

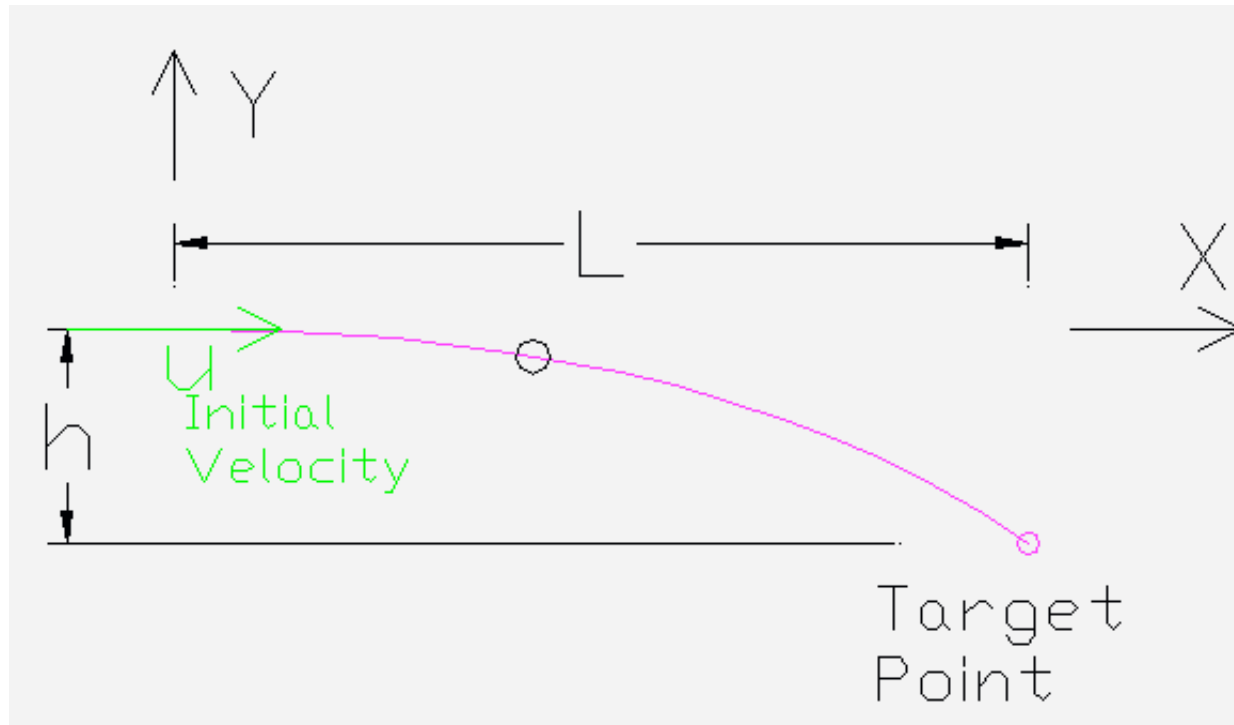
## MECH 2110 - Statics & Dynamics

Chapter D2 Problem 69 Solution

Page 47, Engineering Mechanics - Dynamics, 4th Edition, Meriam and Kraige

Given: A rock of mass  $m$  launched with a horizontal velocity  $u$  and acted upon exclusively by gravity. The rock must cover a horizontal distance,  $L$ , equal to 40 m, while dropping no more than a distance,  $h$ , equal to 10 m.

Find: The minimum acceptable value of the horizontal launch velocity,  $u$ .



The figure shows the particle (black circle) at a position on its path (purple) sometime after the boy has thrown it. The launch and target points are indicated. The relevant distances are indicated. The coordinate axes are shown. The initial horizontal velocity,  $u$ , (green) is shown. Note that as expected, the initial velocity is tangent to the path at the launch point (velocity is always tangent to the path).

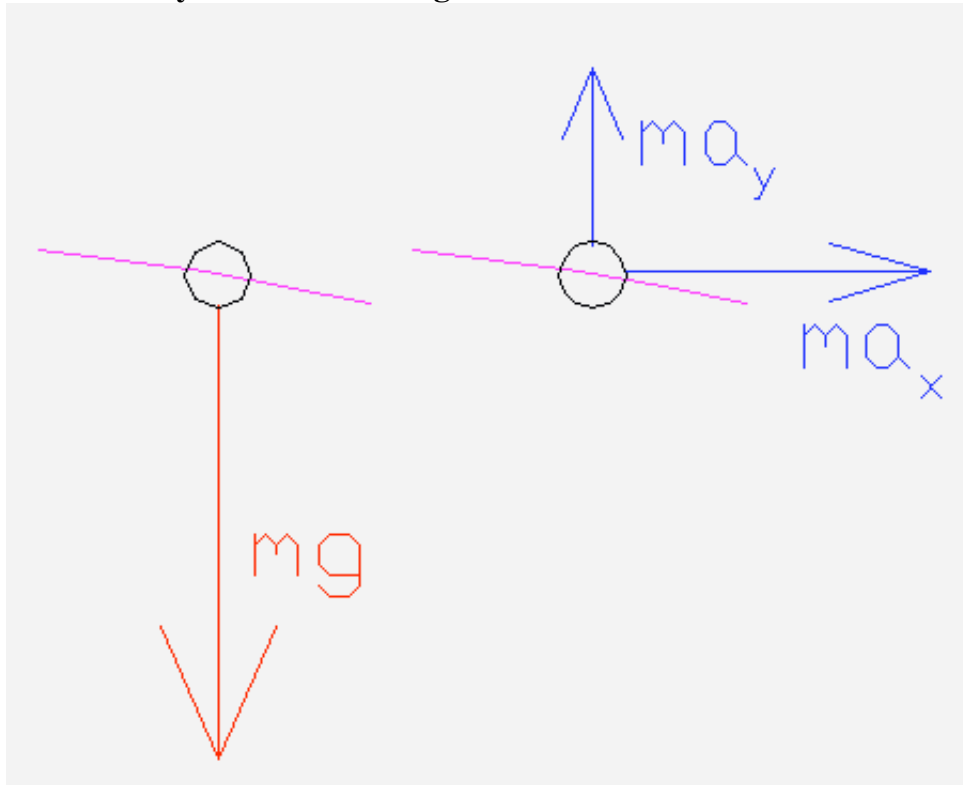
### 0. Observations:

1. Neglect air resistance.
2. From a knowledge of the forces acting on the rock, we can determine the acceleration of the rock.
3. By integrating the acceleration of the rock with respect to time, we can determine the velocity of the rock.
4. By integrating the velocity of the rock with respect to time, we can determine the position of the rock.
5. From the position of the rock, we can determine the minimum necessary launch velocity.
6. The above process is applied so frequently, that many students memorize the results. This is acceptable

as long as you remember the conditions under which the memorized equations are applicable. The full development is presented here to reinforce the basic procedures applicable to any dynamics problem.

**1. Mechanical System** - Rock from the instant of release until it covers a horizontal distance equal to  $L$ .

## 2. Free Body and Motion Diagrams



The figure provides the free body diagram (left) and the motion diagram (right). The rock (black circle in both diagrams) is analyzed at an arbitrary point along its path (purple). A small segment of the path near the position of the particle at the instant of analysis is shown in both diagrams (purple). The free body diagram depicts the forces acting on the particle (red) at the instant of analysis. The only force is the weight directed vertically downward. The motion diagram depicts the product of the particle mass and acceleration at the analysis instant. This product is shown in blue. Not having yet determined these accelerations, we assume that both components are positive (this will turn out to be incorrect as is obvious from the free body diagram). The net force on the particle must equal the product of the particle mass and its acceleration.

## 3. Equations

$$\sum F_X = 0 = m a_X$$

$$\sum F_Y = -m g = m a_Y$$

## 4. Solve

From the X-force equation:

$$a_X = 0$$

Recalling the relationship between velocity x-component, acceleration x-component, and time:

$$dv_X/dt = a_X = 0$$

Separating variables and integrating (noting that the initial value of the x-component of the velocity is  $u$ ):

$$\int_u^{v_X} dv_X = \int_0^t 0 dt$$

$$v_X|_u^{v_X} = 0$$

$$v_X - u = 0$$

$$v_X = u$$

Recalling the relationship between position x-component, velocity x-component, and time:

$$dx/dt = v_X$$

$$= u$$

Separating variables and integrating (noting that the initial x-coordinate of the rock is 0):

$$\int_0^x dx = \int_0^t u dt$$

$$x|_0^x = u t|_0^t$$

$$x = u t$$

The time required to reach an x-coordinate of u can now be evaluated:

$$L = u t_L$$

$$t_L = L/u$$

From the Y-Force equation:

$$a_Y = -g$$

Recalling the relationship between velocity y-component, acceleration y-component, and time:

$$dv_Y/dt = a_Y = -g$$

Separating variables and integrating (noting that the initial velocity is horizontal so that the initial value of the y-component of the velocity is 0):

$$\int_0^{v_Y} dv_Y = \int_0^t -g dt$$

$$v_Y|_0^{v_Y} = -g t|_0^t$$

$$v_Y - 0 = -g t$$

$$v_Y = -g t$$

Recalling the relationship between position y-component, velocity y-component, and time:

$$dy/dt = v_Y$$

$$= -g t$$

Separating variables and integrating (noting that the initial y-coordinate of the rock is 0):

$$\int_0^y dy = \int_0^t -g t dt$$

$$y|_0^y = -1/2 g t^2|_0^t$$

$$y = -1/2 g t^2$$

The y-coordinate of the rock after it has traveled a horizontal distance L can now be evaluated:

$$y_L = -1/2 g t_L^2$$

$$= -1/2 g (L/u)^2$$

Enforcing our requirement that the y-coordinate of the rock at this horizontal position be greater than or

equal to  $-h$ :

$$-h \leq -\frac{1}{2} g (L/u)^2$$

At the minimum acceptable speed, the  $y$ -coordinate of the rock at this horizontal position will be equal to the critical value:

$$-h = -\frac{1}{2} g (L/u)^2$$

$$u = \left\{ \frac{1}{2} g L^2/h \right\}^{1/2}$$

$$= \left\{ \frac{1}{2} 9.81 \text{ m/s}^2 (40 \text{ m})^2 / 10 \text{ m} \right\}$$

$$= 28.0 \text{ m/s}$$

### Results

Minimum launch velocity =  $u = 28.0 \text{ m/s}$

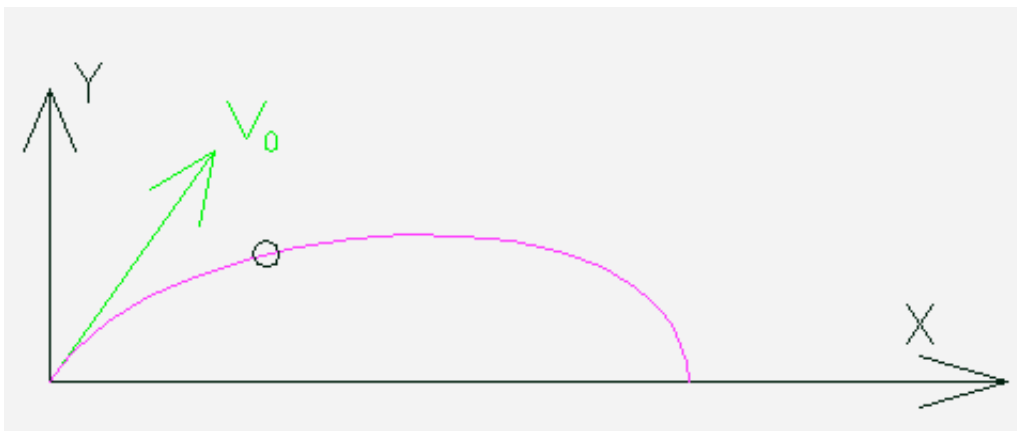
# MECH 2110 - Statics & Dynamics

## Chapter D2 Problem 95 Solution

Page 53, Engineering Mechanics - Dynamics, 4th Edition, Meriam and Kraige

Given: A particle of mass  $m$  is ejected into an experimental fluid. The particle is acted upon by gravity and a drag force that is proportional to the speed of the particle and directed oppositely to the velocity of the particle. The particle is ejected into the fluid with an initial speed of  $v_0$  and makes an angle of  $\mathbf{q}$  with the horizontal ( $x$ -axis). The gravitational force is directed in the  $-y$  direction and has magnitude  $m g$ . The drag force, expressed in vector form, is given by  $-m k \mathbf{v}$ , where  $k$  is a constant.

Find: The time history of the  $x$  and  $y$ -components of the velocity. The time history of the  $x$  and  $y$ -components of the position. The terminal velocity.



The figure shows the particle (black circle) at a position on its path (purple) sometime after ejection into the fluid. The coordinate axes are shown. The initial velocity,  $v_0$ , (green) is shown. This velocity makes an angle  $\mathbf{q}$  with the  $X$ -axis. Note that as expected, the initial velocity is tangent to the path at the ejection point (velocity is always tangent to the path).

### 0. Observations:

1. The gravitational force can be expressed in vector form as:  $-m g \mathbf{j}$

2. The drag force can be expressed in vector form as:

$$\mathbf{F}_D = -m k \mathbf{v} = -m k v_x \mathbf{i} - m k v_y \mathbf{j} \quad \{ v_x, v_y \text{ are the } x \text{ and } y\text{-components of the velocity} \}$$

3. The mass times the acceleration vector can be expressed in vector form as:

$$m \mathbf{a} = m a_x \mathbf{i} + m a_y \mathbf{j} \quad \{ a_x, a_y \text{ are the } x \text{ and } y\text{-components of the acceleration} \}$$

4. The components of the acceleration, velocity, and acceleration vectors are related as follows:

$$\begin{aligned} dv_x/dt &= a_x & dv_y/dt &= a_y \\ dx/dt &= v_x & dy/dt &= v_y \end{aligned}$$

5. At the instant of ejection, the  $x$  and  $y$ -components of the velocity can be related to the speed and the angle  $\mathbf{q}$ .

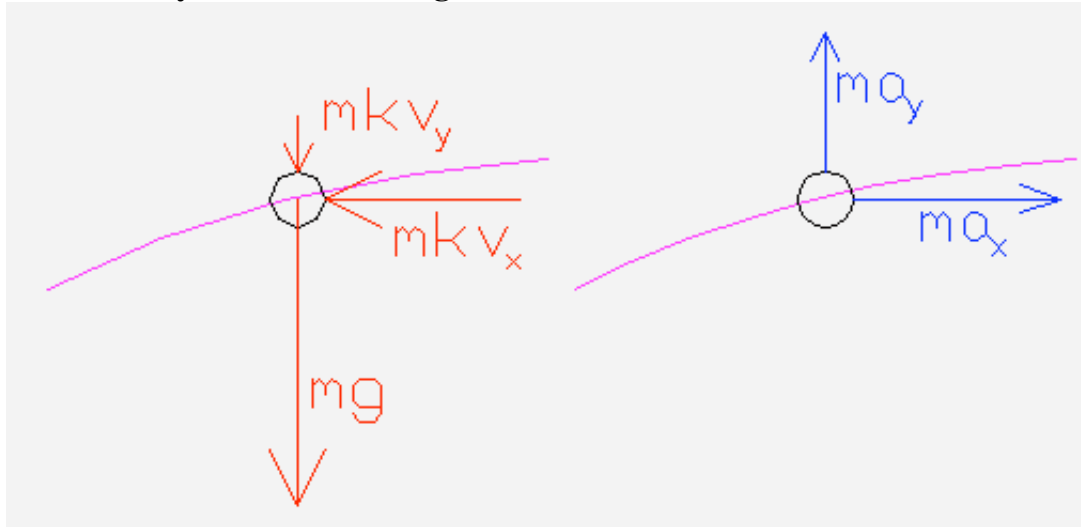
$$v_{x0} = v_0 \cos(\mathbf{q})$$

$$v_{y0} = v_0 \sin(\theta)$$

6. The ejection position of the particle will be chosen as the coordinate origin. Thus the x and y-coordinates of the particle at the instant of ejection are both 0.

**1. Mechanical System** - The particle, from the instant of ejection into the fluid onward. The instant of ejection will be taken as time,  $t$ , equal to zero.

## 2. Free Body and Motion Diagrams



The figure provides the free body diagram (left) and the motion diagram (right). The particle (black circle in both diagrams) is analyzed at an arbitrary point along its path (purple). A small segment of the path near the position of the particle at the instant of analysis is shown in both diagrams (purple). For simplicity, the instant of analysis has been chosen where the particle is moving to the right and upward, that is both the x and y-components of the velocity are positive. The free body diagram depicts the forces acting on the particle (red) at the instant of analysis. These forces include gravity and the drag force. The drag force is shown acting opposite to the velocity. The velocity is always tangent to the path, therefore the drag force must also be tangent to the path. This explains why the x-component of the drag force has been depicted with a larger arrow than was the y-component. The motion diagram depicts the product of the particle mass and acceleration at the analysis instant. This product is shown in blue. Not having yet determined these accelerations, we assume that both components are positive (this will turn out to be incorrect as is obvious from the free body diagram). The net force on the particle must equal the product of the particle mass and its acceleration.

## 3. Equations

$$\sum F_X = -m k v_x = m a_x$$

$$\sum F_Y = -m g - m k v_y = m a_y$$

## 4. Solve

Dividing the above equations by the particle mass:

$$a_x = -k v_x$$

$$a_y = -g - k v_y$$

Recalling the definition of acceleration:

$$a_x = dv_x/dt = -k v_x$$

$$a_y = dv_y/dt = -g - k v_y$$

Separating variables in the x-equation and integrating:

$$\int_{v_x0}^{v_x} dv_x/v_x = - \int_0^t k dt$$

Integrating and using the above expression for the initial x-component of the velocity:

$$\ln(v_x) - \ln(v_0 \cos(\alpha)) = -k t$$

Evaluating and simplifying:

$$\ln(v_x) - \ln(v_0 \cos(\alpha)) = -k t$$

$$\ln(v_x / v_0 \cos(\alpha)) = -k t$$

$$v_x / v_0 \cos(\alpha) = \exp(-k t)$$

$$v_x = v_0 \cos(\alpha) \exp(-k t)$$

As t get large, the term with the negative exponent gets smaller and smaller, approaching zero. The terminal x-component of the velocity is zero.

Separating variables in the y-equation and integrating:

$$\int_{v_y0}^{v_y} dv_y / \{g + k v_y\} = - \int_0^t dt$$

Integrating and using the above expression for the initial y-component of the velocity:

$$1/k \ln(g + k v_y) - 1/k \ln(g + k v_0 \sin(\alpha)) = -t$$

Evaluating and simplifying:

$$\ln(g + k v_y) - \ln(g + k v_0 \sin(\alpha)) = -k t$$

$$\ln([g + k v_y] / [g + k v_0 \sin(\alpha)]) = -k t$$

$$[g + k v_y] / [g + k v_0 \sin(\alpha)] = \exp(-k t)$$

$$g + k v_y = [g + k v_0 \sin(\alpha)] \exp(-k t)$$

$$v_y = -g/k + [g/k + v_0 \sin(\alpha)] \exp(-k t)$$

As t get large, the term with the negative exponent gets smaller and smaller, approaching zero. The terminal y-component of the velocity is -g/k.

Recalling the definition of velocity:

$$dx/dt = v_x = v_0 \cos(\alpha) \exp(-k t)$$

$$dy/dt = v_y = -g/k + [g/k + v_0 \sin(\alpha)] \exp(-k t)$$

Separating variables in the x-equation and integrating:

$$\int_0^x dx = \int_0^t v_0 \cos(\alpha) \exp(-k t) dt$$

$$x - 0 = -1/k v_0 \cos(\alpha) \exp(-k t) + 1/k v_0 \cos(\alpha)$$

$$x = 1/k v_0 \cos(\alpha) \{ 1 - \exp(-k t) \}$$

Separating variables in the y-equation and integrating:

$$\int_0^y dy = \int_0^t \{-g/k + [g/k + v_0 \sin(\alpha)] \exp(-k t)\} dt$$

$$y|_0^y = \{ -g/k t - 1/k [ g/k + v_0 \sin(\mathbf{q}) ] \exp(-k t) \} |_0^t$$

$$y = -g/k t + 1/k [ g/k + v_0 \sin(\mathbf{q}) ] [ 1 - \exp(-k t) ]$$

Can you plot the path of the particle based upon the above results?

### Results

#### Velocity Components

$$v_x = v_0 \cos(\mathbf{q}) \exp(-k t)$$

$$v_y = -g/k + [ g/k + v_0 \sin(\mathbf{q}) ] \exp(-k t)$$

#### Position Components

$$x = 1/k v_0 \cos(\mathbf{q}) \{ 1 - \exp(-k t) \}$$

$$y = -g/k t + 1/k [ g/k + v_0 \sin(\mathbf{q}) ] [ 1 - \exp(-k t) ]$$

#### Terminal Velocity Components

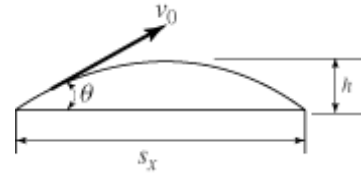
$$v_{xt} = 0$$

$$v_{yt} = -g/k \quad \{ \text{Falling straight down with a constant speed of } g/k \}$$

### Problem 12-81

The nozzle of a garden hose discharges water at the rate  $v_0$ . If the nozzle is held at ground level and directed at angle  $\theta$  from the ground, determine the maximum height reached by the water and the horizontal distance from the nozzle to where the water strikes the ground.

**Given:**  $v_0 := 15 \frac{\text{m}}{\text{s}}$        $\theta := 30\text{deg}$        $g := 9.81 \frac{\text{m}}{\text{s}^2}$



**Solution:**

$$a_x = 0 \qquad a_y = -g$$

$$v_x = v_0 \cdot \cos(\theta) \qquad v_y = -g \cdot t + v_0 \cdot \sin(\theta)$$

$$s_x = v_0 \cdot \cos(\theta) \cdot t \qquad s_y = \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$$

Maximum height       $0 = -g \cdot t + v_0 \cdot \sin(\theta)$        $t := \frac{v_0 \cdot \sin(\theta)}{g}$        $t = 0.76 \text{ s}$

$$h := \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t \qquad \mathbf{h = 2.87 \text{ m}}$$

Horizontal distance       $0 = \frac{-1}{2} \cdot g \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$        $t := \frac{2 \cdot v_0 \cdot \sin(\theta)}{g}$        $t = 1.53 \text{ s}$

$$d := v_0 \cdot \cos(\theta) \cdot t \qquad \mathbf{d = 19.86 \text{ m}}$$

### Problem 12-85

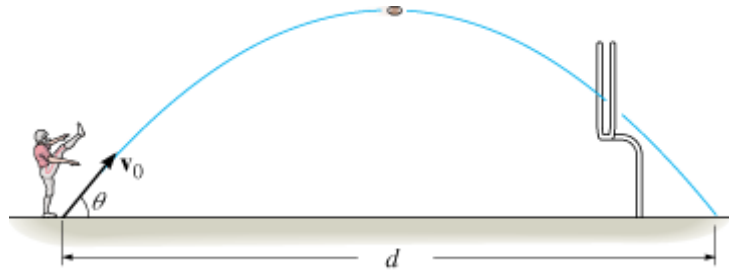
From a videotape, it was observed that a pro football player kicked a football distance  $d$  during a measured time of  $t$ . Determine the initial speed of the ball and the angle at which it was kicked.

**Given:**

$$d := 126\text{ft}$$

$$t := 3.6\text{s}$$

$$g := 32.2 \frac{\text{ft}}{\text{s}^2}$$



**Solution:**

Guesses     $\theta := 30\text{deg}$      $v_0 := 10 \frac{\text{ft}}{\text{s}}$

Given     $0 = \frac{-g}{2} \cdot t^2 + v_0 \cdot \sin(\theta) \cdot t$      $d = v_0 \cdot \cos(\theta) \cdot t$

$$\begin{pmatrix} \theta \\ v_0 \end{pmatrix} := \text{Find}(\theta, v_0)$$

$$v_0 = 67.71 \frac{\text{ft}}{\text{s}}$$

$$\theta = 58.87 \text{ deg}$$

**Problem 12-90**

A golf ball is struck with a velocity  $v_A$  as shown. Determine the distance  $d$  to where it will land.

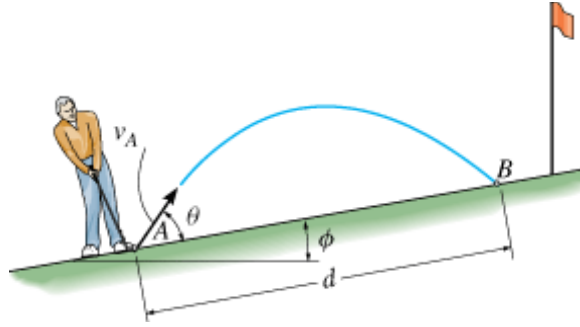
**Given:**

$$v_A := 80 \frac{\text{ft}}{\text{s}}$$

$$\theta := 45\text{deg}$$

$$\phi := 10\text{deg}$$

$$g := 32.2 \frac{\text{ft}}{\text{s}^2}$$



**Solution:**

**Guesses**     $t := 3\text{s}$      $d := 1\text{ft}$

**Given**     $d \cdot \cos(\phi) = v_A \cdot \cos(\theta + \phi) \cdot t$      $d \cdot \sin(\phi) = \frac{-g}{2} \cdot t^2 + v_A \cdot \sin(\theta + \phi) \cdot t$

$$\begin{pmatrix} t \\ d \end{pmatrix} := \mathbf{Find}(t, d) \quad t = 3.57 \text{ s} \quad \mathbf{d = 166.24 \text{ ft}}$$

### Problem 12-93

The ball at A is kicked with a speed  $v_A$  and at an angle  $\theta_A$ . Determine the point  $(x, -y)$  where it strikes the ground. Assume the ground has the shape of a parabola as shown.

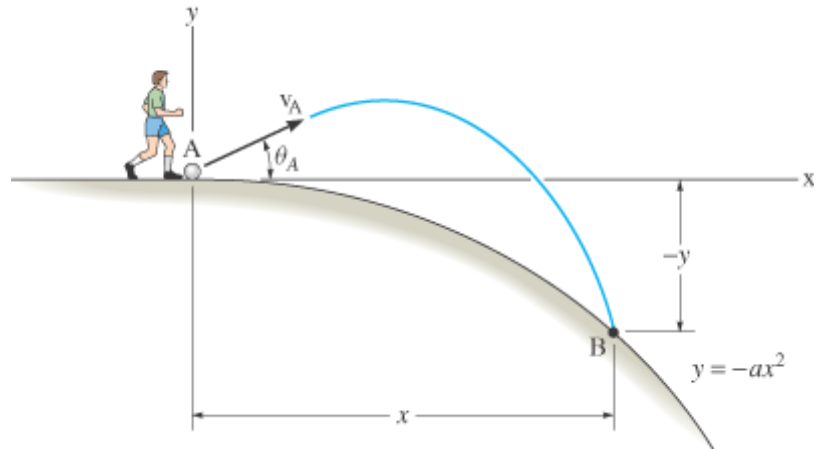
**Given:**

$$v_A := 10 \frac{\text{ft}}{\text{s}}$$

$$\theta_A := 30\text{deg}$$

$$a := 0.04\text{ft}^{-1}$$

$$g := 32.2 \frac{\text{ft}}{\text{s}^2}$$



**Solution:**

**Guesses**     $x := 10\text{ft}$      $y := -10\text{ft}$      $t := 3\text{s}$

**Given**     $x = v_A \cdot \cos(\theta_A) \cdot t$      $y = \frac{-g}{2} \cdot t^2 + v_A \cdot \sin(\theta_A) \cdot t$      $y = -a \cdot x^2$

$$\begin{pmatrix} x \\ y \\ t \end{pmatrix} := \mathbf{Find}(x, y, t) \quad t = 0.38 \text{ s} \quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3.31 \\ -0.44 \end{pmatrix} \text{ft}$$



$$\mathbf{x = 3.31 \text{ ft}}$$

### Problem 12-95

Determine the horizontal velocity  $v_A$  of a tennis ball at  $A$  so that it just clears the net at  $B$ . Also, find the distance  $s_C$  where the ball strikes the ground.

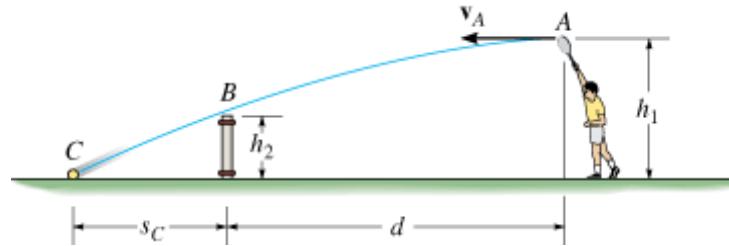
Given:

$$h_1 := 7.5 \cdot \text{ft}$$

$$h_2 := 3 \cdot \text{ft}$$

$$d := 21 \cdot \text{ft}$$

$$g := 32.2 \frac{\text{ft}}{\text{s}^2}$$



Solution:

Guesses  $t_B := 1 \text{ s}$   $t_C := 1 \text{ s}$   $v_A := 1 \frac{\text{ft}}{\text{s}}$   $s_C := 1 \text{ ft}$

Given  $d = v_A \cdot t_B$   $d + s_C = v_A \cdot t_C$

$$h_2 = \frac{-g}{2} \cdot t_B^2 + h_1 \quad 0 = \frac{-g}{2} \cdot t_C^2 + h_1$$

$$\begin{pmatrix} t_B \\ t_C \\ v_A \\ s_C \end{pmatrix} := \text{Find}(t_B, t_C, v_A, s_C) \quad \begin{pmatrix} t_B \\ t_C \end{pmatrix} = \begin{pmatrix} 0.53 \\ 0.68 \end{pmatrix} \text{ s} \quad v_A = 39.72 \frac{\text{ft}}{\text{s}} \quad s_C = 6.11 \text{ ft}$$