1. An air-standard cycle with variable specific heats is executed in a closed system and is composed of the following four processes:

1–2 Isentropic compression from 100 kPa, 27°C to 800 kPa.

2–3 $v$ = constant heat addition to 1800 K

3–4 Isentropic expansion to 100 kPa

4–1 $P$ = constant heat rejection to the initial state.

(a) Show the cycle on $P – v$ and $T – s$ diagrams.
(b) Calculate the net work per unit mass.
(c) Determine the thermal efficiency.

The procedure to the solution will be worked out. State 1 is fixed by $T_1 (= 300 K)$, $P_1$. Process 1–2 is isentropic, and $P_2$ is specified. Specific heat is not assumed constant, so the air tables are used to find state 2 via

$$s_2 – s_1 = 0 = s_2^0 – s_1^0 – R \ln \left( \frac{P_2}{P_1} \right)$$

Solve for $s_2^0$ and interpolate for $T_2$ and $u_2$, and

$$w_{1-2} = q_{1-2} = u_1 – u_1$$

$T_3$ is given. Get $u_3$ from tables:

$$q_{2-3} – u_{2-3} = u_3 – u_2$$

Process 2–3 is constant volume, so

$$P_3 = P_2 \left( \frac{T_3}{T_2} \right)$$

and 3–4 is isentropic, with specified $P_4$:

$$s_4 – s_3 = 0 = s_4^0 – s_3^0 – R \ln \left( \frac{P_4}{P_3} \right)$$

Solve for $s_4^0$, interpolate to get properties at 4, and

$$w_{3-4} = q_{3-4} = u_3 – u_4$$

Process 4–1 is constant pressure, and recall that a constant $P$ process has

$$q_{4-1} = h_1 – h_4$$

in addition, the constant pressure process, coupled with and IG, has

$$w_{4-1} = P(u_4 – v_1) = R(T_4 – T_1)$$

The net work is

$$w_{net} = w_{1-2} + w_{2-3} + w_{3-4} + w_{4-1}$$

It is critically important that the signs are correct on all of the process works. You should check that the net work equals the net heat:

$$q_{net} = q_{1-2} + q_{2-3} + q_{3-4} + q_{4-1}$$

The heat addition is during 2–3: $q_H = q_{2-3}$, and

$$\eta = \frac{w_{net}}{q_H}$$

2. An air-standard cycle is executed in a closed system and is composed of the following four processes:

1–2 Isentropic compression from 100 kPa, 27°C to 1 MPa

2–3 $P$ = constant heat addition in the amount of 2800 kJ/kg

3–4 $v$ = constant heat rejection to 100 kPa

4–1 $P$ = constant heat rejection to the initial state

(a) Show the cycle on $P – v$ and $T – s$ diagrams.
(b) Calculate the maximum temperature of the cycle.
(c) Determine the thermal efficiency.

Assume constant specific heats at room temperature.

Answers: 3360 K, 21%.

The solution is sketched out. Process 1–2 is the same as before with the same solution procedure. Process 2–3 is constant pressure with specified $q_H = q_{2-3}$, and

$$q_{2-3} = h_3 – h_2$$

Enthalpy $h_2$ is known from the $T_2$ obtained from the process 1–2 analysis. Use the above equation to solve for $h_3$ and interpolate in the tables to get properties at 3. The rest of the cycle follows an analysis procedure similar to the previous problem: in each process, you know the beginning state, and you apply 1st law and/or process relations to obtain the final state.

3. Consider a Carnot cycle executed in a closed system of 0.003 kg air. The temperature limits of the cycle are 300 K and 900 K, and the minimum and maximum pressures in the cycle are 20 and 2000 kPa. Assuming constant specific heats, determine the net work output per cycle.
4. An ideal gas Carnot cycle uses air as the working fluid and receives heat from 1027°C. The cycle operates at 1500 cycles/minute, and has a compression ratio of 12. Determine the heat rejection temperature, the thermal efficiency, and the amount of high-temperature heat input, per cycle, required for a power output of 500 kW.

Answers: 481 K, 63%, 31.8 kJ.

The last two problems involve the Carnot cycle. The Carnot cycle is impractical due to the constant-\( T \) heat addition and rejection processes, yet it does attain the theoretical maximum thermal efficiency of

\[
\eta_{\text{rev}} = 1 - \frac{T_L}{T_H}
\]

In the assigned problems the Carnot cycle uses an ideal gas as the working fluid. You may assume that the gas has constant specific heats. The isentropic compression and expansion processes will then have

\[
\left( \frac{v_1}{v_2} \right)^{k-1} = \frac{T_2}{T_1} = \frac{T_H}{T_L} = \frac{T_3}{T_4} = \left( \frac{v_4}{v_3} \right)^{k-1}
\]

from which one can see that

\[
\frac{v_1}{v_2} = \frac{v_4}{v_3} \equiv r
\]

where \( r \) is the compression ratio. The same arguments can be applied to the pressures instead of the volumes, which results in

\[
\frac{P_2}{P_1} = \frac{P_3}{P_4} = \left( \frac{T_H}{T_L} \right)^{k/(k-1)} = r^k
\]

Note that \( P_2 \) and \( P_4 \) are the maximum and minimum pressures in the cycle. So

\[
\frac{P_2}{P_3} = \frac{P_2}{P_4} \cdot \frac{P_3}{P_4} = \left( \frac{T_H}{T_L} \right)^{k/(k-1)}
\]

and since \( T_2 = T_3 = T_H \),

\[
\frac{P_2}{P_3} = \frac{v_3}{v_2}
\]

The heat added during the isothermal process 2–3 is

\[
Q_H = m \cdot RT_H \cdot \ln \left( \frac{v_2}{v_3} \right)
\]

The above equations should give you enough information to complete problem #3. In problem #4, use the isentropic relations and the given compression ratio to obtain \( T_L \) from the given \( T_H \). The power output is the work per cycle times the number of cycles per second, i.e.,

\[
\dot{W} = \frac{RPM}{60} \cdot W_{\text{net}}
\]

\[
W_{\text{net}} = \eta Q_H
\]