• 75 minutes, 30 points, 3 questions, weighted as marked.
• Closed book and notes, with the exception of one side of one 8½ ×11 in. page (any font or margins) as a reference. Unlimited blank scratch paper is allowed.
• Turn in only the attached exam papers. Write only on the front side of each page. Page backs will NOT be scored. Page backs may be used for scratch paper.
• Answers must be presented in an organized manner – disorganized answers will be taken to indicate randomness or incompleteness, and scored as such (you are encouraged to use scratch paper and recopy).
1. (10 points) In a recent test of a “turf” style tire on packed dirt, the following data of lateral force v. slip angle are gathered:

<table>
<thead>
<tr>
<th>Slip angle, degrees</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>10</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral force, N</td>
<td>90</td>
<td>140</td>
<td>175</td>
<td>220</td>
<td>245</td>
<td>250</td>
</tr>
</tbody>
</table>

What is the appropriate cornering stiffness for this tire/road combination?

\[
C_\alpha = \left. \frac{\partial F_y}{\partial \alpha} \right|_{\alpha=0} \approx \frac{90 \text{ N} - 0}{2^\circ - 0} = 45 \text{ N/}^\circ
\]
2. (10 points) A go-kart (of the fun kart category) has a solid rear axle and a single mechanically-actuated disk brake on the rear, with no brakes at all on the front. The wheelbase is 56 in., the weight with driver is 380 lb., the weight split is 30% on the front, and the vertical center of gravity is 20 in. above the ground. The rear wheel has a rolling radius of 9 in., the brake disk has a diameter of 8 in., the brake pad has a friction coefficient of 0.38, and the pad is 2 in. wide (radially). The big knobby rear tires, acting on packed dirt, behave as if their peak braking friction coefficient is independent of vertical load, with a value of 0.74. Find the clamping force necessary to lock the rear wheels.

\[ md = F_{\text{braking, max}} = \mu_{\text{peak}} R_r \]

\[ R_r = \frac{a}{l} mg - \frac{h}{l} md \]

\[ F_{\text{braking, max}} = (0.74) \frac{0.7(56 \text{ in.})}{(56 \text{ in.}) + 0.74(20 \text{ in.})} (380 \text{ lb.}) = 155.7 \text{ lb.} \]

\[ \tau_{\text{braking}} = r_w F_{\text{braking, max}} = r_{\text{disk}} F_{\text{friction}} = r_{\text{disk}} (2 \text{ pads}) \mu_{\text{pad}} F_{\text{clamp}} \]

\[ F_{\text{clamp}} = \frac{(9 \text{ in.})(155.7 \text{ lb.})}{(3 \text{ in.})(2 \text{ pads})(0.38)} = 614.6 \text{ lb.} \]
3. (10 points) The front tires of the go-kart (prob.2 above) are capable of a peak lateral force equal to 0.54 of the tire vertical load (on packed dirt). The extreme load case for the front wheelset comes from all of the front load (still at 30% f/r weight split) being transferred to one side (the outside) of the GV (it has no suspension at all, and so it lifts a front wheel rather easily). If the front wheelset configuration allows the front wheel bearings to be 5 in. apart (center to center), where should the bearings be located, relative to the wheel center-plane, to minimize the bearing radial and thrust loads?

\[ R_f = \frac{b}{l} mg = (0.3)(380 \text{ lb}) = 114 \text{ lb.} \]

\[ F_{y,\text{max}} = (0.54)(114 \text{ lb}) = 61.6 \text{ lb.} \]

\( R_f \) and \( F_{y,\text{max}} \) are applied at the center of the tire patch. Locate the outer bearing a distance \( d \) inside the wheelplane. Then the other bearing is located at \((d+5\text{ in.})\) inside the wheelplane. Sum the forces:

\[ R_f = R_{b,o} + R_{b,i} \]

\[ F_{y,\text{max}} = A_{b,o} + A_{b,i} \]

where \( R_b \) are the bearing radial loads (inside and outside) and \( A_b \) are the bearing axial loads. Take a moment about the intersection of the wheel axle centerline with the wheelplane (choosing this location so that \( A_b \) and \( R_f \) do not appear):

\[ F_{y,\text{max}} r_w = R_{b,o} d + R_{b,i} (d + 5 \text{ in.}) \]

\[ R_{b,i} = \frac{1}{5 \text{ in.}} (F_{y,\text{max}} r_w - R_f d) \]

\( A_b \) is distributed between inside and outside bearings only by the relative axial stiffness of the bearing supports, and is only indirectly related to bearing separation. So axial loading drops out of the problem. \( R_b \) can be minimized, between inside and outside, on a variety of plans. Here, it is chosen to make the \( R_b \) equal for inside and outside:

\[ R_{b,i} = \frac{R_f}{2} \]

\[ d = r_w \frac{F_{y,\text{max}}}{R_f} - 5 \text{ in.} = (5 \text{ in.}) \frac{61.6 \text{ lb}}{114 \text{ lb.}} - 2.5 \text{ in.} = 0.2 \text{ in.} \]

Or, the outer bearing center is placed 0.2 in. inside the wheelplane, and the inner bearing is centered 5.2 in. inside the wheelplane.

Other minimization schemes can be derived, although the \( R_b \) should both have the same direction – otherwise, magnitude is wasted opposing each other.