1(20 Points). Suppose that 27% (p = 0.27) of all drivers in Alabama receive at least one driving citation per year. (a) Compute the Pr that exactly 4 of the 18 randomly selected drivers receive at least one citation next year to 4 decimals. (b) Compute the Pr that at most 1 of the 18 drivers receive at least one citation to 4 decimals? (c) Compute the Pr for at least 2 drivers (in 18 randomly selected) to 4 decimals.

2(30 Points). A company is interviewing candidates to fill exactly 4 engineering jobs. The Pr that an interview is successful and the candidate accepts the offer is 20%. (a) Compute the Pr that the 1st success occurs at the 9th interview to exactly 4 decimals. (b) Compute the Pr that the 4th success occurs at the 13th interview to 4 decimals. (C) Compute the Pr that at least 5 interviews are needed, to fill all 4 positions, to exactly 4 decimals

3(35 Points). The discrete random variable, \(X\), represents the number of times that a network system is down during any one month. From past data, it has been determined that the pmf (probability mass function) of the random variable (rv) \(X\) is given by the table below.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>p(x)</td>
<td>0.20</td>
<td>0.23</td>
<td>0.25</td>
<td>0.12</td>
<td>0.09</td>
<td>0.06</td>
<td>C</td>
</tr>
</tbody>
</table>

(a) Compute the value of the constant C to ensure that \(p(x)\) is a pmf. Give the modal point of \(X\), and obtain the cdf value of \(X\) at \(x = 2\). (b) Compute the population mean, variance (to 4 decimals), and the CV (coefficient of variation) of the rv \(X\). (c) Given that a given month has more than 2 downtime, compute the Pr that there will be at least 5 downtimes during the same month to 4 decimals. Recall that \(P(A \mid B) = P(A \cap B) / P(B)\). (d) The loss in net profit from the system downtime is given by \(Y(x) = 40.0 + 75X - 35(X - 2.05) + 5(2X - 4.10)^2\), measured in hundreds of dollars. Compute the expected value of the monthly loss in net profit, i.e., compute the \(E[Y(x)]\).

4(15 Points). The arrivals of aircrafts at a small airport is a Poisson process at an average rate of \(\lambda = 3.5\) aircrafts per hour. (a) Compute the Pr that exactly 5 aircrafts make landing in the next hour to 5 decimals. (b) Use Table A.2, page 666, to approximate the Pr of more than 5 but at most 10 arrivals in the next 2 hours to 3 decimals.
1. (a, 10 points) \( n = 18, \ p = 0.27 \)
\[
b(4; 18, 0.27) = 18 \binom{4}{18} 0.27^4 0.73^14 = 0.1985
\]
(b, 5 points) \( P(X_{\text{Bin}} \leq 1) = P_0 + P_1 =
\]
\[
= \binom{18}{1} 0.73 0.27 + 0.73 0.27^2 = 0.0265
\]
(c, 5 points) \( P(X \geq 2) = 1 - \binom{18}{1} 0.73 0.27 = 1 - 0.0265 = 0.9735
\]

2. (a, 10 points) \( q(9; 0.20) = (0.80)(0.20) = 0.0336 \)
(b, 10 points) \( \mu_b(13; 4, 0.20) =
\]
\[
= 12 \binom{3}{4} 0.20^3 0.80^4 = 0.0472
\]
(c, 10 points) \( P(X_{\text{int}} \geq 5) = 1 - P(X_{\text{int}} = 4)
\]
\[
= 1 - (0.20) = 0.9984
\]

3. (a, 5 points) \( C = 0.05 \)
\[\text{Mode} = 2 \]
\[F_X(a + k = 2) = 0.68 \]
(b, 15 points) \( \mu = E(X) = \sum x \cdot p(x) = 2.05 \)
\[E(X^2) = \sum x^2 \cdot p(x) = 7.05 \]
\[\sqrt{(X)} = \sigma^2 = 7.05 - 2.05 \approx 2.8475 \]
3 (b, continued) $T_X = 1.6875$

$C V_X = \frac{T_X}{\mu} = 82.315\%$

(c, 5 points) $Pr(X \geq 5 | X > 2) = \frac{P(X \geq 5)}{P(X > 2)} = 0.11/0.32 = 0.3438$

(d, 10 points) $E[Y(X)] = 40 + 75 E(X)$

$= 35 E(X-\mu) + 5 E\left[2(X-\mu)^2\right]$

$= 40 + 75(2.05) + 0 + 5 \times 4 \times 2.8475 = 250.700$

4 (a, 10 points) $\theta(5; 3.5) = 3.5 \frac{5}{5!} e^{-3.5}$

$= 0.13217$

(b, 5 points) $Pr(5 < X \leq 10) = F_X(10; 7) - F_X(5; 7) = 0.901 - 0.301 = 0.60$