• 50 minutes, 2 questions, 20 points, weighted as marked.
• Open book. Open notes. No cell phones, computers, or other communication.
• Unlimited blank scratch paper is allowed.
• Turn in only the attached exam papers.
• Do not detach any of the exam papers.
• Do not write on the backs of the exam papers (backs WILL NOT be scored).
• OK to use backs as scratch paper.
• Disorganized answers will be regarded as disorganized solutions and scored as such. Better to work it out on scratch and then copy onto the exam paper.
1. (10 points) Statics. Strings of Christmas lights are to be wrapped in spirals around a tall pine tree. Each string is wrapped around the tree in a spiral pattern from the base of the tree to as high as the string will reach, carefully positioned, and stapled to the tree in several places along the string by a human installer. A ladder is used to reach each point on the tree at which the light string is placed and/or stapled. The closer to the ladder is to vertical, the easier it is for a person on the ladder to reach the side of the tree and manipulate the string (at points below the top of the ladder - and hence, the whole tree decoration job requires fewer ladder repositioning steps and less overall labor time). But if the ladder is too close to vertical, then it may fall over backwards as a person climbs it. Find the minimum angle from the vertical at which the ladder may be placed, so that it will not fall over backwards when a person starts to climb. For design evaluation purposes, consider the ladder to be 20 ft. long, weigh 30 lb., and the ladder climber to be a 95th percentile male (weight 190 lb., center of gravity 3.17 ft. above ground and 1.16 ft. out of the plane of the ladder). In use, the ladder angle will be checked with an inclinometer whose precision is 0.5°.

At tipover, tree reaction is vanishing. Sum moments about the foot of the ladder:

\[ \sum M = 0 \mid \text{tipover} = W_l \left( \frac{l}{2} \sin \theta \right) + W_p \left( h \tan \theta - d \cos \theta \right) \]

Difficult to solve explicitly. Try small angle approximation:

\[ 0 = W_l \left( \frac{l}{2} \theta \right) + W_p \left( h \theta - d \right) \]

\[ \theta = \frac{W_p d}{W_l \frac{l}{2} + W_p h} = \frac{(190 \text{ lb})(1.16 \text{ ft})}{(30 \text{ lb}) \frac{1}{2}(20 \text{ ft}) + (190 \text{ lb})(3.17 \text{ ft})} = 0.2443 \text{ rad} = 14^\circ \]
2. (10 points) Dynamics. A small offroad racecar competes in a Hill Climb Event. The Event course is a steady 50% grade (22.5° angle of inclination), 100 ft. long, from a standing start on the incline. The car stays in its lowest gear throughout the course, and use of this gear provides 378 lb of total thrust to the tire patches of the driven wheels. The weight of the car plus driver (600 lb) acting through the course angle of inclination, plus the rolling resistance of the offroad tires on the deformable surface of the course, adds up to a total drag of 349 lb resisting the thrust. How long will it take the car to complete the course?

\[ \sum F = ma = \text{Thrust} - \text{Drag} \]
\[ a = \frac{\text{Thrust} - \text{Drag}}{m} = \frac{378 \text{ lb} - 349 \text{ lb}}{(600 \text{ lbm}) \left( \frac{\text{slug}}{32.2 \text{ lbm}} \right)} = 1.556 \text{ ft/s}^2 \]

\[ a = \frac{d^2 x}{dt^2} = \text{constant} \]

\[ \int_0^x d \left( \int_0^x dx \right) = a \int_0^t \left( \int_0^t dt \right) dt ; \quad x(0) = 0 \]

\[ x = \frac{1}{2} at^2 ; \quad t = \sqrt{\frac{2x}{a}} \]

\[ t = \sqrt{\frac{2(100 \text{ ft})}{1.556 \text{ ft/s}^2}} = 11.3 \text{ s} \]