General Rules for Constructing a FFD in a Prime Base

By S. Maghsoodloo

1. A b^{k-p} FFD implies that we have k main factors each at b levels but can afford to conduct only b^{-p}th fraction of the entire b^k runs (k > 2). For example, a 2^{7-3} FFD implies th~~at~~ we have 7 factors but can afford only $2^{4(=7-3)}$ experimental runs, or only $(2^{-3} =1/8)^{th}$ fraction of the entire 2^7 = 128 FLCs. Because $2^4 = 16$ there will be 16 rows of FLCs for the design matrix X.
   While a 3^{6-2} FFD implies that we have 6 factors each at 3 levels but we can afford only a $3^{-2} = (1/9)^{th}$ fraction of the entire $3^6 = 729$ FLCs, i.e., our design matrix, X, consists of only $3^{4(=6-2)} = 81$ distinct FLCs.

2. The exponent (k-p) implies that we can write only (k-p) arbitrary columns and must select the p independent generators containing maximum number of letters that also involve all k factors. For example, for the 2^{7-3} FFD, we must select 3 independent generators such as g_1 = ABCD, g_2 = CDEF, g_3 = ADFG. While, the 3^{6-2} FFD requires 2 independent generators (because p = 2), such as $g_1 = ABC^2D$ and $g_2 = CDE^2F^2$ that involve all 6 factors.

3. Because a b^{k-p} FFD divides the entire b^k runs into b^p blocks, then each effect will have exactly $b^p – 1$ aliases because only one BLK is studied and the remaining $b^p – 1$ BLKs are not studied. For example, the 2^{7-3} FFD has 128 distinct FLCS only 16 of which are

studied. There are 8 BLKs of 16 FLCs each; the 8 BLKs carry 7 df and hence the design must have 7 one-df generators, only 3 of which are independent. While the 3^{6-2} FFD divides the 729 FLCs into 9 BLKS, each with 81 FLCs, but only one BLK is studied and the other 8 are not investigated, and hence each effect has 8 aliases, one alias per BLK that has not been studied. The 9 BLKs carry 8 df and hence all the generators must have 8 df so that we must have 4 generators each with 2 df.

4. **Each generator yields exactly (b-1) alias('s) for each effect in base b. For example, the other 4 generators of the 2^{7-3} FFD with independent generators g_1 = ABCD, g_2 = CDEF, g_3 = ADFG are $g_4 = g_1 \times g_2$ = ABEF, $g_5 = g_1 \times g_3$ = BCFG, $g_6 = g_2 \times g_3$ = ACEG, and $g_7 = g_1 \times g_2 \times g_3$ = BDEG. Each of these 7 generators yields exactly one alias for each effect. While, for the base-3 example 3^{5-2} FFD with independent generators $g_1 = ABC^2D$ and $g_2 = CDE^2F^2$, we need 2 more generators because each column in base-3 has 2 df. The other 2 generators are $g_3 = g_1 \times g_2 = ABD^2E^2F^2$ and $g_4 = g_1 \times g_2^2 = ABCE^2F^2$. For example, the generator $g_1 = ABC^2D$ in base-3 yields the 2 aliases AB^2CD^2 and BC^2D for the factor A.**

5. **The resolution, *R*, of the b^{k-p} FFD is the minimum number of letters amongst all its generators.**

6. **The principal BLK in any FFD is the one for which all contrast functions of the generators have zero values.**