v_1 \) as shown. The putty adheres to and travels with the right-hand end mass. Determine the angular velocity \( \theta_2 \) of the assembly just after impact. The pivot at \( O \) is frictionless, and all three masses may be assumed to be particles.

Solution. If we ignore the angular impulses associated with the weights during the collision process, then system angular momentum about \( O \) is conserved during the impact.

\[
(H_O)_1 = (H_O)_2
\]

\[
mv_1l = (m + 2m)(l \dot{\theta}_2)l + 4m(2l \dot{\theta}_2)2l
\]

\[
\dot{\theta}_2 = \frac{v_1}{19l} \text{ CW}
\]

Ans.

Note that each angular-momentum term is written in the form \( mvd \), and the final transverse velocities are expressed as radial distances times the common final angular velocity \( \dot{\theta}_2 \).

Sample Problem 3/27

A small mass particle is given an initial velocity \( v_0 \) tangent to the horizontal rim of a smooth hemispherical bowl at a radius \( r_0 \) from the vertical centerline, as shown at point \( A \). As the particle slides past point \( B \), a distance \( h \) below \( A \) and a distance \( r \) from the vertical centerline, its velocity \( v \) makes an angle \( \theta \) with the horizontal tangent to the bowl through \( B \). Determine \( \theta \).

Solution. The forces on the particle are its weight and the normal reaction exerted by the smooth surface of the bowl. Neither force exerts a moment about the axis \( O-O \), so that angular momentum is conserved about that axis. Thus,

\[
(H_O)_1 = (H_O)_2
\]

\[
mv_0r_0 = mvr \cos \theta
\]

Also, energy is conserved so that \( E_1 = E_2 \). Thus

\[
[T_1 + V_1 = T_2 + V_2]
\]

\[
\frac{1}{2} mv_0^2 + mgh = \frac{1}{2} mv^2 + 0
\]

\[
v = \sqrt{v_0^2 + 2gh}
\]

Eliminating \( v \) and substituting \( r^2 = r_0^2 - h^2 \) give

\[
v_0r_0 = \sqrt{v_0^2 + 2gh} \sqrt{r_0^2 - h^2} \cos \theta
\]

\[
\theta = \cos^{-1} \left( \frac{1}{\sqrt{1 + \frac{2gh}{v_0^2}} \sqrt{1 - \frac{h^2}{r_0^2}}} \right)
\]

Ans.

Helpful Hint

The angle \( \theta \) is measured in the plane tangent to the hemispherical surface at \( B \).
Sample Problem 3/33

The flatcar moves with a constant speed \(v_0\) and carries a winch which produces a constant tension \(P\) in the cable attached to the small carriage. The carriage has a mass \(m\) and rolls freely on the horizontal surface starting from rest relative to the flatcar at \(x = 0\), at which instant \(X = x_0 = b\). Apply the work-energy equation to the carriage, first, as an observer moving with the frame of reference of the car and, second, as an observer on the ground. Show the compatibility of the two expressions.

Solution. To the observer on the flatcar, the work done by \(P\) is

\[ U_{rel} = \int_0^X P \, dx = Px \quad \text{for constant } P \]

The change in kinetic energy relative to the car is

\[ \Delta T_{rel} = \frac{1}{2} m (\dot{x}^2 - 0) \]

The work-energy equation for the moving observer becomes

\[ [U_{rel} = \Delta T_{rel}] \]

\[ Px = \frac{1}{2} m x^2 \]

To the observer on the ground, the work done by \(P\) is

\[ U = \int_0^X P \, dX = P(X - b) \]

The change in kinetic energy for the ground measurement is

\[ \Delta T = \frac{1}{2} m (\dot{X}^2 - v_0^2) \]

The work-energy equation for the fixed observer gives

\[ [U = \Delta T] \]

\[ P(X - b) = \frac{1}{2} m (\dot{X}^2 - v_0^2) \]

To reconcile this equation with that for the moving observer, we can make the following substitutions:

\[ X = x_0 + x, \quad \dot{X} = v_0 + \dot{x}, \quad \ddot{X} = \ddot{x} \]

Thus,

\[ P(X - b) = Px + P(x_0 - b) = Px + m \ddot{x} (x_0 - b) \]

\[ = Px + m \ddot{x} v_0 \dot{t} = Px + mv_0 \dot{x} \]

and

\[ \dot{X}^2 - v_0^2 = (v_0^2 + \dot{x}^2 + 2v_0 \dot{x}) - v_0^2 = \dot{x}^2 + 2v_0 \dot{x} \]

The work-energy equation for the fixed observer now becomes

\[ Px + mv_0 \dot{x} - \frac{1}{2} m \ddot{x}^2 + mv_0 \dot{x} \]

which is merely \( Px = \frac{1}{2} m \dot{x}^2 \), as concluded by the moving observer. We see, therefore, that the difference between the two work-energy expressions is

\[ U - U_{rel} = T - T_{rel} = mv_0 \dot{x} \]

Helpful Hints

1. The only coordinate which the moving observer can measure is \(x\).

2. To the ground observer, the initial velocity of the carriage is \(v_0\) so its initial kinetic energy is \(\frac{1}{2} mv_0^2\).

3. The symbol \(t\) stands for the time of motion from \(x = 0\) to \(x = x\). The displacement \(x_0 - b\) of the carriage is its velocity \(v_0\) times the time \(t\) or \(x_0 - b = v_0 \dot{t}\). Also, since the constant acceleration \(a\) times the time equals the velocity change, \(\ddot{x} t = \dot{x}\).