5. A flow of 2 kg/s of saturated liquid R-114 at 200 kPa is mixed with 3 kg/s of R-114 at 200 kPa and 85°C. The liquid and vapor phases of the mixture are separated, and each exit separately at 200 kPa.

Find the mass flow rates of the exiting liquid and vapor streams. Note that this problem involves two exits and two inlets.

\[ \dot{m}_1 \cdot h_1 = \dot{m}_3 \cdot h_3 + \dot{m}_2 \cdot h_2 \]

→ know \( \dot{m}_1 \) and \( T_1 \) → get \( h_1 \) (condensed liquid)

\( h_1 \) (condensed liquid) \( \approx h_f \) at \( T \)

→ exit states: say that \( p_e = p_3 = \text{given} \):

Say \( p_e = p_3 = 1 \text{ kPa} \)

→ state 3 is a saturated liquid at \( p_e \), \( \text{state 2 is a saturated vapor at } p_e \)

\[ \begin{align*}
\dot{m}_3 & = \frac{\dot{m}_1 h_1}{h_f} \\
\dot{m}_2 & = \frac{\dot{m}_1 h_1}{h_v} \quad h_f @ p_e \\
\dot{m}_2 & = \frac{\dot{m}_1 (1-x)}{h_v} \quad h_v @ p_e
\end{align*} \]

\[ \begin{align*}
\dot{m}_3 & = \frac{\rho_1}{\rho_f} h_f \\
\dot{m}_2 & = \frac{\rho_1}{\rho_v} h_v
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\end{align*} \]

The Second Law of Thermodynamics

Consider this simple heat exchanger problem

\[ \begin{align*}
T_{in} &= 100 \text{ C} & T_{in} &= 100 \text{ C} \\
T_{out} &= 50 \text{ C} & T_{out} &= 50 \text{ C}
\end{align*} \]

First law analysis:

\[ \dot{m}_R (h_2 - h_1)_{in} + \dot{m}_C (h_2 - h_1)_{out} = 0 \]

\[ \Delta h = \dot{m} c \Delta T \]

\[ \dot{m}_R = \dot{m}_C = 1 \text{ kg/s} \]

\[ T_{out} = T_{in} + (T_{in} - T_{out}) = 120 \text{ C} \]

What if we now ‘reversed’ the problem:

\[ \begin{align*}
T_{out} &= 100 \text{ C} & T_{out} &= 100 \text{ C} \\
T_{in} &= 50 \text{ C} & T_{in} &= 50 \text{ C}
\end{align*} \]

First law analysis is the same:

\[ T_{in} = T_{out} + (T_{in} - T_{out}) = 120 \text{ C} \]

What is wrong with this picture?

- Nothing is wrong, as far as the first law is concerned.
- Yet something is obviously incorrect: heat would have to flow from the cold to the hot stream.
- We need some way (besides our common sense) of analytically proving that this process is impossible.

Consider this problem for an isothermal air compressor:

\[ \begin{align*}
T_1 &= 20 \text{ C} & T_2 &= 20 \text{ C} \\
P_1 &= 1 \text{ atm} & P_2 &= 7 \text{ atm}
\end{align*} \]

\( q = w = h_2 - h_1 = 0 \) (isothermal ideal gas)

\[ q = 0 = -200 \text{ kJ/kg} \]

What is the exit pressure?

- A poorly-designed compressor could have \( P_2 = P_1 = 1 \text{ atm} \): no compression at all.
- What is the ‘best-possible’ performance? How high could \( P_2 \) be?
- There is nothing in the first law that will tell us this information.

A ‘perfect’ power cycle:

\[ \begin{align*}
T_1 &= 20 \text{ C} & T_2 &= 20 \text{ C} \\
P_1 &= 1 \text{ atm} & P_2 &= 7 \text{ atm}
\end{align*} \]

Given:

1: saturated liquid at 20 kPa: \( h_1 = 251.4 \text{ kJ/kg} \)
2: compressed liquid: \( h_2 = 256.4 \text{ kJ/kg} \)
3: superheated: \( h_3 = 3456.5 \text{ kJ/kg} \)

First law analysis:

\[ \begin{align*}
\eta &= h_3 - h_2 = 3200.1 \text{ kJ/kg} \\
w &= h_2 - h_1 = 3200.1 \text{ kJ/kg} \\
w_{net} &= w + w_s = 3200.1 \text{ kJ/kg} \\
\eta &= \frac{w_{net}}{w} = 100\%
\end{align*} \]
This ‘perfect’ cycle converts all of the heat input into useful work.

- There may be technical reasons why the cycle would be impractical...
- Yet there is nothing in the first law which makes this cycle impossible.
- This cycle, however, is impossible: it is a perpetual motion machine.

All of these issues will be covered by the second law of thermodynamics.

In general, the second law will:
- Put a direction on time: it will determine the direction of a process.
- Determine the ideal performance of devices (turbines, compressors).
- Establish the maximum thermal efficiency of a heat engine (i.e., a power cycle).
- Quantify the work ‘potential’ of a source of heat.
- And countless other things...

The first law is very simple concept: energy is conserved. It is mathematically formulated as an equality.

The second law will be a more difficult concept: it will be a non-conservation law, mathematically formulated as an inequality.

![Thermodynamic cycle diagram](image)

A heat engine:
- Operates in a cycle.
- During the cycle, it draws an amount of heat $Q_H$ from a high temperature thermal reservoir at temperature $T_H$.
- It converts a fraction of $Q_H$ into useful work, $W_{net}$.
- And it rejects the remaining fraction as heat, $Q_L$, to a low temperature reservoir at $T_L$.

Since the heat engine operates in a cycle,

$$Q_{net} = Q_H - Q_L = W_{net}$$

- $Q_H$, $Q_L$, and $W_{net}$ are all taken to be positive quantities.
- The thermal efficiency $\eta$ is defined as

$$\eta = \frac{W_{net}}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

- A heat engine that did not reject heat ($Q_L = 0$) would have $\eta = 1$: is this possible?
- It turns out that it would violate the basic postulates of the second law.

![Heat engine diagram](image)

A 100% efficient heat engine:
- Work can be transformed into any form of energy – including heat.
- We can then take $W_{net}$ and use it to transfer heat back into the high-temperature reservoir.

- The system (as a whole) is continuously out of equilibrium.
- This would be a perpetual motion machine of the second kind.

If a heat engine cannot have $\eta = 100\%$, what is the best possible $\eta$?

The maximum thermal efficiency would occur for a reversible heat engine.

Reversible:
- A process is reversible if it can be reversed and brought back to the initial state, with no effect on the system or the environment.
- A reversible process, when reversed, would leave no history that it ever occurred.
- Sources of irreversibility:
  - Friction
  - Non-equilibrium processes (i.e., free expansion)
  - Mixing
  - Heat transfer over a finite temperature difference

A reversible heat engine could be reversed, and the magnitudes of $Q_H$, $Q_L$, and $W_{net}$ will remain the same and point in the opposite direction:

![Reversible heat engine diagram](image)

The engine is now a heat pump (as in a refrigerator).

Say we have two heat engines, both operating between the same reservoirs.
- One engine ($A$) is reversible, the other is not.
- Both reject the same amount of heat $Q_L$ to the $T_L$ reservoir.
- Assume that $B$ has a larger thermal efficiency than $A$ – is this possible?

Since $Q_L$ is the same for both and $\eta_B > \eta_A$,

$$1 - \frac{Q_L}{Q_H} > 1 - \frac{Q_L}{Q_H}$$

or

$$Q_{net,B} > Q_{net,A}$$

And

$$W_{net,B} = Q_{net,B} - Q_L > W_{net,A}$$

Engine $A$ is reversible, so now reverse it...
The magnitudes of $Q_{H,A}$ and $W_{ext,A}$ are the same - only in the reverse direction.

The net heat transfer to the $T_L$ reservoir is zero.

A net amount of heat is transferred from the $T_H$ reservoir:

$$Q_{H,net} = Q_{H,B} - Q_{H,A} > 0$$

and a net amount of work is produced by the combined engines:

$$W_{net} = W_{ext,B} - W_{ext,A} > 0$$

The overall effect is a perpetual motion machine.

A reversible heat engine will have the maximum thermal efficiency of all possible heat engines operating between the same two thermal reservoirs.

So

$$\eta_{rev} = 1 - \frac{Q_L}{Q_H} = \frac{Q_H}{Q_H} = f(T_L, T_H)$$

What is this function $f$?

It has to have certain properties: say we ‘stack’ two reversible heat engines on top of each other:

- one engine (A) operates between temperatures $T_H$ and $T_M$,
- the other engine (B) operates between $T_M$ and $T_L$.

And

$$\frac{Q_A}{Q_H} = f(T_M, T_H)$$

and

$$\frac{Q_B}{Q_H} = f(T_L, T_M)$$

Since

$$\frac{Q_L}{Q_H} = \frac{Q_M}{Q_H} - \frac{Q_L}{Q_H}$$

it follows that

$$f(T_L, T_H) = f(T_M, T_H) \cdot f(T_L, T_M)$$

Many functions have this property, i.e.,

$$f(T_L, T_H) = g(T_L) \cdot g(T_H)$$

where $g$ is an arbitrary function.

So what is $g(T)$?

At this point we really need to consider a different question: what makes the absolute temperature scale unique?

A relative temperature scale (i.e., °C) is defined by two arbitrary points and an arbitrary division. Zero Celsius = H₂O freezing point at 1 atm, 100 °C = boiling point at 1 atm.

What defines absolute zero?

The absolute temperature scale is defined so that a reversible heat engine, exchanging heat $Q_H$ and $Q_L$ between thermal reservoirs at absolute temperatures $T_H$ and $T_L$, will have

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H}$$

The theoretical maximum efficiency of a heat engine operating between $T_H$ and $T_L$ is therefore

$$\eta_{rev} = 1 - \frac{T_L}{T_H}$$

Say we construct (somehow) a reversible heat engine that operates between 100°C and 0°C.

We carefully measure the thermal efficiency of the engine, and get

$$\eta = 0.268$$

$$\frac{Q_L}{Q_H} = 1 - \eta = 0.732$$

where $T_L$ and $T_H$ are absolute temperatures.

We can arbitrarily assign 100 points between $T_L$ and $T_H$. So

$$0.732 = \frac{T_L}{T_H} = 100$$

or

$$T_L = 273 \text{ K}$$

There is another ‘definition’ of absolute temperature:

$$Pr = RT$$

Is this consistent with the 2nd law, heat engine definition?

Yes. See Sec. 7.8 of the text.