Accelerated Life Testing (ALT)

Reference : Section 13.6 of Ebeling (2nd) **Maghsoodloo**

In most instances component reliability is so high that placing even n = 100 units on test may not yield any failures for a test duration, of say, more than 5000 hours. If a new product is being developed, such long testing times cannot be tolerated because the new product has to get to the global market in due time, or else lack of market share may occur. In such cases, the experimenter has no choice but to use accelerated testing procedures to induce failures in order to estimate component TTF (or reliability).

Accelerated life testing (ALT), in combination with DOE (design of experiments), is conducted by subjecting n identical units to stresses well beyond what the units on test will experience under normal operating conditions. Such high stresses for ALT that accelerate failure mechanism may be applied in many forms: very high, or very low temperatures, humidity levels well beyond normal operating conditions, excessive usage, very high levels of voltage, extreme cycling between low and high levels beyond what is considered normal operating conditions, excessive force, excessive vibrations, ten times more units on test than needed, etc, etc.

As Elsayed (Reliability Engineering, E.A. Elsayed, Chapter 6, Addison Wesley Longman, INC., 1996) points out in the beginning of section 6.2 of his text, the underlying assumption is that the failure mechanism under ALT conditions is, except for a multiplicative factor, similar to failure mechanism under normal operating conditions. Put differently, ALT is based on the principle that a unit under accelerated test will exhibit the same behavioral statistical pattern in a short testing time under very high stresses as it will exhibit in a much longer time at normal operating stresses. For example, if the underlying failure distribution is W(δ, θ, β), then under ALT the change in the shape parameter β will be much smaller than the changes in minimum life δ and scale parameter $\theta - \delta$. That is to say, under ALT the change in β (or overall process variability, or CV) will be negligible compared to changes in δ and the characteristic life θ as compared to normal operating conditions. There are 3 different physical models that have been developed in the past 115 years that can be used to estimate the MTTF under normal operating conditions (OP = normal operating conditions) from ALT

data, where subscripts will be used to designate statistics computed under high stressed conditions.

(1) The Arrhenius Model

This is the most commonly used model relating TTF to high thermal stresses. Thermal stresses occur in solid state diffusion, chemical reactions, many semiconductor failure mechanism, battery life, etc. The underlying distribution of TTF_{OP} (TTF under normal operating conditions) in almost all these cases is exponential, Weibull, or lognormal (i.e., all positively skewed pdfs). The Arrhenius rate law that describes the (failure) rate, r, at which reaction to temperature of the test unit occurs is given below.

$$r = C' e^{-E_a/(k\,T)} \qquad (114)$$

where C′ is a constant which is characteristic of the failure mechanism of the item under test, E_a = the activation energy needed to induce failure measured in eV (= electron volts; close to vaporization energy for metals and chemical bond energies for polymers), k = the Boltzman's constant = 8.6171×10^{-5} eV/Kelvin (Note that Google.com gives k = 8.616×10^{-5}, while L. W. Condra, p. 232, gives k = 8.617×10^{-5}) and T = the temperature in Kelvin = Centigrade + 273.15 . In RE engineering, the Arrhenius model is also used to measure the impact of temperature on reliability because we make the assumption that the TTF is inversely proportional to the reaction (failure) rate, r, given in equation (114), i.e.,

$$TTF = C e^{E_a/(k\,T)} \qquad (115)$$

where $C \neq 1/C'$ is the constant of proportionality characteristic of the product under test. The Arrhenius model is applicable when the product $r_1\times TTF_1 = r_2 \times TTF_2$, where r_1 and r_2 are reaction rates at testing temperatures T_1 and T_2, respectively. The relationship $Rate_1\times TTF_1 = Rate_2 \times TTF_2$ implies that $r \times TTF$ will practically stay constant over the range of temperature applicability, and as a result $r_{OP}\times TTF_{OP} \cong r_s\times TTF_s$, where TTF_{OP} represents TTF under normal operating conditions and TTF_s represent component life under (accelerated) stressed conditions. Thus, $A_f = \dfrac{TTF_{OP}}{TTF_s} = \dfrac{r_s}{r_{OP}} = \dfrac{C' e^{-E_a/(k\,T_s)}}{C' e^{-E_a/(k\,T_{OP})}} = e^{E_a(\frac{1}{T_{OP}} - \frac{1}{T_s})/k} \rightarrow$

$$\mathbf{TTF_{OP} = TTF_S}\ e^{E_a\left(\frac{1}{T_{OP}} - \frac{1}{T_S}\right)/k} \quad \textbf{(116a)}$$

Eq. (116a) shows that the acceleration factor for the Arrhenius model is given by

$$\mathbf{A_f = TTF_{OP} / TTF_S} = e^{E_a\left(\frac{1}{T_{OP}} - \frac{1}{T_S}\right)/k} \quad \textbf{(116b)}$$

Note that the smaller the required activation energy E_a is, the more rapidly the unit on test will fail resulting in smaller A_f value. Note that Ebeling uses AF for A_f; either notation is prevalent.

<u>The Example 6.10 on page 380 of E. A. Elsayed.</u> In this example n microelectronic devices (the value of n not specified by the author) are put on accelerated test at T_S = 200 Celsius = 200 + 273.15 = 473.15 Kelvin and the $MTTf_S$ of the n units was approximately equal to 4000 hours. The operating temperature T_{OP} = 50 ^{o}C = 323.15K, and the required activation energy was 0.191 eV. Thus, the sample $MTTf_{OP}$ = $MTTf_S\ e^{E_a\left(\frac{1}{T_O} - \frac{1}{T_S}\right)/k}$ = 4000× $e^{0.191\left(\frac{1}{323.15} - \frac{1}{473.15}\right)/8.6171\times10^{-5}}$ = 35191.33024 hours, which is almost identical to Elsayed's answer in the middle of his page 380. The value of acceleration factor is A_f = 35191.33024 /4000 = 8.79783256.

<u>Example 22.</u> The TTF of n = 10 samples under an accelerated temperature of T_S = 100 Centigrade are $t_{(i)}$: 130, 140, 160, 180, 185, 195, 205, 205, 240, and 260 hours. The measurement of interest is the thermo-compression bond between two dissimilar metals, the strength of which reduces in time by the formation of voids by solid-state diffusion which has an activation energy of 0.90 eV. The normal operating temperature is T_{OP} = 25 Celsius. The sample statistics are $\bar{x}_s$ =190, S_S = 40.8248290, and cv_S = 21.487% showing that the accelerated data is obviously not exponentially (i.e., IFR) and if it is Weibull, then the slope $\beta \cong 5.0$ (see my Table 1 on p. 12). Most probably, the accelerated data is lognormally distributed. The use of equation (116a) yields the normal operating sample mean to failure

$MTTf_{OP}$ =190 $e^{0.90\left(\frac{1}{298.15} - \frac{1}{373.15}\right)/(8.6171\times10^{-5})}$ **= 190×1142.3450161 = 217045.5531 hours →**

A_f = 1142.3450161. If we wish to be more conservative about our estimate of MTTF in normal operating use, we could estimate it as $MTTf_{OP} = 130\,e^{0.90(\frac{1}{298.15}-\frac{1}{373.15})10^5/8.6171}$ = 148504.8521 hours, giving an acceleration factor of A_f = 148504.8521/190 =781.6044847, where 130 is the value of the 1st order statistic, $x_{(1)} = t_1$, under stressed condition. Note that we are using MTTf as the sample MTT failure.

It is reported in the literature that the value of E_a ranges in the interval 0.30 − 0.60 for semiconductor failures, for intermetallic diffusion (like in Example 22) it ranges in the interval 0.90 −1.1 eV, and for silicon junction defects E_a = 0.80 eV. The question arises how high the stressed temperature should be for a unit under accelerated test so that the resulting stressed life can be extrapolated to the expected life under normal operating conditions. Almost all metals change properties when the testing temperature exceeds 50% of their melting temperature T_m. Therefore, the accelerated testing temperature, T_s, must not exceed $0.50\times T_m$; to be on the safe side, T_s should be set below $0.45T_m$.

<u>Example 23.</u> The lifetimes of n = 50 PC components under an accelerated temperature of T_s = 100 °C yielded the sample mean $\bar{x}_s$ = 232.2 hours and a standard deviation of S_s = 82.8 hours, with E_a = 0.85 eV and T_{OP} = 27 °C. The use of equation (116b) gives an acceleration factor of $A_f = e^{E_a(\frac{1}{300.15}-\frac{1}{373.15})/k}$ = 619.695651 giving an estimated $MTTf_{OP} = A_f \times \bar{x}_s$ = 619.695651×232.2 = 143893.3301 hours ≅ 16.42618 years. Since the sample size n > 20, then we may use the normal approximation to the SMD of $\bar{x}_s$ to obtain an approximate lower 95% CI for the $MTTf_S$, given by $L_S = 232.2 - 1.645\times 82.8/\sqrt{n}$ = 212.937563 hours → L_{OP} = $Life_{OP}$= 619.695651×212.938 =131956.75253264 hours → 15.0635562252 years ≤ $MTTf_{OP} < \infty$ at the 95% confidence level. Note that this normal approximation would not be permissible unless n > 20. Methods of analysis for the exponential, Weibull, and lognormal underlying distribution of TTF_S, for any n, are given by Wayne Nelson, (1990), " Accelerated Testing, Wiley, New York, ISBN: 0-471-52277-5.

Determination of the Acceleration Factor A_f (or AF) Using Linear Regression

In order to use regression to estimate A_f, the Arrhenius model must first be linearized as shown below. From equation (115), TTF = $Ce^{E_a/(kT)}$, which can be linearized by taking the natural logarithm of both sides only once. This leads to y = ln(TTF) = ln(C) + E_a/(kT) = ln (C) + E_a x , where x = 10^5/(8.6171T), k = the Boltzman's constant = 8.6171×10^{-5} eV/Kelvin and T must be in units of Kelvin. I used the data provided by Boris Gnedenko *et al* (Statistical Reliability Engineering, Wiley, Example 5.2 on pp. 171-172, ISBN: 0-471-12356-0) and W. Nelson (1990), which are listed for your convenience below, to estimate C and E_a using regression analysis. The experiment from the above two authors involved a new Class-H motor insulation with a design temperature of T_{OP} = 180 °C = 453.15 Kelvin, where n = 40 units were equally and randomly divided and tested to failure at the accelerated temperatures 190, 220, 240, and 260 Celsius. The accelerated times to failure, TTF_S, in hours are provided in Table 5.2 of B. Gnedenko and duplicated herein atop the next page. I used Minitab to regress y on x, where x = 10^5/(8.6171T_{Kelvin}), and its output is provided below; further, in these notes MTTf denotes the sample MTTF.

Regression Analysis: y = ln(TTF) versus x =10^5/(8.6171*Kelvin)

The regression equation is
y = - 7.28 + 0.649 x

Predictor	Coef	SE Coef	T	P
Constant	-7.2834	0.7719	-9.44	0.000
x	0.64936	0.03316	19.58	0.000

S = 0.255738 R-Sq = 91.0% R-Sq(adj) = 90.7%

The above regression output clearly shows that $\hat{y} = -7.2834 + 0.64936x$ is an excellent model because R^2_{Model} = 91% so that ln(C) = $-$7.2834 → C = 0.000686846 and E_a = 0.64936 eV.

To extrapolate the expected life to the operating temperature of 180 °C = 453.15 K, we insert x_{OP} = 100000/(8.6171×453.15) = 25.609251 into our regression model →

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	1	25.073	25.073	383.36	0.000
Residual Error	38	2.485	0.065		
Total	39	27.558			

Table 5.2 of Boris Gnedenko *et al* (page 171; Stressed Hours to failure data)

190 °C	220 °C	240 C°	260 °C
7228 hours	1764 hours	1175 hours	600 hours
7228	2436	1175	744
7228	2436	1521	744
8448	2436	1569	744
9167	2436	1617	912
9167	2436	1665	1128
9167	3108	1665	1320
9167	3108	1713	1464
10511	3108	1761	1608
10511	3108	1953	1896

T_{OP} = 180°C → T_{OP} = 453.15 Kelvin.

MTTf = 8782.20 2637.6000 1581.4000 1116.0000

S = 1244.0117 453.5654 244.2745 439.2357

$\widehat{CV}$ = **14.165%** $\widehat{CV}$ = **17.196%** $\widehat{CV}$ = **15.447%** $\widehat{CV}$ = **39.358%**

Unusual Observations

Obs	x	y	Fit	SE Fit	Residual	St Resid
11	23.5	7.4753	7.9973	0.0416	-0.5220	-2.07R
39	21.8	7.3827	6.8509	0.0635	0.5319	2.15R
40	21.8	7.5475	6.8509	0.0635	0.6966	2.81R

R denotes an observation with a large standardized residual.
Predicted Values for New Observations

New Obs	Fit	SE Fit	95% CI	95% PI
1	9.3462	0.0883	(9.1674, 9.5249)	(8.7985, 9.8939)

Values of Predictors for New Observations
New Obs x = 25.6

$\hat{y}_{OP} = -7.2837 + 0.64936x_{OP} = 9.3462 \rightarrow \widehat{MTTF}_{OP} = MTTf_{OP} = e^{9.3462} = 11455.2108$

hours. I will next convert the above regression model $\hat{y} = -7.2837 + 0.64936x$, $\ln(TTF_{OP})$ =

ln (C) + E_a /(k T) to the Arrhenius format:

$$\widehat{MTTF_{OP}} = 0.000686846\, e^{0.64936\times 10^5/(8.6171\, T_{OP})}, \qquad (117)$$

where 0.000686846 = $e^C = e^{-7.2837}$ and the temperature T_{OP} must be measured in Kelvin. Inserting T_{OP} = 180 + 273.15 = 453.15 into equation (117) again yields $\widehat{MTTF_{OP}}$ (180 °C) = 11455.21083 hours. The acceleration factors from 180 to 190, from 180 to 220, and to 240 °C, are A_f = 11455.211/8782.200 = 1.30437, 4.34404, and 7.243715, respectively, where 8782.200 = Sample MTTf(at 190°C). Further, I attempted to improve the above model by adding the regressors x^2 and x^3 to the model, unfortunately the R^2_{Model} improved by a small amount to 92.3% but all the coefficients in the model became highly insignificant (i.e., a worthless model).

Exercise 30. **Boris Gnedenko *et al* mention on their page 172 that the failure data at 260 Celsius in the above experiment looks very suspicious because it exhibits much higher variability of times TF than the other 3 accelerated temperatures. That is to say, the failure mechanism at 260 °C is different from failure modes at lower temperatures. This implies that in real-life testing, the accelerated load should not exceed the point that changes the underlying distribution family and/or the shape. Repeat my analysis of the above experiment but remove the data at 260 C.**

ANS: $\widehat{MTTF}_{OP}$ (180 °C) = $MTTf_{OP}$ > 12000 hours.

(2) The Inverse Power Law (IPL)

This law is generally used when the TTF is inversely proportional to the applied (accelerated) stress, and the underlying lifetime distribution is almost always Weibull, or perhaps lognormal. As in the case of Arrhenius model, the IPL model is applicable only when there is a single type of stress, which in most cases is voltage accelerated stress, alternating temperature stress, or mechanical vibration in order to induce fatigue failure. The general form of the IPL is given by

$$TTF_S = C/S^b \qquad (118a)$$

where C > 0 and the exponent b > 0 are constants characteristics of the items under test, and S is the applied (accelerated) stress. The value of the exponent b = [2, 3] for metals and electronic solder joints, b = [4, 10] for microelectronic parts, and b = [4, 7] for intermetallic fatigue failures. Note that the model in Eq. (118a) can also be expressed as $TTF_s = Ce^{-bS}$.

<u>Example 22 (borrowed from L. W. Condra, RE Improvement with DOE, pp 236-237, Marcel Dekker, ISBN: 0-8247-0527-0).</u> A sample of n electronic solder joints are placed on accelerated fatigue-testing at a displacement of S = 0.0008 inches with a $MTTf_s$ = 10 cycles. Assuming that under normal use the maximum displacement is S_o = 0.00005 inches and the exponent b = 2.50, our objective is to estimate $MTTF_o$. We need to compute the value of the

$$A_f = MTTf_{OP}/\, MTTf_s = \frac{C/\,S_{OP}^b}{C/\,S^b} = S^b/\,S_{OP}^b = (0.0008/0.00005)^{2.5} = 1024 \rightarrow MTTf_{OP} =$$

1024×10 = 10,240 cycles to failure.

Elsayed provides another form of IPL given in his equation (6.53) in the middle of page 384 which is of the form

$$TTF_s = C'(V_{OP} / V_s)^b \qquad (118b)$$

where V_{OP} is the standard specified (voltage) operating stress, V_s is the accelerated voltage stress, and the constant C′ is characteristic of the product under test , fabrication method, etc.

<u>The Example 6.12 on pages 385-387 of Elsayed.</u> In this experiment two samples of 20 CMOS integrated circuits each are put on accelerated life test, where V_s represents accelerated electric field stresses at 10 and 25 eV. The underlying distribution of TTF is assumed Weibull and there is only one stress factor, namely electric field, and hence the IPL is a plausible model for TTF_s. The normal operating stress is at V_{OP} = 5 eV. For your convenience I have duplicated Table 6.6 of E. A. Elsayed on his page 385 below. I first used the data under the two accelerated stress levels, V_1 = 10 and V_2 = 25 eV, to obtain the MLEs of the Weibull parameters β and θ. Using methods of Chapter 15 the MLEs are $\hat{\beta}_{1s}$ = 1.836028, $\hat{\theta}_{1s}$ = 9343.5856011 hours, and at V_2 = 25 eV, $\hat{\beta}_{2s}$ = 1.981834234, $\hat{\theta}_{2s}$ = 3916.9661061541 hours. These MLEs under stressed conditions are exactly consistent with those of Elsayed's given at

the bottom of his page 386. It seems that if the data set is W(0, θ, β), then a rough value of the Weibull slope is close to $\beta \cong 1.910$. However, it is not clear what the estimate of the characteristic life is at normal operating stress 5 eV, because $\hat{\theta}_{1S}$ = 9343.585601 hours and $\hat{\theta}_{2S}$ = 3916.9661061541 hours were obtained under accelerated testing conditions. Elsayed provides the ML estimation procedure on his pages 384-385, but as of right now, I am not sure how the equations were arrived at. Perhaps, the book by Mann, N. R. , *et al,* (1974), Methods for Statistical Analysis of Reliability and Life Data, New York: John Wiley, explains it. Therefore, I will obtain the least-squares (LS) estimate of θ. In order to obtain a LS

Table 6.6 of Elsayed page 385 (TTF_s under accelerated testing condition)

10 eV	1037.39 hours, 3218.11, 3407.17, 3520.36, 3879.49, 3946.45, 6635.54, 6941.07, 7849.78, 8452.49
10 eV	9003.08, 9124.50, 9365.93, 9642.53, 10429.50, 10470.60, 11162.90, 12204.50, 12476.90, 23198.30 hours TTF_s ; note that the normal operating stress = eV_{OP} = 5.0; CV = 0.583127
25 eV	809.10, 1135.93, 1151.03, 1156.17, 1796.53, 1961.23, 2366.54, 2916.91, 3013.68, 3038.61 hours TTF_s ; CV = 0.546156
25 eV	3802.88, 3944.15, 4095.62, 4144.03, 4305.32, 4630.58, 4720.63, 6265.99, 6916.16, 7113.82 hours

estimate of θ, I first linearized the IPL model, $TTF_s = C/ S^b$, by taking the natural logarithm of both sides. This leads to $y = \ln(TTF_s) = \ln(C) - b\times\ln(S)$, where S is at 2 levels, 10 and 25 eV. I used Minitab to regress $y = \ln(TTF_s)$ on $x = \ln(S)$, with the following output.

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The regression equation is
y = 11.0 - 0.941 x,  x = ln(S)

Predictor      Coef    SE Coef       T      P
Constant    11.0065     0.6383   17.24  0.000
x           -0.9406     0.2281   -4.12  0.000

S = 0.6609    R-Sq = 30.9%    R-Sq(adj) = 29.1%

Analysis of Variance

Source           DF       SS        MS       F      P
Regression        1   7.4281    7.4281   17.01  0.000
Residual Error   38  16.5968    0.4368
Total            39  24.0248
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Unusual Observations

Obs	x	y	Fit	SE Fit	Residual	St Resid
1	2.30	6.979	8.841	0.148	-1.862	-2.89R

I must caution the reader before using the above Minitab model for extrapolation! You must observe that the value of R^2_{Model} = 30.9% is woefully too small to be an adequate model due to the fact that there is too much within (or experimental error) variability in the data. The data under level 1 of stress (10 eV) ranges from 1037.39 stressed hours to 23198.30 hours to failure, which is huge, but still the regression is highly significant. The above model cannot be improved because there are only 2 levels of stress factor and hence regression can have only one *df* and any attempt to improve it by adding regressors such as x^2, x^3 and 1/x to the model will be futile because the design does not provide but one *df* for studying effects. Hence, we have to extrapolate with a model whose R^2_{Model} = 30.9%. The estimate of the constant $\hat{C}$ = $e^{11.0065}$ = 60264.591 which is quite different from Elsayed's answer of 5362.25 given in the middle of his page 386, but the estimate of the exponent $\hat{b}$ = 0.9406 is very close to Elsayed's answer of 0.95318 listed in the middle of page 386. To estimate the $MTTF_{OP}$ at 5 eV, we insert x_{OP} = ln(5) =1.609438 into the regression model. This yields $\hat{y}_{OP}$ (1.60944) = 11 – 0.9406×1.60944 = 9.49266 → $\widehat{MTTF_{OP}}$ = $e^{9.49266}$ = 13262.06124 hours, which is fairly close to Elsayed's answer of 12140 hours atop his page 387, but quite different from 16065 hours given in the middle of his page 386. Since the Weibull mean E(T) = $\theta\,\Gamma(1+\frac{1}{\beta})$, then $\hat{\theta}_{OP}$ = 13262.06124 $/\,\Gamma(1+\frac{1}{1.910})$ = 14947.92193 hours. The two acceleration factors are A_{f1} = 13262.06124 /8298.3295 = 1.59816036, where 8298.3295 = $\bar{y}(\text{at } 10\,\text{eV})$ and A_{f2} = 13262.06124 /3464.2455 = 3.82827.

(3) The Eyring Model

Both the Arrhenius and IPL models include only the effect of one accelerated stress. The Eyring model contains two stress factors, one of which is always temperature stress, and the other can be any stress type such as electric field, voltage, humidity, mechanical stress (load per area), even temperature cycling, or electrical current stress. The rate of reaction (or rate of failure) to the two stresses is given by

$$r = C_1 e^{-E_a/(kT)} \times S^b \tag{119a}$$

where r is the reaction rate to the two stresses; r may be thought of the parameter λ if the underlying distribution is exponential, but $r \cong 1/\theta$ if the TTF is W(0, θ, β), and if TTF is lognormal, then $r = 1/T_{0.50}$ (the reciprocal of median life). Thus, from (119a) we deduce that

$$TTF_s = C e^{E_a/(kT_s)} / S^b = C e^{E_a/(kT_s)} \times S^{-b} \tag{119b}$$

The values of E_a and exponent b can be obtained empirically once accelerated data are available. For electronic applications, $b \cong 2$ to 3 and $E_a = 0.90$ eV, and C is a constant characteristic of the product and testing conditions. Equation (119b) implies that under normal operating conditions the TTF is given by

$$TTF_{OP} = C e^{E_a/(kT_{OP})} / S_{OP}^b \tag{119c}$$

Combining equations (119 b&c) yields

$$A_f = \frac{TTF_{OP}}{TTF_s} = \frac{e^{E_a/(kT_{OP})} / S_{OP}^b}{e^{E_a/(kT_s)} / S^b} = (S/S_{OP})^b \, e^{(E_a/k)(1/T_{OP} - 1/T_s)} \tag{120}$$

Note that A_f must be directly proportional to E_a because larger activation energy required to induce failure in the test unit generally implies longer $MTTF_{OP}$. Note that, like Ebeling, some sources use the $TTF_s = C e^{E_a/(kT_s)} \times e^{-bS}$, but this will not impact the value of AF = A_f.

Example 24. L. W. Condra (RE Improvement with DOE, 2nd edition, Marcel Dekker) reports (on his p. 239) the results of an accelerated life testing experiment of n (unspecified) microelectronic circuits conducted at the standard accelerated temperature stress of 85 °C and a standard accelerated relative humidity (RH) of S = 85%. (He refers to this type of accelerated testing as Temperature-Humidity Operating Bias test.) The sample $MTTf_s$ is reported to be 800 hours and normal operating conditions are T_{OP} = 40 °C and RH_{OP} = 60%. The Model (120) when the 2nd stress represents S = RH (relative humidity) is referred to as Peck's relationship. Peck, D. S. (1986) *"Proc. International RE Physics Symposium, 24, pp. 44-45*, reports an exponent value of $b \cong 2.70$ and an activation energy of $E_a = 0.79$ eV, but Condra in his example uses the rough values of b = 3 and $E_a = 0.90$ eV. I will use Peck's values

in equation (120) to estimate the acceleration factor A_f.

$$A_f = (85/60)^{2.7} e^{(0.79\times10^5/8.6171)(1/313.15 - 1/358.15)} = 101.3770$$

which yields $\widehat{MTTF_{OP}}$ = 101.3770×800 hours = 81101.6013 hours = 9.2582 years.

The above estimated value of $\widehat{MTTF_{OP}}$ = 9.2582 years does not conform well with that of Condra's 16.6 years. If we use Condra's values of b = 3 and E_a = 0.90 eV in equation (120), we obtain A_f = 187.780224 and an estimated $\widehat{MTTF_{OP}}$ = 187.780224 × 800 = 150224.179364 hours = 17.148878923 years. I tried to obtain Condra's answer of A_f = 182 by using his values of T_{OP} = 313 and T_s = 358 in equation (120) but I still got an answer of A_f = 188.5450005 which is not quite equal to Condra's answer of 182.

The reader should be careful about interpreting the values of $\widehat{MTTF_{OP}}$ because if the underlying distribution is exponential, then $\widehat{MTTF_{OP}}$ is an estimate of MTTF; if the underlying distribution is Weibull, then $\widehat{MTTF_{OP}}$ is an estimate of the characteristic life t_c = θ, and if the underlying distribution is lognormal, then $\widehat{MTTF_{OP}}$ is an estimate of the median life. Furthermore, the farther the operating conditions are from the stressed conditions, the less accurate the regression estimates of b and E_a become. This problem gets compounded when the baseline distribution is very highly skewed and /or there are outliers in the data.

<u>Example 6.13 on pages 387-388 of Elsayed.</u> The data listed in Table 6.7 atop page 388 of Elsayed presents the results of an ALT with 8 FLCs (factor level combinations) of Temperature and Voltage stresses. For your convenience, I am providing Elsayed's data below. The normal operating temperature T_{OP} = 30 °C = 303.15 Kelvin and the operating voltage is V_{OP} = 25 volts. Instead of using Elsayed's parametric approach to estimate $MTTF_{OP}$, I linearized the Eyring Model (119b) as follows: y = ln(TTF_s) = ln (C)– b ln(V_s) + E_a x, where x = 100000/ (8.6171T_s), and then I regressed y on ln(V_s) and x. The Minitab output is given below Table 6.7.

Table 6.7 of E. A. Elsayed (his p. 388). **T_{OP} = 30 °C; 60°C and 70°C are accelerated-stress Temperatures; V_{OP}= 25 volts. The accelerated voltages are 50, 100, 150, and 200 volts.**

Voltage / Temperature	50 v	100	150	200 volts
60°C	1800 hours	1500	1200	1000
70°C	1500 hrs TF	1200	1000	800 hours

The regression equation is

y = 2.25 - 0.427 LV + 0.200 x, LV = $\ln(V_s)$, x = 100000/(8.6171T_S), y = $\ln(TTF_s)$

```
Predictor      Coef    SE Coef      T      P
Constant      2.253      1.405   1.60  0.170
LV         -0.42674    0.03955 -10.79  0.000
x           0.19973    0.04057   4.92  0.004

S = 0.05824   R-Sq = 96.6%   R-Sq(adj) = 95.2%

Analysis of Variance

Source          DF       SS       MS      F      P
Regression       2  0.47708  0.23854  70.32  0.000
Residual Error   5  0.01696  0.00339
Total            7  0.49404

Source  DF   Seq SS
LV       1  0.39488
x        1  0.08220
```

In the above Minitab output, LV = $\ln(V_s)$ and y = $\ln(TTF_s)$. Note that the value of E_a from the above regression output is $E_a \cong 0.20$ which is identical to the reported value by Elsayed near the bottom of his page 387. In order to estimate the $MTTF_{OP}$, I used extrapolation (which is generally not a good idea in regression analysis) in the above regression model, which has an excellent R^2_{Model}, as follows: $\hat{y}_{OP}(30\ ^\circ C,\ 25\ volts) =$ 2.253 – 0.42674 ln(25) + 0.20 × 100000/(8.6171×303.15) = 8.535542 → $\widehat{MTTF_{OP}}$ = $e^{8.535542}$ = 5092.5911 hours, which is a bit larger than L_{OP} = 4484.11 hours reported by Elsayed in the middle of his page 388. There are 8 different values of A_f because there are 8 (= 2×4) FLCs of the two stresses. For

example, the value of A_f from normal operating conditions (30°C, 25 volts) to stress FLC (60 °C, 150 volts) is $\hat{A}_f$ = 5092.5911 / 1200 = 4.24383. To verify the adequacy of the Eyring model to the data, we also need to estimate this last acceleration factor A_f from equation (120) as follows: $\hat{A}_f$ (Model) = $(\frac{150}{25})^{0.42674} e^{(19973/8.6171)(1/303.15 - 1/333.15)}$ = 4.27663, which is fairly consistent with the previous value of 4.24383.

<u>Example 6.7 of Elsayed on pages 369-371.</u> This experiment makes no assumptions about the underlying distributions of Times TF and uses regression to estimate the MTTF by extrapolation. I used the data in Table 6.1 of Elsayed on his page 370 to regress the TTF on stress factor Temperature in Kelvin, and stress factor electric field measured in units of eV. For your convenience, I am duplicating Elsayed's Table 6.1 on the next page.

The resulting Minitab output is given below:

Regression Analysis: TTF versus Temperature (units will not impact results), eV

The regression equation is
TTF = 6062 - 17.8 T + 160 eV

Predictor	Coef	SE Coef	T	P
Constant	6062.15	3.56	1703.55	0.000
T	-17.8487	0.0134	-1329.01	0.000
eV	160.159	0.225	712.68	0.000

S = 1.163   R-Sq = 100.0%   R-Sq(adj) = 100.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	3967238	1983619	1.466E+06	0.000
Residual Error	19	26	1		
Total	21	3967264			

The number of distinct predictor combinations equals the number of parameters. No degrees of freedom for lack of fit.

Cannot do pure error test because there only 3 levels and the model absorbs the entire 2df so that there is no df left for LOF.

Source	DF	Seq SS
T	1	3280158
eV	1	687080

Table 6.1 (Example 6.7 data Atop Page 370 of E. A. Elsayed; OP =25^{o}C, 5 eV)

Temperature C°	100	100	100	100	100	100				
Electric Field (eV)	10	10	10	10	10	10				
Stressed TTF (TTF_s)	1000 hours	1002	1003	1004	1005	1006 hours TF				

CV(100,10) = 0.002153

Temperature C°	150	150	150	150	150	150				
Electric Field (eV)	10	10	10	10	10	10				
Stressed TTF (TTF_s)	110	110.5	110.7	111	111.4	111.8 hours TF_s				

CV(150,10) = 0.005816

Temperature C°	200	200	200	200	200	200	200	200	200	200
Electric Field (eV)	15	15	15	15	15	15	15	15	15	15
Stressed TTF (TTF_s)	19	19	19.1	19.2	19.3	19.32	19.38	19.4	19.44	19.49

CV(200,15) = 0.00929953

Unusual Observations

Obs	T	TTF	Fit	SE Fit	Residual	St Resid
17	373	1000.00	1003.33	0.47	-3.33	-3.14R
22	373	1006.00	1003.33	0.47	2.67	2.51R

R denotes an observation with a large standardized residual

Regression Analysis: Lnts versus x, lnEF = ln(ELCF); ELCF =Electric field.

The regression equation is

Lnts = - 11.7 + 0.599 x - 0.0333 lnEF

Predictor	Coef	SE Coef	T	P
Constant	-11.6518	0.0659	-176.80	0.000
x	0.599347	0.001125	532.97	0.000
lnEF	-0.03327	0.01515	-2.20	0.041

S = 0.00715762 R-Sq = 100.0% R-Sq(adj) = 100.0%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	58.910	29.455	574939.34	0.000
Residual Error	19	0.001	0.000		
Total	21	58.911			

Source	DF	Seq SS
x	1	58.910
lnEF	1	0.000

Predicted Values for New Observations
New

Obs	Fit	SE Fit	95% CI	95% PI
1	11.62293 $\rightarrow$ MTTf$_{OP}$ = $e^{11.62293}$ = 111628.313,	0.00703 ,	(11.60822, 11.63764)	(11.60193, 11.64393)

Values of Predictors for New Observations
New

Obs	x	lnEF
1	38.9	1.61

To estimate the MTTF$_{OP}$ from the above regression model at the normal operating temperature T$_{OP}$ = 25°C = 298.1500 Kelvin, and 5 eV, we insert these values into the model as follows: $\widehat{\mathbf{MTTF_{OP}}}$ = 6062.15 − 17.8487×298.15 + 160.159×5 = 1541.176608 hours which is quite consistent with that of Elsayed's atop his page 371. The value of the acceleration factor from normal operating conditions (25°C, 5 eV) to stress levels (100°C, 10 eV) is equal to $\hat{\mathbf{A}}_{\mathbf{f1}}$ = 1541.176608/$\mathbf{MTTf(100^oC, 10\,eV)}$ = 1541.176608/1003.33333 = 1.53605642. Similarly, the value of the acceleration factor from normal operating conditions (25°C, 5 eV) to stress levels (200 °C, 15 eV) is given by $\hat{\mathbf{A}}_{\mathbf{f2}}$ = 1541.176608/19.263 = 80.0071. Using the Eyring model (the parametric approach), $\widehat{\mathbf{MTTF_{OP}}}$ = $e^{11.62293}$ = 111628.3133, which is quite different from the nonparametric approach of 1541.176608. The value of Equation (120) gives $\hat{\mathbf{A}}_{\mathbf{f2}}$(Model) = $(\frac{15}{5})^{0.03325}e^{(59937.7/8.6171)(1/298.15 - 1/373.15)}$ = 112.3912, which is not consistent with $\hat{\mathbf{A}}_{\mathbf{f2}}$ = 80.0071 and may warrant the rejection of the Eyring model because the discrepancy between the two methods of computing MTTf$_{OP}$ should not be so drastic. It has been reported that Eyring model does not work well in Accelerated-testing with electrical stresses, and is mostly suited for Temperature and Mechanical stresses. Finally, Note that one reason behind the discrepancy between the Eyring Model and the Nonparametric approach could be the fact that the CV[at (200,15)] is more than 3 times the CV(100,10), perhaps an indication Over-acceleration.

Time-Compression and Constant-Stress Models (see pp. 350-355 of Ebeling)

As Ebeling states on his p. 350 the TsTF (Times-to-Failure) can be accelerated by

increasing cycles per hr, or increasing hours of usage, or both. In such higher than normal stress environment, it is assumed that life under operating conditions, t_{OP}, is linearly related to stressed life t_s, i.e., $t_{OP} = AF \times t_s = A_f \times t_s$. For example, if the number normal operating cycles for a system per day is 25, but on a 24-hr period we do the experimentation at 500 cycles, then the AF = A_f =500/25 = 20. On the other hand, if the daily use of the system per week is 20 hrs, but during one week we do 168 hrs of testing, then AF = 168/20 = 8.40. Further, if the underlying distribution is Weibull, then it is assumed that its shape β parameter stays in tact under accelerated testing. Ebeling provides a good example (13.11, p.351), where a mechanical part is cycled 100 cycles per hour, where the operating number of cycles for the part is 5 per hour; thus, Af = AF = 100/5 =20. He further states that a Weibull with θ_s = 1000 hrs and β_s = 2.5 was found to be a good-fit for the data. Thus, an estimate of $\theta_{OP} = 20 \times 1000 = 20000$ hrs, leading to $\hat{R}(t_{OP}) = \exp[-(t_{OP}/20000)^{2.5}]$. If one wishes to be very conservative, s/he could replace the θ_s = 1000 hrs with $t_s^{(1)}$, which is the value of the 1st-order statistic under stressed conditions.

For a constant-stress model, see the Example 13.13 and 13.14 on page 353, where in Example 13.13, $Temp_s = 120^oC$, while operating conditions Temperature is $Temp_{OP} = 25^oC$, where one unit was tested to failure and found to have a stressed life t_s = 500 hrs (note that Ebeling gives a $MTTF_s$, which is not common usage for one unit being tested). He does not state how he arrived at AF = 15 (it seems he assumes this value). Thus, under this assumption, $MTTF_{OP} = 15 \times 500 = 7500$ hrs. I checked all the values in Ebeling's Example 13.14 on p. 353, but it seems that the n = 30 TsTF were obtained at θ_s = 100, or else AF would not equal to 3. I further checked His experimental results in Example 13.15, pp. 354-355 and the correct values of $\hat{\theta}$ and $\hat{\beta}$ are tabulated below, where the normal operating conditions is 2 psi. Note that Ebeling's θ-hats are very close to mine (and those of Minitab's), but his last 2 estimates of shape are inaccurate. It seems that the last two stress-levels (9 and 10 psi) may have changed the shape of the underlying distribution, an indication of over-accelerating.

Stress Levels	7 psi	8 psi	9 psi	10 psi
β-hat	2.679476	2.585545	3.745004	3.868121
θ-hat	4647.35998	3435.2533	1910.6474	*1103.3577*

ALT Chapter Summary

1. **The acceleration factor For the Arrhenius Model is given by**

$$A_f = e^{E_a\left(\frac{1}{T_{OP}} - \frac{1}{T_S}\right)/k} \quad \rightarrow \quad MTTF_{OP} = A_f \times MTTF_S,$$

where k = Boltzman's constant = 8.6171×10^{-5}. Two cases exit: (a) The required activation energy E_a to induce failure is known, (b) E_a is not known and has to be empirically estimated from accelerated data. For Semiconductor failure $0.30 \le E_a \le 0.60$; for intermetallic diffusion $0.90 \le E_a \le 1.10$; for silicon junction defects $E_a = 0.80$ eV. Generally, $0.15 \le Ea \le 1.20$.

(a) Assume $E_a = 0.288$ and normal operating temperature is 25 °C and accelerated testing is done at 125 °C. Then T_{OP} = 25 + 273.15 = 298.15K and T_S = 125 + 273.15 = 393.1500K →

$$A_f = e^{0.288\left(\frac{1}{298.15} - \frac{1}{393.15}\right)10^5/8.6171}$$ **= 15 (see the Example 13.3 p.353 of Ebeling).**

Note that A_f is an increasing function of E_a because larger values of E_a imply that more energy is required to induce failure which in turn would lead to higher $MTTF_{OP}$. Note that most sources use the conversion Kelvin = °C + 273.15 and a few may use Kelvin = °C + 273.16.

(b) E_a is unknown.

Identify at least two stressed temperature levels, such as 50°C and 75°C (< $0.45T_m$, T_m = melting Temp) and obtain stressed failure data. Linearize the Arrhenius model TTF_S = $c\,e^{E_a/(k\,T_S)}$ and regress $\ln(TTF_S)$ on $x = 10^5/(8.6171T_s)$; then the rough estimate of E_a is given by the slope of the regression line. However, one must be cognizant of the fact that extrapolation is classical regression violates regression assumptions and is generally frowned upon. But then when there are no information about E_a (physical or otherwise), then the

regression approach would be the only way to obtain a statistically unsound manner of obtaining a rough estimate of the activation energy E_a.

2. The IPL : $TTF_s = C/S^b \rightarrow$ **Larger values of b induce higher failure rate reaction and smaller TTF. The value of b = [2, 3] for metals and electronic solder joints, b = [4, 7] for intermetallic fatigue failure, and b = [4, 10] for microelectronic parts, and very rarely b lies outside the range [2, 20].**

(a) b is known $\rightarrow A_f = S^b/S_{OP}^b$

For example, suppose the normal operating voltage is S_{OP} = 110V, stressed voltage is S = 220 volts and b = 2.8. Then $A_f = (220/110)^{2.8} = 6.9644$.

(b) b is unknown. First linearize $TTF_s = C/S^b \rightarrow$

$y = \ln(TTF_s)$, $x = \ln(S)$, **y-intercept=** $\ln(C)$, **and** $\hat{b} = -$ **slope of the LS line.**

3. The Eyring Model : $TTF_s = C\, e^{E_a/(kT_s)} / S^b = C\, e^{E_a/(kT_s)} \times S^{-b}$

$$A_f = (S/S_{OP})^b\, e^{(E_a/k)(1/T_{OP} - 1/T_S)}$$

$$= (S/S_{OP})^b \times e^{(10^5 E_a/8.6171)(1/T_{OP} - 1/T_S)}$$ **; always has two stress factors.**

(a) Both E_a and b are known. **A_f can easily be computed from the above equation.**

(b) At least one is unknown. Use stressed data to extrapolate to estimate b and E_a.

Note that this extrapolation often does not provide adequate and /or reasonable estimates of E_a and b, which implies that the Eyring model does not fit the data, and/or regression assumptions are grossly violated. Further, extrapolation is always on poor statistical ground and is used in accelerated testing because there are no other options, i.e., the constants b and E_a are unknown and testing under normal operating conditions involves well over thousands of hours. It should be emphasized that the Eyring model does not always work well with electrical stresses. Last but not least, in real-life situation, the accelerated load should not exceed the point that changes the base-line distribution and/or the shape.