

**Computing the Probabilities of all Outcomes when 5 Cards Are Drawn at random W/O Replacement (and then also with replacement) from a Well-Shuffled Deck of 52 Cards**

Let  $A_i$  represent the event that exactly  $i$  cards that are same,  $i = 0, 2, 3, 4$ . The index  $i$  can equal 5 only in the case of with replacement.

1(Drawing W/O Replacement). Then, the event  $A_0$  represents an outcome such as {5J96A}, where none of the cards are the same.

Thus,

$$P_0 = \Pr(A_0) = \frac{52 \times 48 \times 44 \times 40 \times 36}{52 \times 51 \times 50 \times 49 \times 48} = 0.507082833133,$$

i.e., roughly 51% of the times all the 5 cards will be distinct. Note that the above hands include straights, flushes and straight-flushes.

1(With Replacement). It is best to think of this situation as drawing 5 cards at random from 5 distinct deck of well-shuffled of 52 cards, one card from each deck of 52 cards.

$$P_0 = \Pr(A_0) = \frac{52 \times 48 \times 44 \times 40 \times 36}{52 \times 52 \times 52 \times 52 \times 52} = 0.41595182242$$

2(W/O Replacement). The event  $A_2$  represents an outcome such as {5J9JA}, where exactly 2 cards are the same and the other 3 are distinctly different, i.e.,  $A_2$  represents exactly one-pair.

$$P_2 = \Pr(A_2) = \frac{4 \times 3 \times 48 \times 44 \times 40}{52 \times 51 \times 50 \times 49 \times 48} \times ({}_5C_2) \times 13 = 0.42256902761.$$

The multiplier  ${}_5C_2$  is needed because of (JJA59), (JAJ59), (JA5J9), ...,

(A59JJ), of which there are 10 combinations, i.e., this is how many ways we can place the 2 jacks in the 5 drawings.

2(With Replacement). (5 decks of card; exactly one card drawn at random at random from each)

$$\Pr(A_2) = (4/52)^2(48/52)(44/52)(40/52) \times ({}_5C_2) \times 13 = 0.462168692$$

3(W/O Replacement). The event  $A_3$  represents an outcome such as {5J9JJ}, where exactly 3 cards are the same and the other 2 are distinctly different, i.e.,  $A_3$  represents exactly 3-of-a-kind.

$$P_3 = \Pr(A_3) = \frac{4 \times 3 \times 2 \times 48 \times 44}{52 \times 51 \times 50 \times 49 \times 48} \times ({}_5C_3) \times 13 = 0.021128451380$$

3(With Replacement).

$$\Pr(A_3) = (4/52)^3(48/52)(44/52) \times ({}_5C_3) \times 13 = 0.04621686916$$

4(W/O Replacement). The event  $A_4$  represents an outcome such as {JJ9JJ}, where exactly 4 cards are the same and the other per force is different, i.e.,  $A_4$  represents exactly 4-of-a-kind.

$$P_4 = \Pr(A_4) = \frac{13 \times ({}_4C_4) \times ({}_{48}C_1)}{{}_{52}C_5} = 0.0002400960384154$$

Or: 
$$P_4 = \Pr(A_4) = \frac{4 \times 3 \times 2 \times 1 \times ({}_5C_4) \times 48 \times 13}{52 \times 51 \times 50 \times 49 \times 48}$$

4(With Replacement).

$$\Pr(A_4) = (4/52)^4(48/52) \times ({}_5C_4) \times 13 = 0.00210076677987$$

5(Only With Replacement).

$$\Pr(A_5) = (4/52)^5 \times 13 = 0.00003501277966$$

**6(W/O Replacement).** Let the event  $A_{22}$  represent an outcome such as {5J9J5}, where there are exactly 2 different pairs but the 5<sup>th</sup> card is distinctly different from each of the 2 pairs, i.e.,  $A_{22}$  represents exactly Two-pairs.

$$P_{22} = \Pr(A_{22}) = \frac{{}_{13}C_2 \times ({}_4C_2) \times ({}_4C_2) \times 44}{{}_{52}C_5} = 0.04753901561.$$

**6(With Replacement).**

$$P_{22} = \Pr(A_{22}) = (4/52)^2(48/52)({}_{13}C_2) \times ({}_5C_2) \times ({}_3C_2) = 0.06932530374$$

**7(W/O Replacement).** Let the event  $A_{23}$  represent an outcome such as {5J5J5}, which is called a full-house.

$$P_{23} = \Pr(A_{23}) = \frac{{}_{13}C_2 \times ({}_4C_3) \times ({}_4C_2) \times 2}{{}_{52}C_5} = 0.001440576.$$

**7(With Replacement).**

$$P_{23} = \Pr(A_{22}) = ({}_{13}C_2) \times 2 \times ({}_5C_2) \times (4/52)^2(4/52)^3 = 0.00420153356.$$