1. You pour 0.5 L of boiling water into a 5 L glass jar. After a few seconds – during which the steam rising from the liquid has displaced all of the air in the jar – you tightly cap the jar and place the jar in the refrigerator. At this moment the system can be assumed to be a saturated mix, with 1/10 of the total volume in the liquid phase. The system cools to 5°C. Calculate how much heat was transferred to the system.

In the lectures I made this problem out to be more difficult than it actually was. Taking the liquid density to be 1000 kg/m³ (close enough), the mass of the water is \( m = \rho V = 0.5 \text{ kg} \) (recall 1 L = 10⁻³ m³). The total volume is 5 L, and the specific volume is

\[
v = \frac{5 \times 10^{-3}}{0.5} = 0.01 \text{ m}^3/\text{kg}
\]

State 1 is a saturated mixture at 100°C, and

\[
x_1 = \frac{v - v_f}{v_f} = 5.36 \times 10^{-3}, \quad u_1 = u_f + x_1 u_f = 430.2 \text{ kJ/kg}
\]

with saturation properties evaluated at 100°C. The specific volume is constant during the process. The final state will be a saturated mixture at 5°C, and

\[
x_2 = \frac{v - v_f}{v_f} = 6.12 \times 10^{-5}, \quad u_2 = u_f + x_1 u_f = 21.2 \text{ kJ/kg}
\]

with saturation properties evaluated at 5°C. There is no work, and

\[
Q_{1-2} = m(u_2 - u_1) = -204.5 \text{ kJ}
\]

If we simply assumed that the system was 0.5 of liquid water, going from 100 to 5°C, then the heat transfer would be

\[
Q_{1-2} \approx m C(T_2 - T_1) = 0.5 \cdot 4.186 \cdot 95 = -199 \text{ kJ}
\]

The slightly larger heat transfer for the precise calculation is due to the fact that some of the vapor condenses during the process, and this condensation releases latent heat.

2. A rigid container of volume 0.1 m³ contains 5 kg of water at 20°C. The maximum allowable pressure of the container is set at 5 MPa. The system is now connected to a heat source, which transfers heat at a rate of 500 W to the container. How long will the container survive?

The specific volume is \( v = 0.1/5 = 0.02 \text{ m}^3/\text{kg} \), and it is constant because the container is rigid. The initial state is a saturated mixture,

\[
x_1 = \frac{v - v_f}{v_f} = 3.29 \times 10^4, \quad u_1 = u_f + x_1 u_f = 84.8 \text{ kJ/kg}
\]

and the final state is also a saturated mix at 5 MPa, with \( x_2 = 0.49 \) and \( u_2 = 1859 \text{ kJ/kg} \). The total heat transfer is

\[
Q_{1-2} = m(u_2 - u_1) = 8871 \text{ kJ}
\]

and the time until explosion is

\[
\Delta t = \frac{Q_{1-2}}{Q} = \frac{8871}{0.5} = 17742 \text{ s} = 4.93 \text{ hours}
\]

3. A system contains 2 kg water at 500°C and 500 kPa. Heat is now transferred from the water in a constant pressure process. At the end of process 3000 kJ of heat has been removed from the water. Find the initial and final volumes of the system.

The initial state is superheated with \( h_1 = 3484 \text{ kJ/kg} \), \( v_1 = 0.711 \text{ m}^3/\text{kg} \), and

\[
V_1 = m v_1 = 1.42 \text{ m}^3
\]

For a constant pressure, control mass process the heat transfer is

\[
Q_{1-2} = m(h_2 - h_1) \rightarrow h_2 = h_1 + \frac{Q_{1-2}}{m} = 1984 \text{ kJ/kg}
\]

Make sure you use the correct sign for heat transfer: it is negative for this problem. The properties \( h_2 \) and \( P_2 = P_1 \) fix state 2. This is a saturated mixture, since \( h_2 < h_g \), and

\[
x_2 = \frac{h_2 - h_f}{h_f} = 0.637, \quad v_2 = v_f + x_2 v_f = 0.239 \text{ m}^3/\text{kg}
\]

and

\[
V_2 = m v_2 = 0.479 \text{ m}^3
\]

4. Air is contained in a piston–cylinder apparatus. Initial conditions are \( V_1 = 0.05 \text{ m}^3 \), \( P_1 = 120 \text{ kPa} \), and \( T_1 = 300 \text{ K} \). Heat is now added to the gas, and the air expands in a constant pressure process. At the end of the process the volume has doubled. Determine the final temperature and the heat transfer to the air. Use the air tables in the back of the book to solve this problem.

Assume the air is an ideal gas. Since mass is constant,

\[
\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}
\]

and since \( P_2 = P_1 \),

\[
T_2 = T_1 \frac{V_2}{V_1} = T_2 \cdot 2 = 600 \text{K}
\]

For a constant pressure process, the heat transfer is given by

\[
q_{1-2} = h_2 - h_1
\]

and for an ideal gas, \( h \) is a function of temperature only. From table A-17,

\[
h_1 = 300.2 \text{ kJ/kg}, \quad h_2 = 607.0 \text{ kJ/kg}
\]
and

\[ q_{1-2} = 306.8 \text{ kJ/kg} \]

The mass of the air is

\[ m = \frac{P_1 V_1}{RT_1} = \frac{120 \cdot 0.05}{0.287 \cdot 300} = 0.0697 \text{ kg} \]

and

\[ Q_{1-2} = m q_{1-2} = 21.38 \text{ kJ} \]

5. Air is contained in an insulated piston–cylinder apparatus. Initial conditions are \( V_1 = 0.1 \text{ m}^3, P_1 = 120 \text{ kPa}, \) and \( T_1 = 300 \text{ K}. \) The air is now slowly compressed, and the volume at the final state is half the initial volume. Assuming the specific heat of the air is constant with \( C_P = 1.005 \text{ kJ/kg K}, \) calculate the final temperature of the gas and the work done to the gas.

This adiabatic, ideal gas process corresponds to a polytropic process with exponent \( k = C_P/C_V. \) Use \( k = 1.4 \) for air, and \( C_V = C_P - R = 1.005 - 0.287 = 0.718 \text{ kJ/kg K}. \) The final temperature is

\[ T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{k-1} = 300 \cdot (2)^{0.4} = 395.9 \text{ K} \]

Make sure you use absolute temperature in the ideal gas relations. Since the system is adiabatic, the first law reduces to

\[ w_{1-2} = u_1 - u_2 \]

The substance is an ideal gas, and \( u \) is a function of temperature only. Assuming the specific heats are constant, then

\[ w_{1-2} \approx C_v(T_1 - T_2) = -68.82 \text{ kJ/kg} \]

Work is negative because the system is being compressed (work done on the system is negative, per our sign convention). The system mass is

\[ m = \frac{P_1 V_1}{RT_1} = 0.139 \text{ kg} \]

and

\[ W_{1-2} = m w_{1-2} = -9.59 \text{ kJ} \]

6. A rigid, insulated tank is internally divided into two subvolumes, A and B, each of equal volume. Side A contains 1 kg of an ideal gas at 100°C and 1 MPa pressure, while side B contains a vacuum. The partition is now suddenly removed, and the gas in A undergoes a free expansion to fill the entire tank. Find the final temperature of the gas. Note that a free expansion involves zero work.

There is no heat transfer because the system is insulated, and there is no work because the process is a free expansion. The first law reduces to

\[ u_2 = u_1 \]

and since internal energy is a function solely of temperature for an ideal gas, the temperature of the gas stays constant: \( T_2 = 100°C = 373 \text{ K}. \)

7. Repeat the problem, except now take side A to initially contain 1 kg of saturated water vapor at 1 MPa initial pressure. Use the H2O code to solve this problem. Does the water behave like an ideal gas?

Water is not an ideal gas, so two properties are needed to fix the final state. These are

\[ u_2 = u_1 = u_g(1 \text{ MPa}) = 2583.7 \text{ kJ/kg} \]

and

\[ v_2 = 2v_1 = 2v_g(1 \text{ MPa}) = 0.3888 \text{ m}^3/\text{kg} \]

Use the H2O code to find this state: \( T_2 = 164.7°C. \) The initial temperature is \( T_1 = T_{sat}(1 \text{ MPa}) = 179.9°C, \) and as you can see, the temperature did not remain constant during the process, as it would have if the water behaved as an ideal gas.