

**Computing the Probabilities of all Outcomes when 5 Cards Are Drawn at random W/O Replacement from a Well-Shuffled Deck of 52 Cards**

Let  $A_i$  represent the event that exactly  $i$  cards that are same,  $i = 0, 2, 3, 4$ .

(1) Then, the event  $A_0$  represents an outcome such as {5J96A}, where none of the cards are the same. Thus,

$$P_0 = \Pr(A_0) = \frac{52 \times 48 \times 44 \times 40 \times 36}{52 \times 51 \times 50 \times 49 \times 48} = 0.507082833133,$$

i.e., roughly 51% of the times all the 5 cards will be distinct. Note that the above hands includes straights, flushes and straight-flushes.

(2) The event  $A_2$  represents an outcome such as {5J9JA}, where exactly 2 cards are the same and the other 3 are distinctly different, i.e.,  $A_2$  represents exactly one-pair.

$$P_2 = \Pr(A_2) = \frac{4 \times 3 \times 48 \times 44 \times 40}{52 \times 51 \times 50 \times 49 \times 48} \times ({}_5C_2) \times 13 = 0.42256902761.$$

(3) The event  $A_3$  represents an outcome such as {5J9JJ}, where exactly 3 cards are the same and the other 2 are different, i.e.,  $A_3$  represents exactly 3-of-a-kind.

$$P_3 = \Pr(A_3) = \frac{4 \times 3 \times 2 \times 48 \times 44}{52 \times 51 \times 50 \times 49 \times 48} \times ({}_5C_3) \times 13 = 0.021128451380$$

(4) The event  $A_4$  represents an outcome such as {JJ9JJ}, where exactly 4 cards are the same and the other per force is different, i.e.,  $A_4$  represents exactly 4-of-a-kind.

$$P_4 = \Pr(A_4) = \frac{13 \times ({}^4C_4) \times ({}_{48}C_1)}{{}_{52}C_5} = 0.0002400960384154$$

(5) Let the event  $A_{22}$  represent an outcome such as {5J9J5}, where there are exactly 2 pairs but the 5<sup>th</sup> card is distinctly different from each of the 2 pairs, i.e.,  $A_{22}$  represents exactly Two-pairs.

$$P_{22} = \Pr(A_{22}) = \frac{{}_{13}C_2 \times ({}^4C_2) \times ({}^4C_2) \times 44}{{}_{52}C_5} = 0.04753901561.$$

(6) Let the event  $A_{23}$  represent an outcome such as {5J5J5}, which is called a full-house.

$$P_{23} = \Pr(A_{23}) = \frac{{}_{13}C_2 \times ({}^4C_3) \times ({}^4C_2) \times 2}{{}_{52}C_5} = 0.001440576.$$

Note that the following Pr

$$\frac{{}_{13}C_1 \times ({}^4C_2) \times ({}_{48}C_3)}{{}_{52}C_5} = 0.519087635$$

is simply the sum of  $P_2 + 2P_{22} + P_{23}$ .