Chapter 5
Equilibrium

5.1 Equilibrium Equations

A body is in equilibrium when it is stationary or in steady translation relative to an inertial reference frame. The following conditions are satisfied when a body, acted upon by a system of forces and moments, is in equilibrium:

1. the sum of the forces is zero

\[ \sum F = 0. \quad (5.1) \]

2. the sum of the moments about any point is zero

\[ \sum M_P = 0, \forall P. \quad (5.2) \]

If the sum of the forces acting on a body is zero and the sum of the moments about one point is zero, then the sum of the moments about every point is zero.

**Proof.** The body shown in Figure 5.1, is subjected to forces \( F_{Ai}, i = 1, ..., n \), and couples \( M_j, j = 1, ..., m \). The sum of the forces is zero

\[ \sum F = \sum_{i=1}^{n} F_{Ai} = 0, \]

and the sum of the moments about a point \( P \) is zero

\[ \sum M_P = \sum_{i=1}^{n} r_{PAi} \times F_{Ai} + \sum_{j=1}^{m} M_j = 0, \]

where \( r_{PAi} = \overrightarrow{PA}_i, i = 1, ..., n \). The sum of the moments about any other point \( Q \) is

\[ \sum M_Q = \sum_{i=1}^{n} r_{QAi} \times F_{Ai} + \sum_{j=1}^{m} M_j = \]
Fig. 5.1 Body subjected to forces $F_{Ai}$ and couples $M_j$.

\[
\sum_{i=1}^{n} (r_{QP} + r_{PAi}) \times F_{Ai} + \sum_{j=1}^{m} M_j = \\
r_{QP} \times \sum_{i=1}^{n} F_{Ai} + \sum_{i=1}^{n} r_{PAi} \times F_{Ai} + \sum_{j=1}^{m} M_j = \\
r_{QP} \times 0 + \sum_{i=1}^{n} r_{PAi} \times F_{Ai} + \sum_{j=1}^{m} M_j = \\
\sum_{i=1}^{n} r_{PAi} \times F_{Ai} + \sum_{j=1}^{m} M_j = \sum M_P = 0.
\]

A body subjected to concurrent forces $F_1, F_2, \ldots, F_n$ and no couples. If the sum of the concurrent forces is zero,

\[
F_1 + F_2 + \ldots + F_n = 0,
\]

the sum of the moments of the forces about the concurrent point is zero, so the sum of the moments about every point is zero. The only condition imposed by equilibrium on a set of concurrent forces is that their sum is zero.
5.2 Supports

5.2.1 Planar Supports

The *reactions* are forces and couples exerted on a body by its supports. The following force convention is defined: $F_{ij}$ represents the force exerted by link $i$ on link $j$.

*Pin Support*

Figure 5.2 shows a pin support. A beam 1 is attached by a smooth pin to a ground bracket 0. The pin passes through the bracket and the beam. The beam can rotate about the axis of the pin. The beam cannot translate relative to the bracket because the support exerts a reactive force that prevents this movement. The pin support is not capable of exerting a couple. Thus a pin support can exert a force on a body in any direction. The force of the pin support 0 on the beam 1 at point $A$, Figure 5.3, is expressed in terms of its components in plane

$$ F_{01} = F_{01x} + F_{01y}, $$

Fig. 5.2 Pin joint

Fig. 5.3 Pin joint forces
The directions of the reactions $F_{01x}$ and $F_{01y}$ are positive. If one determine $F_{01x}$ or $F_{01y}$ to be negative, the reaction is in the direction opposite to that of the arrow. The force of the beam 1 on the pin support 0 at point A, Figure 5.3, is expressed

$$
F_{10} = F_{10x} \hat{i} + F_{10y} \hat{j} = -F_{01x} \hat{i} - F_{01y} \hat{j},
$$

where $F_{10x} = -F_{01x}$ and $F_{10y} = -F_{01y}$. The pin supports are used in mechanical devices that allow connected links to rotate relative to each other.

Roller Support

Figure 5.4(a) represents a roller support which is a pin support mounted on rollers.

![Fig. 5.4 Roller support](image)

The roller support 0 can only exert a force normal (perpendicular) to the surface 1 on which the roller support moves freely, Fig. 5.4(b)

$$
F_{01} = F_{01y} \hat{j}.
$$

The roller support cannot exert a couple about the axis of the pin and it cannot exert a force parallel to the surface on which it translates. Figure 5.4(c) shows other schematic representations used for the roller support. A plane link on a smooth surface can also modeled by a roller support. Bridges and beams can be supported in this way and they will be capable of expansion and contraction.

Fixed Support

Figure 5.5 shows a fixed support or built-in support. The body is literally built into a wall. A fixed support 0 can exert two components of force and a couple on the link 1

$$
F_{01} = F_{01x} \hat{i} + F_{01y} \hat{j}, \quad \text{and} \quad M_{01} = M_{01z} \hat{k}.
$$
5.2 Supports

5.2.2 Three-Dimensional Supports

Ball and Socket Support

Figure 5.6 shows a ball and socket support, where the supported body is attached to a ball enclosed within a spherical socket. The socket permits the body only to rotate freely. The ball and socket support cannot exert a couple to prevent rotation. The ball and socket support can exert three components of force
\[ \mathbf{F}_{21} = F_{21x}\hat{i} + F_{21y}\hat{j} + F_{21z}\hat{k}. \]

Bearing Support

The type of bearing shown in Fig. 5.7(a) supports a circular shaft while permitting it to rotate about its axis, z-axis. In the most general case, as shown in Fig. 5.7(b), the bearing can exert a force on the supported shaft in each coordinate direction, \( F_{21x}, F_{21y}, F_{21z} \), and can exert couples about axes perpendicular to the shaft, \( M_{21x}, M_{21y} \), but cannot exert a couple about the axis of the shaft. Situations can occur in which the bearing exerts no couples, or exerts no couples and no force parallel to the shaft axis as shown in Fig. 5.7(c). Some radial bearings are designed in this way for specific applications.
5.3 Free-Body Diagrams

Free-body diagrams are used to determine forces and moments acting on simple bodies in equilibrium. The beam in Figure 5.8(a) has a pin support at the left end \( A \) and a roller support at the right end \( B \). The beam is loaded by a force \( F \) and a moment \( M \) at \( C \). To obtain the free-body diagram first the beam is isolated from its supports. Next, the reactions exerted on the beam by the supports are shown on the the free-body diagram, Figure 5.8. Once the free-body diagram is obtained one can apply the equilibrium equations.

The steps required to determine the reactions on bodies are
1. draw the free-body diagram, isolating the body from its supports and showing the forces and the reactions;
2. apply the equilibrium equations to determine the reactions.

For two-dimensional systems, the forces and moments are related by three scalar equilibrium equations

\[
\sum F_x = 0, \tag{5.3} \\
\sum F_y = 0, \tag{5.4} \\
\sum M_P = 0, \forall P, \tag{5.5}
\]
One can obtain more than one equation from Eq. (5.5) by evaluating the sum of the moments about more than one point. The additional equations will not be independent of Eqs. (5.3)-(5.5). One cannot obtain more than three independent equilibrium equations from a two-dimensional free-body diagram, which means one can solve for at most three unknown forces or couples.

**Free-Body Diagrams for Kinematic Chains**

A free-body diagram is a drawing of a part of a complete system, isolated in order to determine the forces acting on that rigid body. The vector \( \mathbf{F}_{ij} \) represents the force exerted by link \( i \) on link \( j \) and \( \mathbf{F}_{ij} = -\mathbf{F}_{ji} \). Figure 5.9 shows the joint reaction forces for a pin joint, Fig. 5.9 (a), and a slider joint, Fig. 5.9 (b). Figure 5.10 shows various free-body diagrams that are considered in the analysis of a slider-crank mechanism Fig. 5.10 (a). In Fig. 5.10 (b), the free body consists of the three moving links isolated from the frame 0. The forces acting on the system include an external driven force \( \mathbf{F}_0 \), and the forces transmitted from the frame at joint \( A, \mathbf{F}_{01} \), and at joint \( C, \mathbf{F}_{03} \). Figure 5.10(c) is a free-body diagram of the two links 1 and 2 and Fig. 5.10(d) is a free-body diagram of the two links 0 and 1. Figure 5.10(e) is a free-body diagram of crank 1 and Fig. 5.10(f) is a free-body diagram of slider 3.

The force analysis can be accomplished by examining individual links or a subsystem of links. In this way the joint forces between links as well as the required input
force or moment for a given output load are computed.

For three-dimensional systems, the forces and moments are related by six scalar equilibrium equations

\[ \sum F_x = 0, \quad \sum F_y = 0, \quad \sum F_z = 0, \quad \sum M_x = 0, \quad \sum M_y = 0, \quad \sum M_z = 0. \]  
\[ (5.6) \]
One can evaluate the sums of the moments about any point. Although one can obtain other equations by summing the moments about additional points, they will not be independent of these equations. For a three-dimensional free-body diagram one can obtain six independent equilibrium equations and one can solve for at most six unknown forces or couples.

A body has redundant supports when the body has more supports than the minimum number necessary to maintain it in equilibrium. Redundant supports are used whenever possible for strength and safety. Each support added to a body results in additional reactions. The difference between the number of reactions and the number of independent equilibrium equations is called the degree of redundancy.

A body has improper supports if it will not remain in equilibrium under the action of the loads exerted on it. The body with improper supports will move when the loads are applied.

5.4 Two-Force and Three-Force Members

A body is a two-force member if the system of forces and moments acting on the body is equivalent to two forces acting at different points.

For example a body is subjected to two forces, \( F_A \) and \( F_B \), at \( A \) and \( B \). If the body is in equilibrium, the sum of the forces equals zero only if \( F_A = -F_B \). Furthermore, the forces \( F_A \) and \( -F_B \) form a couple, so the sum of the moments is not zero unless the lines of action of the forces lie along the line through the points \( A \) and \( B \). Thus for equilibrium the two forces are equal in magnitude, are opposite in direction, and have the same line of action. However, the magnitude cannot be calculated without additional information.

A body is a three-force member if the system of forces and moments acting on the body is equivalent to three forces acting at different points.

**Theorem.** If a three-force member is in equilibrium, the three forces are coplanar and the three forces are either parallel or concurrent.

**Proof.** Let the forces \( F_1, F_2, \) and \( F_3 \) acting on the body at \( A_1, A_2, \) and \( A_3 \). Let \( \pi \) be the plane containing the three points of application \( A_1, A_2, \) and \( A_3 \). Let \( \Delta = A_1A_2 \) be the line through the points of application of \( F_1 \) and \( F_2 \). Since the moments due to \( F_1 \) and \( F_2 \) about \( \Delta \) are zero, the moment due to \( F_3 \) about \( \Delta \) must equal zero,

\[
[n \cdot (r \times F_3)] \cdot n = [F_3 \cdot (n \times r)] \cdot n = 0,
\]

where \( n \) is the unit vector of \( \Delta \). This equation requires that \( F_3 \) be perpendicular to \( n \times r \), which means that \( F_3 \) is contained in \( \pi \). The same procedure can be used to show that \( F_1 \) and \( F_2 \) are contained in \( \pi \), so the forces \( F_1, F_2, \) and \( F_3 \) are coplanar.

If the three coplanar forces are not parallel, there will be points where their lines of action intersect. Suppose that the lines of action of two forces \( F_1 \) and \( F_2 \) intersect at a point \( P \). Then the moments of \( F_1 \) and \( F_2 \) about \( P \) are zero. The sum of the
moments about \( P \) is zero only if the line of action of the third force, \( F_3 \), also passes through \( P \). Therefore either the forces are concurrent or they are parallel.

The analysis of a body in equilibrium can often be simplified by recognizing the two-force or three-force member.

### 5.5 Plane Trusses

A structure composed of links joined at their ends to form a rigid structure is called a truss. Roof supports and bridges are common examples of trusses. When the links of the truss are in a single plane, the truss is called a plane truss. Three bars linked by pins joints at their ends form a rigid frame or noncollapsible frame. The basic element of a plane truss is the triangle, Fig. 5.11. Four, five or more bars pin-connected to form a polygon of as many sides form a nonrigid frame. A nonrigid frame is made rigid, or stable, by adding a diagonal bars and forming triangles. Frameworks built from a basic triangle are known as simple trusses. The truss is statically indeterminate when more links are present than are needed to prevent collapse. Additional links or supports which are not necessary for maintaining the equilibrium configuration are called redundant.

Several assumptions are made in the force analysis of simple trusses. First, all the links are considered to be two-force members. Each link of a truss is straight and has two nodes as points of application of the forces. The two forces are applied at the ends of the links and are necessarily equal, opposite, and collinear for equilibrium. The link may be in tension (\( T \)) or compression (\( C \)), as shown in Fig. 5.12.

The weight of the link is small compared with the force it supports. If the weight of the link is not small, the weight \( W \) of the member is replaced by two forces, each \( W/2 \) one force acting at each end of the member. These weight forces are considered as external loads applied to the pin connections. The connection between the links are assumed to be smooth pin joints. All the external forces are applied at the pin connections of the trusses. For large trusses, a roller support is used at one of the supports to provide for expansion and contraction due to temperature changes and for deformation from applied loads. Trusses and frames in which no such provision is made are statically indeterminate. Two methods for the force analysis of simple trusses will be given. Each method will be explained for the simple truss shown.
in Fig. 5.13(a). The length of the links are $AB = BE = ED = BC = CD = a$. The external force at $E$ is given and has the magnitude of $F$. The free-body diagram of the truss as a whole is shown in Fig. 5.13(b). The external support reactions are usually determined first, by applying the equilibrium equations to the truss as a whole. The reaction force on the truss at the pin support $A$ is $F_A = F/2$ and the reaction force on the truss at the roller support $C$ is $F_C = F/2$.

**Fig. 5.13** Simple truss

**Method of Joints**

This method for calculating the forces in the members consists of writing the conditions of equilibrium for the forces acting on the connecting pin of each joint. The method deals with the equilibrium of concurrent forces, and only two independent equilibrium equations are involved. The analysis starts with any joint where at least one known force exists and where not more than two unknown forces are located. For the truss shown in Fig. 5.13 the analysis begins with the pin at $A$. The force in each link is designated by the two letters defining the ends of the member. The proper directions of the forces should be evident by inspection for this simple case. The free-body diagrams of portions of members $AE$ and $AB$ are also shown in
Fig. 5.14(a). Figure 5.14(a) indicates the mechanism of the action and reaction in the links. The force $AB$ is drawn from the right side and is shown acting away from the pin. The tension (force $AB$) is indicated by an arrow away from the pin and the compression (force $AE$) is indicated by an arrow toward the pin. The magnitudes of $AB$ and $AE$ are obtained from the equations $\sum F_x = 0$ and $\sum F_y = 0$ or

$-AE \sqrt{2}/2 + AB = 0$ and $F_A = AE \sqrt{2}/2 = 0$ or $AE = F \sqrt{2}/2$ and $AB = F/2$.

Joint C is analyzed next, Fig. 5.14(b), since it contains only two unknowns, $CB$ and $CD$

$CB = 0$ and $CD = FC = F/2$.

For joint E the force equilibrium conditions give

$ED + AE \sqrt{2}/2 = 0$ and $EB + AE \sqrt{2}/2 - F = 0$ or $ED = -F/2$ and $EB = F/2$.

The force in the member $ED$ is toward the pin $E$ (compression). For joint B, Fig. 5.14(c), the force equilibrium condition for $y$-axis gives

$BD \sqrt{2}/2 - EB = 0$ or $BD = F \sqrt{2}/2$. 
The correctness of the analysis is checked with the force equilibrium condition for \( x \)-axis

\[
BD \sqrt{2}/2 - AB = 0.
\]

Figure 5.15 shows the free-body diagram of each joint. The method of joints for

![Fig. 5.15 Free-body diagrams of each joint](image)

plane trusses employs only two of the three equilibrium equations because the method involves concurrent forces at each joint. A plane truss is statically determinate internally if \( n + 3 = 2c \), where \( n \) is number of its links and \( c \) is the number of its joints.

**Method of Sections**

The method of sections has the advantage that the force in almost any member may be found directly from an analysis of a section which has cut that link. Since there are only three independent equilibrium equations in plane not more than three members whose forces are unknown should be cut. For the truss shown in Fig. 5.13 for ready reference the external reactions are first computed by considering the truss as a whole. The force in the member \( ED \) will be determined. An imaginary section, indicated by the dashed line, is passed through the truss, cutting it into two parts, Fig. 5.16. This section has cut three links whose forces \( ED, BD, \) and \( BC \) are initially unknown. The left-hand section is in equilibrium under the action of the external force \( F \) at \( E \), the pin support reaction \( F_A = F/2 \), and the three forces exerted \( ED, BD, \) and \( BC \) on the cut members by the right-hand section which has been removed. In general, the forces are represented with their proper senses by a visual approximation of the system in equilibrium. The proper senses will also result from the computations. The sum of the moments about point \( B \) for the left-hand section (LHS) gives

\[
\sum M_B^{\text{LHS}} = EDa + F_A a = 0 \quad \text{or} \quad ED = F_A = F/2.
\]

The sum of the moments about point \( D \) for the right-hand section gives
Fig. 5.16 Method of sections

\[ \sum M_{D}^{RHS} = BC a + F_{C} (0) = 0 \text{ or } BC = 0. \]

The sum of the forces for right-hand section on x-axis is

\[ ED - BD \sqrt{2}/2 = 0 \text{ or } BD = ED \sqrt{2} = F \sqrt{2}/2. \]

5.6 Examples

Example 5.1
A block 1 with the mass \( m \) is on an inclined plane 2 with the angle \( \alpha \) with the horizontal, as shown in Fig. E5.1(a). Find the normal (perpendicular to the plane) and the tangential (parallel to the plane) components of the reaction force of the inclined 2 plane on the block 1. The dimensions of the block are negligible. Numerical application: \( m = 100 \text{ kg}, g = 9.81 \text{ m/s}^2 \), and \( \alpha = 30^\circ \).

Solution
The free-body diagram of the block 1 is shown in Fig. E5.1(b) and the reaction force of the inclined plane on the block is \( F_{21} \).

\[ F_{21} = F_{21n} + F_{21t} \]

The equilibrium equation for the block 1 is

\[ \sum \mathbf{F} = \mathbf{0} \implies \mathbf{G} + F_{21} = \mathbf{0}, \]

or

\[ \mathbf{G} + F_{21n} + F_{21t} = \mathbf{0}. \]

The normal component of the reaction force \( F_{21n} \) is at an angle of \( \alpha = 30^\circ \) with the gravitational force vector \( \mathbf{G} = mg \) and
Example 5.1

\[ F_{21n} = G \cos \alpha = mg \cos \alpha = 100(9.81) \cos 30^\circ = 849.571 \text{ N}. \]

The parallel component to the plane is

\[ F_{21t} = G \sin \alpha = mg \sin \alpha = 100(9.81) \sin 30^\circ = 490.5 \text{ N}. \]

Example 5.2

Figure E5.2(a) shows a block of mass \( m \) supported by two cables \( AB \) and \( AC \). The distance \( BO \) is \( a_1 \), the distance \( OC \) is \( a_2 \) and the distance \( AO \) is \( a_3 \). Find the tension in each cable. Numerical application: \( m=10 \text{ kg}, \ g=9.81 \text{ m/s}^2, \ a_1=3 \text{ m}, \ a_2=5 \text{ m}, \) and \( a_3=1 \text{ m}. \)

Solution

The free-body diagram of the knot at \( A \) is shown in Fig. E5.2(a) with \( mg \) acting vertically down and the tensions in \( AC \) and \( AB \). The force equilibrium equations are

\[
\sum F_x = -T_{AB} \sin \theta_1 + T_{AC} \sin \theta_2 = 0, \tag{5.7}
\]

\[
\sum F_y = T_{AB} \cos \theta_1 + T_{AC} \cos \theta_2 = mg. \tag{5.8}
\]
There are two equations with two unknowns. The problem is therefore statically determinate, i.e., it can be solved. From Eq. (5.7), \( T_{AC} = \frac{\sin \theta_1}{\sin \theta_2} T_{AB} \). Substituting into Eq. (5.8) it results
\[
T_{AB} \cos \theta_1 + \frac{\sin \theta_1}{\sin \theta_2} T_{AB} \cos \theta_2 = mg,
\]
or
\[
T_{AB} = \frac{mg}{\cos \theta_1 + \frac{\sin \theta_1}{\sin \theta_2} \cos \theta_2} = \frac{mg \sin \theta_2}{\cos \theta_1 \sin \theta_2 + \sin \theta_1 \cos \theta_2}.
\]
The trigonometric functions are
\[
\sin \theta_1 = \frac{a_1}{l_{AB}}, \quad \cos \theta_1 = \frac{a_3}{l_{AB}}, \quad \sin \theta_2 = \frac{a_2}{l_{AC}}, \quad \text{and} \quad \cos \theta_2 = \frac{a_3}{l_{AC}},
\]
where \( l_{AB} = \sqrt{a_1^2 + a_3^2} \) and \( l_{AC} = \sqrt{a_2^2 + a_3^2} \).
It results
\[
T_{AB} = mg \frac{a_2 \sqrt{a_1^2 + a_3^2}}{a_3(a_1 + a_2)} = 10(9.81) \frac{5 \sqrt{3^2 + 1^2}}{(1)(3 + 5)} = 193.887 \text{ N},
\]
and in a similar way

\[ T_{AC} = mg \sqrt{\frac{a_1}{a_3(a_1 + a_2)}} = 10(9.81) \frac{3\sqrt{5^2 + 1^2}}{(1)(3+5)} = 187.58 \text{ N.} \]

The same solution could also be obtained by writing an equilibrium moment equation with respect to a point that yields to one unknown. Suppose, for example, the moment equation is written about the point \( B \). Then

\[ \sum M_B = r_{BA} \times (T_{AB} + T_{AC} + G) = r_{BA} \times (T_{AC} + G) = 0, \quad (5.9) \]

where

\[ G = G_j = -mg_j, \quad T_{AB} = T_{ABx} \hat{i} + T_{ABy} \hat{j}, \quad T_{AC} = T_{ACx} \hat{i} + T_{ACy} \hat{j}, \]

and

\[ r_{BA} \times T_{AB} = 0. \quad (5.10) \]

The position vectors of the points \( A, B, \) and \( C \) are

\[ r_A = x_A \hat{i} + y_A \hat{j} = -a_3 \hat{j}, \quad r_B = x_B \hat{i} + y_B \hat{j} = -a_1 \hat{i}, \quad r_C = x_C \hat{i} + y_C \hat{j} = a_2 \hat{i}. \]

Equation (5.9) becomes

\[ \sum M_B = r_{BA} \times T_{AC} + r_{BA} \times G \]

or

\[ (x_A - x_B) T_{AC} \cos \theta_2 - (y_A - y_B) T_{AC} \sin \theta_2 + (x_A - x_B) G = 0. \]

It results

\[ T_{AC} = mg \frac{(x_A - x_B)}{(x_A - x_B) \cos \theta_2 - (y_A - y_B) \sin \theta_2}. \]

The unknown \( T_{AB} \) is calculated from a equilibrium moment equation of the system about the point \( C \).

\[ \sum M_C = r_{CA} \times (T_{AB} + T_{AC} + G) = r_{CA} \times (T_{AB} + G) = 0, \]

and from the previous relation the tension \( T_{AB} \) is calculated. The MATLAB program for the problem is given by

```matlab
% problem 5.2
clear all; clc; close
```
syms a_1 a_2 a_3 T_AB T_AC m g

list={m, g, a_1, a_2, a_3};
listn={10, 9.81, 3, 5, 1};

lAB=sqrt(a_1^2+a_3^2);
lAC=sqrt(a_2^2+a_3^2);

s_theta_1=a_1/lAB;
c_theta_1=a_3/lAB;
s_theta_2=a_2/lAC;
c_theta_2=a_3/lAC;

TAB=[ -T_AB*s_theta_1, T_AB*c_theta_1, 0];
TAC=[ T_AC*s_theta_2, T_AC*c_theta_2, 0];
G=[0, -m*g, 0];

fprintf('Method I 
')
% SF = TAB + TAC + G = 0
fprintf('sum forces = TAB + TAC + G = 0 
')

SF=TAB+TAC+G;
SFx=SF(1);
SFy=SF(2);

sol=solve(SFx, SFy,'T_AB, T_AC');
Tab=eval(sol.T_AB);
Tac=eval(sol.T_AC);

fprintf('T_AB = 
');pretty(simple(Tab)); fprintf('
')
fprintf('T_AC = 
');pretty(simple(Tac)); fprintf('
')

Tabn=subs(Tab, list, listn);
Tacn=subs(Tac, list, listn);

fprintf('T_AB = %g (N) 
', Tabn);
fprintf('T_AC = %g (N) 
', Tacn);

fprintf('
')

fprintf('Method II 
')
rB=[-a_1, 0, 0];
rC=[ a_2, 0, 0];
5.6 Examples

```
 rA=[0,-a_3,0];

 % SM_B = rBA x (TAC+G) = 0
 fprintf('sum M about B = rBA x (TAC+G) = 0 \n')
 SM_B=cross(rA-rB,TAC+G);
 TACs=solve(SM_B(3),'T_AC');
 fprintf('T_AC = 
');pretty(simple(TACs)); fprintf('
')
 TACn=subs(TACs, list, listn);
 fprintf('T_AC = %g (N) \n', TACn);

 % SM_C = rCA x (TAB+G) = 0
 fprintf('sum M about C = rCA x (TAB+G) = 0 \n')
 SM_C=cross(rA-rC,TAB+G);
 TABs=solve(SM_C(3),'T_AB');
 fprintf('T_AB = 
');pretty(simple(TABs)); fprintf('
')
 TABn=subs(TABs, list, listn);
 fprintf('T_AB = %g (N) \n', TABn);

 Method I
 sum forces = TAB + TAC + G = 0
 T_AB =

 \[ \frac{2^{1/2} (a_1 + a_3) m g a_2}{a_3 (a_1 + a_2)} \]

 T_AC =

 \[ \frac{2^{1/2} (a_2 + a_3) m g a_1}{a_3 (a_1 + a_2)} \]

 T_AB = 193.887 (N)
 T_AC = 187.58 (N)
```
Method II

sum M about B = rBA x (TAC+G) = 0

\[ T_{AC} = \frac{\left( a_2 + a_3 \right) m g a_1}{a_3 \left( a_1 + a_2 \right)} \]

\[ T_{AC} = 187.58 \text{ (N)} \]

sum M about C = rCA x (TAB+G) = 0

\[ T_{AB} = \frac{\left( a_1 + a_3 \right) m g a_2}{a_3 \left( a_1 + a_2 \right)} \]

\[ T_{AB} = 193.887 \text{ (N)} \]
Chapter 6
Problems

5.1 The beam shown in Fig. P5.1 is loaded with the concentrated forces $F_1=100$ N and $F_2=500$ N. The following dimensions are given: $a=0.5$ m, $b=0.3$ m, and $l=1$ m. Find the reactions at the supports $O$ and $C$.

![Fig. P5.1 Problem 5.1](image)

5.2 The beam depicted in Fig. P5.2 is loaded with the two concentrated forces with the magnitude $F=200$ lbs. The dimensions of the beam are given: $a=5$ in and $l=1$ ft. Find the reactions at the supports.

![Fig. P5.2 Problem 5.2](image)
5.3 Consider the cantilever beam of Fig. P5.3, subjected to a uniform load distributed, \( w = 100 \text{ N/m} \), over a portion of its length. The dimensions of the beam are: \( a = 10 \text{ cm} \) and \( l = 1 \text{ m} \). Find the support reaction on the beam.

![Fig. P5.3 Problem 5.3](image)

5.4 A smooth sphere of mass \( m \) is resting against a vertical surface and an inclined surface that makes an angle \( \theta \) with the horizontal, as shown in Fig. P5.4. Find the forces exerted on the sphere by the two contacting surfaces.

Numerical application: a) \( m = 10 \text{ kg} \), \( \theta = 30^\circ \), and \( g = 9.8 \text{ m/s}^2 \); b) \( m = 2 \text{ slugs} \), \( \theta = 60^\circ \), and \( g = 32.2 \text{ ft/sec}^2 \).

![Fig. P5.4 Problem 5.4](image)

5.5 The links 1 and 2 shown in Fig. P5.5 are each connected to the ground at \( A \) and \( C \), and to each other at \( B \) using frictionless pins. The length of link 1 is \( AB = l \). The angle between the links is \( \angle ABC = \theta \). A force of magnitude \( P \) is applied at the point \( D \) (\( AD = 2l/3 \)) of the link 1. The force makes an angle \( \theta \) with the horizontal. Find the force exerted by the lower link 2 on the upper link 1.

Numerical application: a) \( l = 1 \text{ m} \), \( \theta = 30^\circ \), and \( P = 1000 \text{ N} \); b) \( l = 2 \text{ ft} \), \( \theta = 45^\circ \), and \( P = 500 \text{ lb} \).
5.6 The shaft shown in Fig. P5.6 turns in the bearings A and B. The dimensions of
the shaft are $a = 6$ in. and $b = 3$ in. The forces on the gear attached to the shaft
are $F_t = 900$ lb and $F_r = 500$ lb. The gear forces act at a radius $R = 4$ in. from the
axis of the shaft. Find the loads applied to the bearings.

5.7 The shaft shown in Fig. P5.7 turns in the bearings A and B. The dimensions of
the shaft are $a = 120$ mm and $b = 30$ mm. The forces on the gear attached to the shaft
are $F_t = 4500$ N, $F_r = 2500$ N, and $F_a = 1000$ N. The gear forces act at a
radius $R = 100$ mm from the shaft axis. Determine the bearings loads.
5.8 The dimensions of the shaft shown Fig. P5.8 are $a = 2$ in. and $l = 5$ in. The force on the disk with the radius $r_1 = 5$ in. is $F_1 = 600$ lb and the force on the disk with the radius $r_2 = 2.5$ in. is $F_2 = 1200$ lb. Determine the forces on the bearings at $A$ and $B$. 

Fig. P5.8 Problem 5.8
5.9 The dimensions of the shaft shown Fig. P5.9 are \( a = 50 \text{ mm} \) and \( l = 120 \text{ mm} \). The force on the disk with the radius \( r_1 = 50 \text{ mm} \) is \( F_1 = 4000 \text{ N} \) and the force on the disk with the radius \( r_2 = 100 \text{ mm} \) is \( F_2 = 2000 \text{ N} \). Determine the bearing loads at \( A \) and \( B \).

![Fig. P5.9 Problem 5.9](image)

5.10 The force on the gear in Fig. P5.10 is \( F = 1.5 \text{ kN} \) and the radius of the gear is \( R = 60 \text{ mm} \). The dimensions of the shaft are \( l = 300 \text{ mm} \) and \( a = 60 \text{ mm} \). Determine the bearing loads at \( A \) and \( B \).

![Fig. P5.10 Problem 5.10](image)
5.11 A torque (moment) of 24 N m is required to turn the bolt about its axis, as shown in Fig. P5.11, where $d = 120$ mm and $l = 14$ mm. Determine $P$ and the forces between the smooth hardened jaws of the wrench and the corners of $A$ and $B$ of the hexagonal head. Assume that the wrench fits easily on the bolt so that contact is made at corners $A$ and $B$ only.