1. A simple vapor refrigeration cycle uses R-134a. The evaporator is maintained at a temperature of \(-10^\circ\text{C}\), and the minimum temperature in the condenser is \(2^\circ\text{C}\) above the ambient temperature. Assuming the compressor is isentropic, calculate the compressor work per unit mass, and the cycle coefficient of performance, for ambient temperatures of 10, 20, and 30°C.

Evaporator exit (state 1): \(h_1 = h_b(\text{sat}(-10^\circ\text{C})) = 244.5\ \text{kJ/kg},\ s_1 = 0.938\ \text{kJ/kg\ K}\). Compressor is isentropic, \(s_2 = s_1\) and \(P_{\text{cond}} = P_{\text{sat}}(T_3)\), where \(T_3\) is the condenser exit temperature, which is \(2^\circ\text{C}\) above the ambient. For \(10^\circ\text{C}\) ambient, then \(T_3 = 12^\circ\text{C},\ P_{\text{cond}} = 443\ \text{kPa}\). We now need to get \(h\) at \(s_2 = 0.938\ \text{kJ/kg\ K},\ P_2 = 443\ \text{kPa}\). An estimate, by inspection, is \(h_2 = 260\ \text{kJ/kg}\). State 3 is a saturated liquid at the condenser pressure: \(h_3 = 68.2\ \text{kJ/kg}\).

\[
w_c = h_2 - h_1 = 15.5\ \text{kJ/kg},\quad q_L = h_1 - h_4 = h_1 - h_3 = 176.3\ \text{kJ/kg}
\]

\[
COP = \frac{q_L}{w_c} = 11.4
\]

2. A heat pump using R-134 operates between the pressures of 200 kPa and 1.2 MPa. The compressor has an isentropic efficiency of 0.8. Calculate the required mass flow of refrigerant, and the power input to the compressor, necessary to provide a heating rate of 30 kW.

State 1 is a saturated vapor at 200 kPa: \(h_1 = 244.5\ \text{kJ/kg},\ s_1 = 0.938\ \text{kJ/kg\ K}\).

State 2s (\(s_{2s} = s_1,\ P_2 = 1.2\ \text{MPa}\)): \(h_{2s} = 280\ \text{kJ/kg}\).

\[
w_{c,s} = h_{2s} - h_1 = 35.5\ \text{kJ/kg},\quad w_c = \frac{w_{c,s}}{\eta_c} = 44.4\ \text{kJ/kg},\quad h_2 = h_1 + w_c = 288.9\ \text{kJ/kg}
\]

State 3: saturated liquid at 1.2 MPa: \(h_3 = 117.8\ \text{kJ/kg}\).

\[
q_H = h_2 - h_3 = 171.1\ \text{kJ/kg},\quad \dot{m} = \frac{30}{q_H} = 0.175\ \text{kg/s}
\]

\[
\dot{W}_c = \dot{m} w_c = 7.78\ \text{kW}
\]

3. A cascade vapor refrigeration system is designed to remove \(\dot{Q}_L = 20\ \text{kW}\) from a cooled space. The temperature of the evaporator in the bottom stage is \(-20^\circ\text{C}\), and the condenser in the top stage is at 1 MPa. Assuming that both compressors are isentropic, and that the bottom stage condenser pressure is equal to the upper stage evaporator pressure, determine the optimum bottom stage condenser pressure which minimizes the power input to the cycle.

The evaporator pressure is \(P_{\text{sat}}(-20^\circ\text{C}) = 133\ \text{kPa}\). I will work out a solution for one intermediate pressure: \(P_1 = 0.2\ \text{MPa}\). This is the pressure of the bottom condenser and the top evaporator.

Bottom cycle: \(h_{1b} = 238.4\ \text{kJ/kg},\ s_{1b} = 0.946\ \text{kJ/kg\ K}\). Compressor is isentropic and \(P_{2b} = 0.2\ \text{MPa}\): \(h_{2b} = 247\ \text{kJ/kg}\). State 3 is a saturated liquid: \(h_{3b} = 117.8\ \text{kJ/kg}\).

Top cycle: State 1 is a saturated vapor at 0.2 MPa: \(h_{1t} = 244.5,\ s_{1t} = 0.938\ \text{kJ/kg\ K}\). Compressor is isentropic, state 2 is at 1 MPa: \(h_{2t} \approx 276\ \text{kJ/kg}\). State 3 is a saturated liquid at 1 MPa: \(h_{3t} = 107.3\ \text{kJ/kg}\).

The 20 kW of heat removal occurs through the bottom stage evaporator:

\[
q_{Lb} = h_{1b} - h_{3b} = 120.6\ \text{kJ/kg},\quad \dot{m}_b = \frac{20}{q_{Lb}} = 0.166\ \text{kg/s}
\]

The total heat transfer from the bottom condenser must match the heat transfer to the top evaporator. This allows determination of the mass flow rate in the top cycle, and the total power input.

\[
w_{c,t} = h_{2t} - h_{1t} = 8.6\ \text{kJ/kg},\quad q_{Hb} = h_{2b} - h_{3b} = 129.2\ \text{kJ/kg}
\]

\[
q_{Lt} = h_{1t} - h_{3t} = 137.2\ \text{kJ/kg},\quad w_{ct} = h_{2t} - h_{1t} = 31.5\ \text{kJ/kg}
\]

\[
\dot{m}_t = \dot{m}_b \frac{q_{Hb}}{q_{Lt}} = 0.156\ \text{kg/s}
\]

\[
\dot{W} = \dot{m}_b w_{cb} + \dot{m}_t w_{ct} = 6.35\ \text{kW}
\]

\[
COP = \frac{\dot{W}}{\dot{W}t} = 3.15
\]