ect: The Solution to STAT 3610 Bonus HW 1

S 2010 S. Maghsoodloo

(a) Prove that the model point of the Student's distribution occurs at $t = 0$ for all $df > 1$.

$f(t) = C(1 + t^2/\nu)^{-(\nu+1)/2}$, $-\infty < t < +\infty$

$C > 0$ where $C = \dfrac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}$.

$f'(t) = \dfrac{df(t)}{dt} = -\dfrac{2Ct}{\nu}(1 + t^2/\nu)^{-(\nu+3)/2} \rightarrow$

$t = 0$ is the solution to $f'(t) = 0$.

$f''(t) = \dfrac{d^2f}{dt^2} = \dfrac{2C}{\nu}(1 + t^2)^{-(\frac{\nu+1}{5})}\left[(1 + 2/\nu)t^2 - 1\right]$

At $t = 0$, $f''(t) < 0 \rightarrow t = 0$ maximizes $f(t)$, i.e., $t = 0$ is the modal point of T_ν.

(b) $f''(t) \overset{\text{set to}}{=} 0 \rightarrow (1 + 2/\nu)t^2 - 1 = 0 \rightarrow$

$(1 + 2/\nu)t^2 = 1 \rightarrow t^2 = \dfrac{1}{1 + 2/\nu} = \dfrac{\nu}{\nu + 2} \rightarrow$

$t_{\text{inflection}} = \pm\sqrt{\nu/(\nu+2)} \rightarrow$ 2 inflection points.

Note that in the limit as $\nu \rightarrow \infty$, $t_{\text{inflection}} \rightarrow \pm 1$, as expected because $F(t_\nu) \rightarrow \Phi(t)$ as $\nu \rightarrow \infty$. Recall that the infection points of the unit normal, $Z \sim N(0, 1)$, occur at ± 1.

DIVOGA