Sample Problem 3/4

The design model for a new ship has a mass of 10 kg and is tested in an experimental towing tank to determine its resistance to motion through the water at various speeds. The test results are plotted on the accompanying graph, and the resistance \( R \) may be closely approximated by the dashed parabolic curve shown. If the model is released when it has a speed of 2 m/s, determine the time \( t \) required for it to reduce its speed to 1 m/s and the corresponding travel distance \( x \).

**Solution.** We approximate the resistance-velocity relation by \( R = kv^2 \) and find \( k \) by substituting \( R = 8 \) N and \( v = 2 \) m/s into the equation, which gives \( k = \frac{8}{2^2} = 2 \) N\cdot m^2/s^2. Thus, \( R = 2v^2 \).

The only horizontal force on the model is the resistance, so that

1. \( [\Sigma F_x = ma_x] \quad -R = ma_x \quad \text{or} \quad -2v^2 = 10 \frac{dv}{dt} \)

We separate the variables and integrate to obtain

\[
\int_0^t dt = -5 \int_0^v \frac{dv}{v^2} \quad t = 5 \left( \frac{1}{v} - \frac{1}{2} \right) \text{ s}
\]

Thus, when \( v = v_0/2 = 1 \) m/s, the time is \( t = 5 \left( \frac{1}{\frac{1}{2}} - \frac{1}{2} \right) = 2.5 \) s. \( \text{Ans.} \)

The distance traveled during the 2.5 seconds is obtained by integrating \( v = \frac{dx}{dt} \). Thus, \( v = 10(5 + 2t) \) so that

2. \( \int_0^x dx = \int_0^{\frac{2.5}{5 + 2t}} 10 \frac{dt}{2} = \frac{10}{2} \ln (5 + 2t) \bigg|_0^{\frac{2.5}{5}} = 3.47 \) m \( \text{Ans.} \)

Sample Problem 3/5

The collar of mass \( m \) slides up the vertical shaft under the action of a force \( F \) of constant magnitude but variable direction. If \( \theta = kt \) where \( k \) is a constant and if the collar starts from rest with \( \theta = 0 \), determine the magnitude \( F \) of the force which will result in the collar coming to rest as \( \theta \) reaches \( \pi/2 \). The coefficient of kinetic friction between the collar and shaft is \( \mu_k \).

**Solution.** After drawing the free-body diagram, we apply the equation of motion in the \( x \)-direction to get

1. \( [\Sigma F_x = ma_x] \quad F \cos \theta - \mu_k N - mg = m \frac{dv}{dt} \)

where equilibrium in the horizontal direction requires \( N = F \sin \theta \). Substituting \( \theta = kt \) and integrating first between general limits gives

\[
\int_0^t (F \cos kt - \mu_k F \sin kt - mg) dt = m \int_0^t dv
\]

which becomes

\[
\frac{F}{k} [\sin kt + \mu_k(\cos kt - 1)] - mgt = mv
\]

For \( \theta = \pi/2 \) the time becomes \( t = \pi/2k \), and \( v = 0 \) so that

2. \( \frac{F}{k} (1 + \mu_k(0 - 1)) - \frac{mg \pi}{2k} = 0 \quad \text{and} \quad F = \frac{mg \pi}{2(1 - \mu_k)} \quad \text{Ans.} \)
EXAMPLE 13.4

A smooth 2-kg collar \( C \), shown in Fig. 13–9\( \text{a} \), is attached to a spring having a stiffness \( k = 3 \, \text{N/m} \) and an unstretched length of 0.75 m. If the collar is released from rest at \( A \), determine its acceleration and the normal force of the rod on the collar at the instant \( y = 1 \, \text{m} \).

SOLUTION

Free-Body Diagram. The free-body diagram of the collar when it is located at the arbitrary position \( y \) is shown in Fig. 13–9\( \text{b} \). Note that the weight is \( W = 2(9.81) = 19.62 \, \text{N} \). Furthermore, the collar is assumed to be accelerating so that “\( a \)” acts downward in the positive \( y \) direction. There are four unknowns, namely, \( N_C, F_s, a, \) and \( \theta \).

Equations of Motion.

\[
\begin{align*}
\sum F_x &= ma_x; \quad -N_C + F_s \cos \theta = 0 \quad (1) \\
\sum F_y &= ma_y; \quad 19.62 - F_s \sin \theta = 2a \quad (2) \\
\end{align*}
\]

From Eq. 2 it is seen that the acceleration depends on the magnitude and direction of the spring force. Solution for \( N_C \) and \( a \) is possible once \( F_s \) and \( \theta \) are known.

The magnitude of the spring force is a function of the stretch \( s \) of the spring; i.e., \( F_s = ks \). Here the unstretched length is \( AB = 0.75 \, \text{m} \), Fig. 13–9\( \text{a} \); therefore, \( s = CB - AB = \sqrt{y^2 + (0.75)^2} - 0.75 \). Since \( k = 3 \, \text{N/m} \), then

\[
F_s = ks = 3\left(\sqrt{y^2 + (0.75)^2} - 0.75\right) \quad (3)
\]

From Fig. 13–9\( \text{a} \), the angle \( \theta \) is related to \( y \) by trigonometry.

\[
\tan \theta = \frac{y}{0.75} \quad (4)
\]

Substituting \( y = 1 \, \text{m} \) into Eqs. 3 and 4 yields \( F_s = 1.50 \, \text{N} \) and \( \theta = 53.1^\circ \). Substituting these results into Eqs. 1 and 2, we obtain

\[
\begin{align*}
N_C &= 0.900 \, \text{N} \quad \text{Ans.} \\
a &= 9.21 \, \text{m/s}^2 \downarrow \quad \text{Ans.}
\end{align*}
\]

NOTE: This is not a case of constant acceleration, since the spring force changes both its magnitude and direction as the collar moves downward.
EXAMPLE 13.6

Determine the banking angle $\theta$ for the race track so that the wheels of the racing cars shown in Fig. 13–12a will not have to depend upon friction to prevent any car from sliding up or down the track. Assume the cars have negligible size, a mass $m$, and travel around the curve of radius $\rho$ with a speed $v$.

**SOLUTION**

Before looking at the following solution, give some thought as to why it should be solved using $t$, $n$, $b$ coordinates.

**Free-Body Diagram.** As shown in Fig. 13–12b, and as stated in the problem, no frictional force acts on the car. Here $N_C$ represents the resultant of the ground on all four wheels. Since $a_n$ can be calculated, the unknowns are $N_C$ and $\theta$.

**Equations of Motion.** Using the $n$, $b$ axes shown,

$$\Sigma F_n = ma_n; \quad N_C \sin \theta = \frac{m v^2}{\rho} \quad (1)$$

$$\Sigma F_b = 0; \quad N_C \cos \theta - mg = 0 \quad (2)$$

Eliminating $N_C$ and $m$ from these equations by dividing Eq. 1 by Eq. 2, we obtain

$$\tan \theta = \frac{v^2}{g \rho}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{g \rho}\right) \quad \text{Ans.}$$

**NOTE:** The result is independent of the mass of the car. Also, a force summation in the tangential direction is of no consequence to the solution. If it were considered, then $a_t = dv/dt = 0$, since the car moves with constant speed. A further analysis of this problem is discussed in Prob. 21–48.
EXAMPLE 13.8

Design of the ski jump shown in the photo requires knowing the type of forces that will be exerted on the skier and his approximate trajectory. If in this case the jump can be approximated by the parabola shown in Fig. 13–14a, determine the normal force on the 150-lb skier the instant he arrives at the end of the jump, point A, where his velocity is 65 ft/s. Also, what is his acceleration at this point?

SOLUTION

Why consider using \( n, t \) coordinates to solve this problem?

Free-Body Diagram. The free-body diagram for the skier when he is at A is shown in Fig. 13–14b. Since the path is curved, there are two components of acceleration, \( a_n \) and \( a_t \). Since \( a_n \) can be calculated, the unknowns are \( a_t \) and \( N_A \).

Equations of Motion.

\[ + \uparrow \Sigma F_n = ma_n; \quad N_A - 150 = \frac{150}{32.2} \left( \frac{(65)^2}{\rho} \right) \]  
\[ \pm \Sigma F_t = ma_t; \quad 0 = \frac{150}{32.2} a_t \]  
\[ \rho = \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{3/2} \left| \frac{d^2y}{dx^2} \right| \bigg|_{x=0} = \left[ 1 + \left( \frac{1}{100} \right)^2 \right]^{3/2} \frac{1}{100} = 100 \text{ ft} \]

Substituting into Eq. 1 and solving for \( N_A \), we have

\[ N_A = 347 \text{ lb} \quad \text{Ans.} \]

Kinematics. From Eq. 2,

\[ a_t = 0 \]

Thus,

\[ a_n = \frac{v^2}{\rho} = \frac{(65)^2}{100} = 42.2 \text{ ft/s}^2 \]

\[ a_A = a_n = 42.2 \text{ ft/s}^2 \quad \text{Ans.} \]

NOTE: Apply the equation of motion in the \( y \) direction and show that when the skier is in mid air the acceleration is 32.2 ft/s\(^2\).
EXAMPLE 13.9

Packages, each having a mass of 2 kg, are delivered from a conveyor to a smooth circular ramp with a velocity of \( v_0 = 1 \) m/s as shown in Fig. 13–15a. If the effective radius of the ramp is 0.5 m, determine the angle \( \theta = \theta_{\text{max}} \) at which each package begins to leave the surface.

**SOLUTION**

**Free-Body Diagram.** The free-body diagram for a package, when it is located at the general position \( \theta \), is shown in Fig. 13–15b. The package must have a tangential acceleration \( a_t \), since its speed is always increasing as it slides downward. The weight is \( W = 2(9.81) = 19.62 \) N. Specify the three unknowns.

**Equations of Motion.**

\[ + \sum F_n = ma_n; \quad -N_B + 19.62 \cos \theta = 2 \frac{v^2}{0.5} \quad (1) \]
\[ + \sum F_t = ma_t; \quad 19.62 \sin \theta = 2a_t \quad (2) \]

At the instant \( \theta = \theta_{\text{max}} \), the package leaves the surface of the ramp so that \( N_B = 0 \). Therefore, there are three unknowns, \( v, a_t, \) and \( \theta \).

**Kinematics.** The third equation for the solution is obtained by noting that the magnitude of tangential acceleration \( a_t \) may be related to the speed of the package \( v \) and the angle \( \theta \). Since \( a_t \), \( ds = v \, dv \) and \( ds = r \, d\theta = 0.5 \, d\theta \), Fig. 13–15a, we have

\[ a_t = \frac{v \, dv}{0.5 \, d\theta} \quad (3) \]

To solve, substitute Eq. 3 into Eq. 2 and separate the variables. This gives \( v \, dv = 4.905 \sin \theta \, d\theta \)

Integrate both sides, realizing that when \( \theta = 0^\circ \), \( v_0 = 1 \) m/s.

\[ \int_{1}^{v} v \, dv = 4.905 \int_{0^\circ}^{\theta} \sin \theta \, d\theta \]

\[ \frac{v^2}{2} \bigg|_{1}^{v} = -4.905 \cos \theta \bigg|_{0^\circ}^{\theta}; \quad v^2 = 9.81(1 - \cos \theta) + 1 \]

Substituting into Eq. 1 with \( N_B = 0 \) and solving for \( \cos \theta_{\text{max}} \) yields

\[ 19.62 \cos \theta_{\text{max}} = \frac{2}{0.5} [9.81(1 - \cos \theta_{\text{max}}) + 1] \]

\[ \cos \theta_{\text{max}} = \frac{43.24}{58.86} \]

\[ \theta_{\text{max}} = 42.7^\circ \quad \text{Ans.} \]

**NOTE:** The speed of the package is increasing because its tangential acceleration is increasing with \( \theta \), Eq. 2.
EXAMPLE 13.10

The 2-lb block in Fig. 13–19a moves on a smooth horizontal track, such that its path is specified in polar coordinates by the parametric equations \( r = (10t^2) \) ft and \( \theta = (0.5t) \) rad, where \( t \) is in seconds. Determine the magnitude of the tangential force \( F \) causing the motion at the instant \( t = 1 \) s.

SOLUTION

Free-Body Diagram. As shown on the block’s free-body diagram, Fig. 13–19b, the normal force of the track on the block, \( N \), and the tangential force \( F \) are located at an angle from the \( r \) and axes. This angle can be obtained from Eq. 13–10. To do so, we must first express the path as by eliminating the parameter \( t \) between \( r \) and \( \theta \). This yields

\[
\tan \psi = \frac{r}{dr/d\theta} = \frac{40\theta^2}{40(2\theta)} \bigg|_{\theta=0.5 \text{ rad}} = 0.25
\]

Because \( \psi \) is a positive quantity, it is measured counterclockwise from the \( r \) axis to the tangent (the same direction as \( \theta \)) as shown in Fig. 13–19b. There are presently four unknowns: \( F, N, a_r \) and \( a_\theta \).

Equations of Motion.

\[
\begin{align*}
\sum F_r &= ma_r; \quad F \cos 14.04^\circ - N \sin 14.04^\circ = \frac{2}{32.2} a_r \quad (1) \\
\sum F_\theta &= ma_\theta; \quad F \sin 14.04^\circ + N \cos 14.04^\circ = \frac{2}{32.2} a_\theta \quad (2)
\end{align*}
\]

Kinematics. Since the motion is specified, the coordinates and the required time derivatives can be calculated and evaluated at \( t = 1 \) s.

\[
\begin{align*}
r &= 10t^2 \bigg|_{t=1 \text{ s}} = 10 \text{ ft} \quad \theta = 0.5t \bigg|_{t=1 \text{ s}} = 0.5 \text{ rad} \\
\dot{r} &= 20t \bigg|_{t=1 \text{ s}} = 20 \text{ ft/s} \quad \dot{\theta} = 0.5 \text{ rad/s} \\
\ddot{r} &= 20 \text{ ft/s}^2 \quad \ddot{\theta} = 0 \\
a_r &= \ddot{r} - r\dot{\theta}^2 = 20 - 10(0.5)^2 = 17.5 \text{ ft/s}^2 \\
a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(0) + 2(20)(0.5) = 20 \text{ ft/s}^2
\end{align*}
\]

Substituting into Eqs. 1 and 2 and solving, we get

\[
F = 1.36 \text{ lb} \quad \text{Ans.} \\
N = 0.942 \text{ lb}
\]

NOTE: The tangential axis is in the direction of \( F \), and the normal axis is in the direction of \( N \).
EXAMPLE 13.11

The smooth 2-kg cylinder $C$ in Fig. 13–20a has a peg $P$ through its center which passes through the slot in arm $OA$. If the arm rotates in the vertical plane at a constant rate $\dot{\theta} = 0.5$ rad/s, determine the force that the arm exerts on the peg at the instant $\theta = 60^\circ$.

**Solution**

Why is it a good idea to use polar coordinates to solve this problem?

**Free-Body Diagram.** The free-body diagram for the cylinder is shown in Fig. 13–20b. The force on the peg, $\mathbf{F}_P$, acts perpendicular to the slot in the arm. As usual, $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are assumed to act in the directions of positive $r$ and $\theta$, respectively. Identify the four unknowns.

**Equations of Motion.** Using the data in Fig. 13–20b, we have

\[ + \Sigma F_r = ma_r; \quad 19.62 \sin \theta - N_C \sin \theta = 2a_r \quad (1) \]
\[ + \Sigma F_\theta = ma_\theta; \quad 19.62 \cos \theta + F_P - N_C \cos \theta = 2a_\theta \quad (2) \]

**Kinematics.** From Fig. 13–20a, $r$ can be related to $\theta$ by the equation

\[ r = \frac{0.4}{\sin \theta} = 0.4 \csc \theta \]

Since $d(\csc \theta) = -(\csc \theta \cot \theta) \, d\theta$ and $d(\cot \theta) = -(\csc^2 \theta) \, d\theta$, then $r$ and the necessary time derivatives become

\[ \dot{\theta} = 0.5 \quad r = 0.4 \csc \theta \]
\[ \ddot{\theta} = 0 \quad \ddot{r} = -0.4(\csc \theta \cot \theta)\dot{\theta} \]
\[ \quad = -0.2 \csc \theta \cot \theta \]
\[ \dddot{r} = -0.2(\csc \theta \cot \theta)(\dot{\theta}) \cot \theta - 0.2 \csc \theta(-\csc^2 \theta)\dddot{\theta} \]
\[ \quad = 0.1 \csc \theta(\cot^2 \theta + \csc^2 \theta) \]

Evaluating these formulas at $\theta = 60^\circ$, we get

\[ \dot{\theta} = 0.5 \quad r = 0.462 \]
\[ \dot{\theta} = 0 \quad \dot{r} = -0.133 \]
\[ \quad = 0.192 \]
\[ a_r = \dot{r} - r\dot{\theta}^2 = 0.192 - 0.462(0.5)^2 = 0.0770 \]
\[ a_\theta = r\dddot{\theta} + 2r\dot{\theta}\dot{\theta} = 0 + 2(-0.133)(0.5) = -0.133 \]

Substituting these results into Eqs. 1 and 2 with $\theta = 60^\circ$ and solving yields

\[ N_C = 19.4 \text{ N} \quad F_P = -0.356 \text{ N} \quad \text{Ans.} \]

The negative sign indicates that $\mathbf{F}_P$ acts opposite to the direction shown in Fig. 13–20b.
EXAMPLE 13.12

A can $C$, having a mass of 0.5 kg, moves along a grooved horizontal slot shown in Fig. 13–21a. The slot is in the form of a spiral, which is defined by the equation $r = (0.1\theta)$ m, where $\theta$ is in radians. If the arm $OA$ is rotating at a constant rate $\dot{\theta} = 4$ rad/s in the horizontal plane, determine the force it exerts on the can at the instant $\theta = \pi$ rad. Neglect friction and the size of the can.

**SOLUTION**

**Free-Body Diagram.** The driving force $\mathbf{F}_C$ acts perpendicular to the arm $OA$, whereas the normal force of the wall of the slot on the can, $N_C$, acts perpendicular to the tangent to the curve at $\theta = \pi$ rad, Fig. 13–21b. As usual, $a_r$ and $a_\theta$ are assumed to act in the positive directions of $r$ and $\theta$, respectively. Since the path is specified, the angle which the extended radial line $r$ makes with the tangent, Fig. 13–21c, can be determined from Eq. 13–10. We have $r = 0.1\theta$, so that $dr/d\theta = 0.1$, and therefore

$$\tan \psi = \frac{r}{dr/d\theta} = \frac{0.1\theta}{0.1} = \theta$$

When $\theta = \pi$, $\psi = \tan^{-1}\pi = 72.3^\circ$, so that $\phi = 90^\circ - \psi = 17.7^\circ$, as shown in Fig. 13–21c. Identify the four unknowns in Fig. 13–21b.

**Equations of Motion.** Using $\phi = 17.7^\circ$ and the data shown in Fig. 13–21b, we have

\[\begin{align*}
\Sigma F_r &= ma_r; & N_C \cos 17.7^\circ &= 0.5a_r \\
\Sigma F_\theta &= ma_\theta; & F_C - N_C \sin 17.7^\circ &= 0.5a_\theta
\end{align*}\]

**Kinematics.** The time derivatives of $r$ and $\theta$ are

\[\begin{align*}
\dot{r} &= 4 \text{ rad/s} & r &= 0.1\theta \\
\ddot{r} &= \dot{\theta} = 0 & \ddot{\theta} &= 0.1(4) = 0.4 \text{ m/s} \\
\dot{\theta} &= 0 & \ddot{\theta} &= 0
\end{align*}\]

At the instant $\theta = \pi$ rad,

\[\begin{align*}
a_r &= \ddot{r} - r\dot{\theta}^2 = 0 - 0.1(\pi)(4)^2 = -5.03 \text{ m/s}^2 \\
a_\theta &= \dot{r} \dot{\theta} + 2\dot{r}\ddot{\theta} = 0 + 2(0.4)(4) = 3.20 \text{ m/s}^2
\end{align*}\]

Substituting these results into Eqs. 1 and 2 and solving yields

\[\begin{align*}
N_C &= -2.64 \text{ N} \\
F_C &= 0.800 \text{ N}
\end{align*}\]

Ans.

What does the negative sign for $N_C$ indicate?
Problem 13-12

The particle of weight $W$ is subjected to the action of its weight and forces $F_1 = (ai+bj+ck)$, $F_2 = (dr^2+er+jf+ck)$ and $F_3 = hd$. Determine the distance the ball is from the origin a time $t$ after being released from rest.

**Given:**

- $a := 2lb$
- $e := -4\frac{lb}{s}$
- $b := 6lb$
- $f := -1lb$
- $c := -2\frac{lb}{s}$
- $h := -2\frac{lb}{s}$
- $d := \frac{1}{s^2}$
- $t := 2s$
- $W := 6lb$
- $g := 32.2\ \text{ft/s}^2$

**Solution:**

**x - direction**

\[
\begin{align*}
a + d \cdot t^2 + h \cdot t &= \frac{W}{g} \cdot a_x \\
a_x &= \frac{g}{W} \cdot (a + h \cdot t + d \cdot t^2) \\
v_x &= \frac{g}{W} \left( a \cdot t + h \cdot t^2 + d \cdot t^3 \right) \\
s_x &= \frac{g}{W} \left( a \cdot t^2 + h \cdot t^3 + d \cdot t^4 \right) \\
s_x &= 14.31 \ \text{ft}
\end{align*}
\]

**y - direction**

\[
\begin{align*}
b + e \cdot t &= \frac{W}{g} \cdot a_y \\
a_y &= \frac{g}{W} \cdot (b + e \cdot t) \\
v_y &= \frac{g}{W} \left( b \cdot t + e \cdot t^2 \right) \\
s_y &= \frac{g}{W} \left( b \cdot t^2 + e \cdot t^3 \right) \\
s_y &= 35.78 \ \text{ft}
\end{align*}
\]

**z - direction**

\[
\begin{align*}
c \cdot t + f - W &= \frac{W}{g} \cdot a_z \\
a_z &= \frac{g}{W} \cdot (f - W + c \cdot t) \\
v_z &= \frac{g}{W} \left( f \cdot t - W \cdot t + \frac{c}{2} \cdot t^2 \right) \\
s_z &= \frac{g}{W} \left( f \cdot t^2 - W \cdot t^2 + \frac{c}{6} \cdot t^3 \right) \\
s_z &= -89.44 \ \text{ft}
\end{align*}
\]

**Total distance**

\[
s := \sqrt{s_x^2 + s_y^2 + s_z^2} \\
s = 97.39 \ \text{ft}
\]
Sample Problem 3/9

Compute the magnitude $v$ of the velocity required for the spacecraft $S$ to maintain a circular orbit of altitude 200 mi above the surface of the earth.

**Solution.** The only external force acting on the spacecraft is the force of gravitational attraction to the earth (i.e., its weight), as shown in the free-body diagram. Summing forces in the normal direction yields

$$[\Sigma F_n = ma_n] \quad G \frac{mm_e}{(R + h)^2} = m \frac{v^2}{(R + h)} \quad v = \sqrt{\frac{Gm_e}{(R + h)}} = R \sqrt{\frac{g}{(R + h)}}$$

where the substitution $gR^2 = Gm_e$ has been made. Substitution of numbers gives

$$v = (3959)(5280) \sqrt{\frac{32.234}{(3959 + 200)(5280)}} = 25,326 \text{ ft/sec} \quad \text{Ans.}$$

**Helpful Hint**

Note that, for observations made within an inertial frame of reference, there is no such quantity as “centrifugal force” acting in the minus $n$-direction. Note also that neither the spacecraft nor its occupants are “weightless,” because the weight in each case is given by Newton’s law of gravitation. For this altitude, the weights are only about 10 percent less than the earth-surface values. Finally, the term “zero-$g$” is also misleading. It is only when we make our observations with respect to a coordinate system which has an acceleration equal to the gravitational acceleration (such as in an orbiting spacecraft) that we appear to be in a “zero-$g$” environment. The quantity which does go to zero aboard orbiting spacecraft is the familiar normal force associated with, for example, an object in contact with a horizontal surface within the spacecraft.

Sample Problem 3/10

Tube $A$ rotates about the vertical $O$-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug $B$ of mass $m$ whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius $b$. Determine the tension $T$ in the cord and the horizontal component $F_\theta$ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is $\omega_0$ first in the direction for case (a) and second in the direction for case (b). Neglect friction.

**Solution.** With $r$ a variable, we use the polar-coordinate form of the equations of motion, Eqs. 3/8. The free-body diagram of $B$ is shown in the horizontal plane and discloses only $T$ and $F_\theta$. The equations of motion are

$$[\Sigma F_r = ma_r] \quad -T = m(\ddot{r} - r \dot{\theta}^2)$$

$$[\Sigma F_\theta = ma_\theta] \quad F_\theta = m(r \ddot{\theta} + 2r \dot{\theta})$$

**Case (a).** With $\dot{r} = +b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr \omega^2 \quad F_\theta = 2mb \omega_0 \omega \quad \text{Ans.}$$

**Case (b).** With $\dot{r} = -b\omega_0$, $\ddot{r} = 0$, and $\ddot{\theta} = 0$, the forces become

$$T = mr \omega^2 \quad F_\theta = -2mb \omega_0 \omega \quad \text{Ans.}$$

**Helpful Hint**

The minus sign shows that $F_\theta$ is in the direction opposite to that shown on the free-body diagram.