MECH 3020: Notes on design project

A few of you have had questions regarding the analysis when a reheater is present in the steam cycle. In general, the reheater will complicate considerably the analysis, yet there may be some real benefit in cycle performance from using a reheater.

The gas cycle analysis, for a chosen $r_P$ and regenerator configuration, will give you the air exit temperature. A regenerator will probably not be appropriate for this cycle: recall that the point of the regenerator is to avoid throwing away a hot air stream from the turbine. In this cycle, the hot air stream is being used to transfer heat to the water, so employing a regenerator simply means that there will be less overall heat transfer available from the air to the water.

The design in the schematic shows the air flow running in series through the reheater and the boiler. If a reheater is present in the vapor cycle design, the maximum steam temperature will occur at the exit of the reheater, and the boiler exit temperature will be at a somewhat lower temperature. This is because the temperature of the air exiting the reheater – and entering the boiler – will be at a lower temperature than the air entering the reheater. On the other hand, when the reheater is not present the temperature of the steam exiting the boiler will be 30°C lower than the air exhaust temperature.

The steam cycle without reheat will be more simple to analyze than the one with reheat. We will therefore assume – initially – that there is no reheater present. For the purposes of illustration, take the air exhaust temperature to be $T_{7a} = 561°C$, corresponding to the $r_P = 8$, no regeneration configuration. The temperature at the boiler exit is now established as $T_{5w} = 531°C$. (I know that this violates the maximum temperature criterion for the steam, so don’t use these results in your design). A boiler pressure is needed to completely fix the state at $5w$; assume here that $P_B = 8$ MPa.

One condition to check first is whether or not a reheater is needed. Recall that the reheater is employed to prevent excess moisture at the turbine exit/condenser inlet; if too much moisture is present, then the steam is expanded to an intermediate pressure, reheated to a superheated state, and expanded again to the condenser pressure. To check this, perform a calculation assuming a complete expansion from $P_B$ to $P_C = 50$ kPa:

\begin{align*}
    h_{5w} &= 3475 \text{ kJ/kg} \\
    s_{8ws} &= s_{5w} : \quad x_{8ws} = 0.88, \quad h_{8ws} = 2371 \text{ kJ/kg} \\
    h_{8w} &= h_{5w} - \eta_T (h_{5w} - h_{8ws}) = 2537 \text{ kJ/kg} : \quad x_{8w} = 0.953
\end{align*}

The quality at the low pressure exit will therefore be (just barely) within the specifications, and reheat is not needed. State $7w$ therefore becomes the same as state $6w$.

The rest of the steam cycle can now be filled in; A FWH is assumed present and operates at 1 MPa,
which will be the pressure of the turbine bleed at state 6w.

\[ s_{6w} = s_{5w} : \quad h_{6w} = 2890 \text{ kJ/kg} \] (4)

\[ h_{6w} = h_{5w} - \eta_T (h_{5w} - h_{6w}) = 2978 \text{ kJ/kg} \] (5)

\[ h_1s = 341 \text{ kJ/kg (sat. liq.)} \] (6)

\[ h_{2w} \approx h_{1s} + v_{1s}(P_{2w} - P_{1w}) = 342 \text{ kJ/kg} \] (7)

\[ h_{3w} = 763 \text{ kJ/kg (sat. liq.)} \] (8)

\[ y = \frac{h_{6w} - h_{2w}}{h_{3w} - h_{2w}} = 0.160 \] (9)

\[ h_{4w} \approx h_{3w} + c_{v3w}(P_{4w} - P_{3w}) = 771 \text{ kJ/kg} \quad \left(T_{4w} = 181^\circ C\right) \] (10)

A relationship linking the air and water mass flow rates can be obtained via analysis of the boiler. The effectiveness equation, given in the problem statement, will have

\[ \dot{Q}_B = \dot{m}_a (h_a(T_{7a}) - h_a(T_{8a})) = \epsilon \dot{m}_a (h_a(T_{7a}) - h_a(T_{4w})) \] (11)

In the above, \( h_a \) denotes the enthalpy for air, and the first part of the equation represents the first law formula for the rate of heat transfer (in this case, the heat transfer from the air). Assume constant specific heats, use \( \epsilon = 0.9 \) from the specs, and

\[ T_{8a} = T_{7a} - \epsilon (T_{7a} - T_{4w}) = 219^\circ C \] (12)

Now use the first law to equate the air and water heat transfer rates in the boiler:

\[ \dot{m}_w(h_{5w} - h_{4w}) + \dot{m}_a c_P(T_{8a} - T_{7a}) = 0 \] (13)

\[ \frac{\dot{m}_w}{\dot{m}_a} \equiv z = 7.53 \text{ kg air/kg water} \] (14)

The total cycle power output can now be obtained:

\[
\dot{W} = \dot{m}_a c_P (T_{4w} - T_{3a} - T_{2a} + T_{1a}) + \dot{m}_w (h_{5w} - y h_{6w} - (1 - y)h_{8w} - w_{HPP} - (1 - y)w_{LP}) \\
= \dot{m}_w \left[z c_P (T_{4w} - T_{3a} - T_{2a} + T_{1a}) + (h_{5w} - y h_{6w} - (1 - y)h_{8w} - w_{HPP} - (1 - y)w_{LP})\right] 
\] (15)

When the reheater is present, the entire boiler+reheater can be viewed as a single air–water heat exchanger which has the same effectiveness rating as the boiler without a reheater. That is, for the reheater case, the effectiveness formula becomes

\[ \dot{Q}_{B+RH} = \dot{m}_a C_P(T_{6a} - T_{8a}) = \epsilon \dot{m}_a C_P(T_{6a} - T_{4w}) \] (16)

which you can use to calculate \( T_{8a} \), as before. The heat transfer balance for the reheater will appear as

\[ (1 - y)\dot{m}_w(h_{7w} - h_{6w}) = \dot{m}_a C_P(T_{6a} - T_{7a}) \] (17)

and the boiler heat transfer balance is given by Eq. (13). The quantity \( y \) is the FWH bleed fraction; this will be zero if no FWH is used in the design. To apply Eq. (13), you need the conditions of the steam at the exit of the boiler, and to get these you need to use \( T_{5w} = T_{7a} - 30^\circ C \). Yet \( T_{7a} \) can only be predicted from Eq. (13), and this equation requires the water properties and flow rates.

Perhaps there is a simpler way around this; nevertheless, it appears that an iteration strategy must be used for the reheater configuration. The temperature \( T_{6a} \) will be obtained from the gas cycle, and assume that \( T_{7a} \) is somewhat lower than \( T_{6a} \) (perhaps 10 or 20 \(^\circ\)C). This allows you to fix \( T_{5w} \). Go through the analysis as before, and end by using Eq. (13) to calculate the air/water mass flow ratio. Equation (17) can now be used to re-calculate \( T_{7a} \), and the process can be repeated. Iterate a few times until the \( T_{7a} \) converges to a constant.