project: HW 1 STAT 3610 S'2010 Maghsoodloo

Exercise 4/p.262 of Devore's 7th Edition

~~Solution: Each part is worth 3 points~~

(a, 3 points) $X$ = ~~At~~ 10 amps Stray-Load Loss for one motor.

$X \sim N(\mu, 9) \rightarrow \bar{x} \sim N(\mu, 9/n)$.

$1-\alpha = 0.95$, $n = 25$, $\bar{x} = 58.3$, $SE(\bar{x}) = 3/5 = 0.60$, $\sigma_x = 3$; $SE(\bar{x}) = 0.60$

$\mu_L = \bar{x} - 1.96 \times 3/\sqrt{25} = 57.1240$, $\mu_U = 59.4760$ CIL = 2.3520

$57.1240 \le \mu \le 59.4760$ at 95% confidence (0 or 1 probability)

(b, 3pts.) $\mu_L = \bar{x} - 1.96 \times 3/\sqrt{100} = 57.7120$ $\mu_U = 58.8880$ CIL = 1.1760

(c) $\mu_L = \bar{x} - 2.5758 \times 0.30 = 57.5273$, $\mu_U = 59.0727$, $1-\alpha = 0.99$

CIL = 1.5465 = CIW; $\sigma_{\bar{x}} = 3/\sqrt{n} = SE(\bar{x})$

(d) $\mu_L = 58.3 - 1.3408 \times 0.30 = 57.8978$, $\mu_U = 58.7022$

The above interval has either 0 or 1 Pr. of containing $\mu$, but we have 82% confidence that it contains $\mu$. $(1-\alpha = 82\%)$

(e) $CIL = (\bar{x} + 2.5758 \times 3/\sqrt{n}) - (\bar{x} - 2.5758 \times 3/\sqrt{n}) = 15.454976/\sqrt{n} = 1$

or CIW = 1 $\sqrt{n} = 15.454976 \rightarrow n = 239$.

Note that the CIL = CIW of a normal population mean $\mu$ with known variance $\sigma^2$ is always given by $CIL = CIW = 2 z_{\alpha/2} \times \sigma/\sqrt{n}$.

In part (a), if we obtain 1000 random samples, each of size $n = 25$ from $N(\mu, 9)$, then roughly 9500 of the corresponding CIs will contain the true value of $\mu$.