• 50 minutes, 2 questions, 20 points, weighted as marked.
• Open book. Open notes. No cell phones, computers, or other communication.
• Unlimited blank scratch paper is allowed.
• Turn in only the attached exam papers.
• Do not detach any of the exam papers.
• Do not write on the backs of the exam papers (backs WILL NOT be scored).
• OK to use backs as scratch paper.
• Disorganized answers will be regarded as disorganized solutions and scored as such. Better to work it out on scratch and then copy onto the exam paper.
1. (10 points) Statics. A 600 cc motorcycle engine on a test stand (essentially a table) runs at full power (57 N·m of torque at 11,000 rpm). Engine torque is transduced to a hydraulic dynamometer through a chain and sprocket set. Dynamometer torque pressurizes water flowing through the dynamometer, which exhausts to atmosphere. Ground reaction loads at the feet of the stand must be known. Write an explicit formula expressing the left and right reaction loads. Make a list of the physical properties which must be measured so that the reaction loads may be calculated (all of the sufficient properties; only the necessary properties).

Assuming that the dynamometer flows are ducted out of the plane of the page, static equilibrium equations show:

\[
\sum F_{\text{vertical}} = 0 = R_L + R_R - g\left(m_{\text{stand}} + m_{\text{engine}} + m_{\text{dyno}}\right)
\]

\[
\sum M_{\text{left foot}} = 0 = R_R l_{\text{stand}} - g\left(m_{\text{stand}} d_{\text{stand}} + m_{\text{engine}} d_{\text{engine}} + m_{\text{dyno}} d_{\text{dyno}}\right) - \tau_{\text{engine}} r_{\text{drive}}
\]

Which is 2 equations for 2 unknowns. The following parameters are required for solution:

- \(d_{\text{stand}}\) horizontal distance from left foot to stand CG
- \(d_{\text{engine}}\) horizontal distance from left foot to engine CG
- \(d_{\text{dyno}}\) horizontal distance from left foot to dynamometer CG
- \(l_{\text{stand}}\) distance from left foot to right foot
- \(m_{\text{stand}}\) mass of stand
- \(m_{\text{engine}}\) mass of engine
- \(m_{\text{dyno}}\) mass of dynamometer
- \(r_{\text{drive}}\) drive ratio of the chain and sprocket set (shown >1 in figure)
2. (10 points) Dynamics. The owner of a relatively athletic Labrador Retriever gives in to her demands and hurls a tennis ball for her. Write an implicit equation (i.e. you don’t have to solve it, but it must be solvable with the information you provide) for the distance from the release of the tennis ball to its first bounce on the ground in terms of the release velocity, direction, and position provided to the ball by the owner. [Hint: the only forces acting on the tennis ball post-release are gravity and aerodynamic drag; the latter is expressed $\frac{1}{2} \rho_{air} V^2 C_D A$, where $A$ is projected area and $V$ is the ball speed; drag coefficient $C_D$ may be taken as 0.7; air density $\rho_{air}$ may be taken as 1.2 kg/m$^3$; and just so you know, ball mass is 66 g, and diameter is 68 mm].

Nomenclature:

- $A$: projected area of the ball
- $C_D$: drag coefficient of the ball
- $g$: acceleration of gravity
- $m$: mass of the ball
- $t$: time
- $u$: $x$ component of velocity
- $u_0$: initial $x$ velocity at thrower release
- $w$: $z$ component of velocity
- $w_0$: initial $z$ velocity at thrower release
- $x$: displacement of ball parallel to ground in thrown direction, positive out, measured from thrower
- $z$: displacement of ball in vertical direction, positive up, measured from ground
- $z_o$: height above ground at thrower release
- $\rho$: air density

Velocity:

$$m \frac{du}{dt} = -\frac{1}{2} \rho C_D A \left( u^2 + w^2 \right) \frac{u}{\sqrt{u^2 + w^2}} = -\frac{1}{2} \rho C_D A u \sqrt{u^2 + w^2}$$

$$m \frac{dw}{dt} = -\frac{1}{2} \rho C_D A \left( u^2 + w^2 \right) \frac{w}{\sqrt{u^2 + w^2}} - mg = -\frac{1}{2} \rho C_D A w \sqrt{u^2 + w^2} - mg$$

Note that the first parenthesis represents $V^2$, and the following fractions represent the sine and cosine of the velocity orientation in the $x$-$z$ plane. These are first order differential equations, and each needs one boundary condition (an initial condition) for solution. These are $u_0$ and $w_0$ (which define $V_o$ and $\theta_o$). These equations can be solved for $u$ and $w$ as functions of $t$ (see Matlab).

Drop:

$$z = \int_{0}^{t_{bounce}} w(t) \, dt ; \quad z(0) = z_o$$

Find $t_{bounce}$ where $z=0$.

Travel:

$$x_{bounce} = \int_{0}^{t_{bounce}} u(t) \, dt ; \quad x(0) = 0$$