OVERBURDEN CORRECTION FACTORS FOR PREDICTING LIQUEFACTION RESISTANCE UNDER EMBANKMENT DAMS

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ABSTRACT

Evaluating liquefaction of cohesionless soils at large depths, such as under embankment dams, requires an understanding of how both the penetration resistance and cyclic resistance ratios are affected by high overburden stresses. SPT-based and CPT-based liquefaction triggering correlations are derived primarily from analyses of liquefaction case histories at relatively shallow depths and therefore must be extrapolated to larger overburden stresses. The overburden correction factor ($K_\sigma$) is used to adjust the cyclic resistance ratio for the effects of overburden stress. Different curves for $K_\sigma$ have been proposed by various researchers and the values associated with these curves can have significant influence on the prediction of liquefaction at large depths. A database of laboratory test results was compiled and evaluated in light of the current understanding of factors which can affect cyclic strength, including overconsolidation ratio, relative density and fines content. Identification of these factors has allowed for classification of data points which were not necessarily representative of liquefiable materials (clayey and/or heavily compacted sands) and may have biased previous relationships. This paper will present the updated database of $K_\sigma$ laboratory test results, discuss important factors which can influence the interpretation of $K_\sigma$ and present a comparison between current design relationships and the updated database. The implications of these findings for evaluating liquefaction triggering at large depths will be discussed.

INTRODUCTION

The evaluation of liquefaction triggering for large dams requires special consideration for the effects of large overburden stresses and initial static shear stresses. Liquefaction triggering evaluations for dams often utilize liquefaction triggering correlations (e.g., Figure 1) which relate the liquefaction resistance of the soil to its Standard Penetration Test (SPT) or Cone Penetration Test (CPT) penetration resistance. These correlations are derived from analyses of liquefaction case histories involving largely level ground conditions and effective overburden stresses ($\sigma'_v$) less than about 150 kPa. Extrapolation of these correlations to the larger overburden stresses and sloping ground conditions encountered within the shells or foundations of large dams is best guided by fundamental studies of how such conditions affect both the cyclic loading response of cohesionless soils and the penetration resistance in cohesionless soils.

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The cyclic strength of sands and other cohesionless soils (i.e., gravels to nonplastic silts) depends on, among other factors, both the soil density [as may be represented by its void ratio or relative density (D_R)] and the imposed effective stress (σ'), which together determine the state of the soil. Seed and Idriss (1971) introduced use of a liquefaction triggering curve to relate the cyclic strength of a soil to the normalized SPT blow count, which was used as a proxy for the D_R and other factors affecting the cyclic strength of the soil. Seed (1983) later introduced the overburden correction factor (K_σ) to account for the variation of a soil's cyclic resistance ratio (CRR) as a function of effective consolidation stress. This factor was defined as:

\[
K_\sigma = \frac{\text{CRR}_{\sigma'_{vc}, \alpha=0}}{\text{CRR}_{\sigma'_{vc}=1, \alpha=0}}
\]  

where CRR_{\sigma'_{vc}, \alpha=0} and CRR_{\sigma'_{vc}=1, \alpha=0} are the cyclic resistance ratios for a static shear stress ratio (\(\alpha\)) of zero and \(\sigma'_{vc}\) equal to the stress level of interest and to 1.0 atm (1 atm = 101.3 kPa), respectively. Both CRR values are assumed to be for soil that is identical in all respects other than consolidation stress (i.e., the same D_R, same fabric, same age, same cementation, and same loading history). The factor K_σ describes the curvature of the cyclic strength envelope with increasing consolidation stress, as illustrated in Figure 2. More recent studies have found the magnitude of this curvature is dependent on D_R and soil type (Vaid and Sivathayalan 1996, Hynes and Olsen 1999, Boulanger 2003).

The most common approach used for developing and applying liquefaction triggering correlations for sandy soils is to normalize the SPT or CPT penetration resistances (i.e., q_c or N_{60}) to a reference \(\sigma'_{vc}\) of 1 atm (assuming the soil's D_R and all other attributes are unchanged) and to correlate this normalized penetration resistance [i.e., q_{c1} or (N_{1})_{60}] to a normalized cyclic strength for this same reference stress (i.e., CRR_{\sigma'_{vc}=1}). The SPT-based liquefaction triggering correlations recommended by Youd et al. (2001) and Idriss and Boulanger (2008), as shown in Figure 1 for a Magnitude 7.5 earthquake, follow this
overburden correction factor $C_N$ as,

$$\left( N_1 \right)_{60} = C_N \left( N \right)_{60}$$  \hspace{1cm} (2)

where $N_{60}$ = SPT blowcount for an energy ratio of 60%, and $(N_1)_{60}$ = the $N_{60}$ value for an equivalent $\sigma'_{vc}$ of 1 atm. The dependence of the liquefaction triggering correlation on the fines content (FC) is accommodated by adding an increment, $\Delta(N_1)_{60}$, to the $(N_1)_{60}$ value to obtain an equivalent clean-sand $(N_1)_{60cs}$ for the same CRR as,

$$\left( N_1 \right)_{60cs} = \left( N_1 \right)_{60} + \Delta \left( N_1 \right)_{60}$$  \hspace{1cm} (3)

The relationship for $\Delta(N_1)_{60}$ depends on the liquefaction triggering correlation, with the following expression (with FC expressed as a percent) being the one recommended by Idriss and Boulanger (2008) for use with their triggering correlation,

$$\Delta \left( N_1 \right)_{60} = \exp\left( 1.63 + \frac{9.7}{FC + 0.01} - \left( \frac{15.7}{FC + 0.01} \right)^2 \right)$$  \hspace{1cm} (4)

The effect on the CRR of variations in $\sigma'_{vc}$, sloping ground conditions, and earthquake magnitude (M) are accounted for through $K_\sigma$, static shear stress correction ($K_\alpha$), and magnitude scaling (MSF) factors as,

$$CRR_{M,\sigma',\alpha} = CRR_{M=7.5,\sigma'=1,\alpha=0} \cdot MSF \cdot K_\sigma \cdot K_\alpha$$  \hspace{1cm} (5)

where $CRR_{M,\sigma',\alpha}$ is the CRR for given values of M, $\sigma'_{vc}$, and $\alpha$, and $CRR_{M=7.5,\sigma'=1,\alpha=0}$ is the value of CRR for $M = 7.5$, $\sigma'_{vc} = 1$ atm, and $\alpha = 0$, as obtained from case history-based correlations.
The roles of the $C_N$ and $K_\sigma$ factors are schematically illustrated in Figure 3 for a hypothetical deep deposit of saturated clean sand at a uniform $D_R = 60\%$. Penetration resistances (SPT $N_{60}$ values in this example) increase with depth (Figure 3b) because the sand's shear strength and stiffness increase with increasing $\sigma'_{vc}$. The $(N_1)_{60}$ values, however, are constant with depth (Figure 3d) because the deposit is uniform in all attributes other than $\sigma'_{vc}$. The $C_N$ factor, which accounts for the effect of $\sigma'_{vc}$ on the measured $N_{60}$ values, is simply the ratio $(N_1)_{60}/N_{60}$ and thus it decreases with increasing $\sigma'_{vc}$ (Figure 3c). The more strongly dependent $C_N$ is on the $\sigma'_{vc}$ the lower the $C_N$ value becomes for $\sigma'_{vc}$ greater than 1 atm. The $\text{CRR}_{M=7.5, \sigma'_{vc}=1, \alpha=0}$ values obtained from a liquefaction triggering correlation using the $(N_1)_{60}$ values are constant with depth (Figure 3e) because the $(N_1)_{60}$ values are constant with depth. However, the $\text{CRR}_{M=7.5, \sigma'_{vc}=0}$ values are shown to decrease with increasing depth and $\sigma'_{vc}$ (Figure 3g) because $K_\sigma$ decreases with increasing $\sigma'_{vc}$ (Figure 3f). The more strongly dependent $K_\sigma$ is on the $\sigma'_{vc}$ the lower the $K_\sigma$ values become for $\sigma'_{vc}$ greater than 1 atm.

The $K_\sigma$ and $C_N$ factors provide the basis for extrapolating the case history data on liquefaction triggering from depths less than about 15 m and $\sigma'_{vc}$ less than about 1.5 atm to the much greater depths and stresses encountered under large embankment dams. The $K_\sigma$ factor can be evaluated through laboratory element tests, whereas the $C_N$ factor can be evaluated using calibration chamber tests, numerical analyses, and in-situ test data. The $K_\sigma$ and $C_N$ factors are not independent, however, because they are both influenced by the same soil properties (often in opposing ways). The interdependency of the $K_\sigma$ and $C_N$ factors are examined in Boulanger (2003) and Boulanger and Idriss (2004).

The purpose of this paper is to examine current $K_\sigma$ relationships against an updated database of laboratory test results defining $K_\sigma$ effects, as described in in Montgomery et al. (2012). Original laboratory test results included in previous databases (e.g., Seed and Harder 1990) are reexamined in light of the current understanding of factors which can affect laboratory measurements of cyclic strength. This paper will describe an updated database of laboratory test results defining $K_\sigma$ effects, discuss important factors which can influence the interpretation of $K_\sigma$, present a comparison between current relationships and the updated database, and discuss implications for practice.

**DATABASE ON $K_\sigma$ RELATIONSHIP**

A number of investigators have researched and recommended $K_\sigma$ relationships (e.g. Seed and Harder 1990, Pillai and Byrne 1994, Vaid and Sivathayalan 1996, Harder and Boulanger 1997, Hynes and Olsen 1999, Youd et al. 2001, Boulanger 2003, Cetin et al. 2004, Bray and Sancio 2006, Idriss and Boulanger 2008, Stamatapolous 2010). Many of these investigations have utilized, or been compared to, a database of cyclic triaxial test results (Figure 4) compiled by Harder (1988) and presented by Seed and Harder (1990).

The Seed and Harder (1990) database had 45 data points from 18 sources and included results for reconstituted soils and undisturbed field samples published in the open literature or contained in engineering project reports. The majority of the data from engineering projects pertained to embankment dams. There is a significant amount of
scatter within these data for a given consolidation stress and much of the scatter is likely due to the variety of soil types and densities represented. Without a clear understanding of the composition of the original samples, it was previously not possible to discern how these effects may have influenced the data and potentially biased previous relationships.

**Current Database for $K_\sigma$**

The original sources for the data presented by Seed and Harder (1990) were reexamined to evaluate the specimen composition, test procedures and strength results for each individual data set. This evaluation was conducted by reviewing the original sources for each of the data sets, which included various dam project files located in the archives of the California Division of Safety of Dams and various publications in the literature. From this information, it was possible to categorize the specimens in terms of composition and to examine patterns in the data. The re-interpreted values of $K_\sigma$ from this study were within a few percent of the values plotted in Seed and Harder (1990), with few exceptions. In addition, data from the more recent laboratory studies on reconstituted sands by Vaid and Thomas (1995), Vaid and Sivalythalyn (1996), Koseki et al. (2005) and Manmatharajan and Sivathayalan (2011) and on frozen sand samples from Duncan Dam by Pillai and Byrne (1994) were included in the current database.

The laboratory CRR for each data set was converted to an equivalent field CRR for comparison with liquefaction triggering relationships. This conversion was performed...
using the methods summarized in Idriss and Boulanger (2008). The CRR from triaxial and torsional shear tests was first corrected for the effects of the coefficient of lateral earth pressure at rest ($K_0$). Selecting an appropriate $K_0$ value for the field is always uncertain, and values of 0.5 and 1.0 were considered to bracket the effects. For the simple shear test results, no correction for $K_0$ was made. The effects of two-dimensional shaking in the field were accounted for by multiplying the laboratory CRR by a correction factor of 0.9. The laboratory CRR were either selected at 15 uniform loading cycles or converted to 15 uniform cycles. Details are discussed in Montgomery et al. (2012).

The $K_\sigma$ results were separated into the four soil type categories plotted separately in Figures 5a through 5d: clean sands (sands with 5% fines or less, 52 data points), sands with varying levels of silty fines (14 data points), clayey sands (2 data points), and well-compacted specimens (12 data points). Properties for each soil are given in Montgomery et al. (2012). Each of the categories is shown with the curve recommended by Seed and Harder (1990) for comparison purposes.

The results for clayey sands (fines contents of 23 to 28%, plasticity indices of 20 to 25) in Figure 5c were considered to be of limited value for examining the liquefaction potential of cohesionless soils. This is because plastic fines influence the response of a soil to cyclic loading (e.g. Boulanger and Idriss 2006, Bray and Sancio 2006). The effects of the fines' plasticity on $K_\sigma$ are not clear, but this data was not considered useful for evaluating the behavior of clean sands or sands with primarily nonplastic silty fines which are traditionally evaluated using liquefaction triggering curves. For these reasons, the data for clayey sands (two data points) were eliminated from further consideration.

The data from well-compacted soils (fines content of 7 to 35%), as plotted in Figure 5d, were also considered to be of limited value for determining the response of liquefiable materials. These soils all had either: (a) been compacted to a relative compaction of 95%
or larger [per Department of Water Resources (20,000 ft-lb/cu. ft.), L.A. Water System (33,750 ft-lb/cu. ft.) or Mod. AASHO standards (56,000 ft-lb/cu ft.)]; (b) been compacted to a D_R of 100%; or (c) had in-situ blow counts of 45. In addition, these soils all exhibited high cyclic triaxial strengths (i.e., without the K_o and 2-D shaking corrections, the CRR_{\sigma_{ec}=1} in 15 uniform loading cycles at an effective consolidation stress of 1 atm was greater than 0.51 for all specimens and greater than 0.77 for about half of the specimens). In addition to the high cyclic strength, these specimens were likely affected by variations in the overconsolidation ratio (OCR) under different levels of confining stress. These specimens likely became highly overconsolidated at low confining stresses due to the large preconsolidation stresses applied to the specimen during compaction (e.g. Frost and Park 2003). As the consolidation stress was increased the OCR would decrease, leading to results which mix the effects of OCR and consolidation stress. These effects may also be present in the undisturbed or moist tamped specimens, as discussed later. Due to the high strengths and possible high OCR effects, these specimens were not considered to be representative of typical liquefiable soils and were eliminated from further evaluation.

Overburden Correction Factors
The remaining data for clean sands (Figure 5a) and sands with varying levels of silty fines (Figure 5b) are considered most applicable for developing design relationships for liquefiable soils. The clean sand data are freshly reconstituted specimens prepared to a range of $D_R$ using wet and dry pluviation, moist tamping and vibration. The data for sands with silty fines are from field samples that were either reconstituted (four sets of specimens) or "undisturbed" (ten sets of specimens). There is still significant scatter in each category, but the range of densities likely contributes to the spread in $K_\sigma$ values.

Changes in soil properties (e.g., $D_R$, OCR, cementation) with consolidation stress can have significant effects on cyclic strengths, but these effects should not be confused with, or mixed with, the intended purpose of the $K_\sigma$ relationship. Changes in properties such as $D_R$, OCR, or cementation can be expected to affect both the CRR and $N_{60}$ values for a soil. The magnitudes of these changes are not well defined, but the underlying presumption of penetration test-based liquefaction procedures is that their effects on CRR are reasonably accounted for by: (1) the simultaneous changes in measured $N_{60}$ values, (2) the applicability of the $C_N$ relationship for obtaining $(N_1)_{60}$ values, (3) the presumed relative uniqueness of the $CRR_{M=7.5,\sigma'_v=1,\alpha=0}$ versus $(N_1)_{60cs}$ correlation, and (4) the applicability of the $K_\sigma$ relationship for obtaining $CRR_{M=7.5,\sigma'_v,\alpha=0}$ values. Thus, the $K_\sigma$ relationship is only intended to account for how confining stress affects CRR values, whereas the effect on CRR from changes in other soil properties are currently accounted for through their effects on $N_{60}$ values. Examples of the potential errors in $K_\sigma$ due to neglecting these other factors are provided in Montgomery et al. (2012).

An effort has been made to remove some of the above-described adverse influences from the updated $K_\sigma$ database, as discussed in Montgomery et al. (2012), but it is not possible to eliminate all of their effects. The moist tamped and possibly the vibrated specimens in the database likely have been influenced by the effects of OCR at low stress levels which would contribute to produce $K_\sigma$ curves with an artificially strong dependence on $\sigma'_v$. The results for undisturbed samples in the database are likely affected by the simultaneous influences of varying cementation, OCR, and $D_R$ as samples were consolidated to different stresses than existed in situ.

**COMPARISON OF DATABASE AND CURRENT RELATIONSHIPS**

One of the benefits of the updated database is the ability to compare current relationships to a large body of laboratory tests performed on a variety of soils at different densities. The relationships by Idriss and Boulanger (2008) and Hynes and Olsen (1999), with the parameters recommended in Youd et al. (2001), were chosen for this comparison because they both account for the dependence of $K_\sigma$ on $D_R$.

To compare the database to the design relationships it was necessary to find a common measure of denseness for each of the specimens. No one measure of denseness (e.g., $D_R$ or $(N_1)_{60}$) was common to all data sets. One approximate measure of denseness common to all the soils is the $CRR_{M=7.5,\sigma'_v=1,\alpha=0}$. If the liquefaction triggering curve is assumed to be unique for all soils in the current database, this reference CRR can be related to an equivalent $(N_1)_{60cs}$ and then to a $K_\sigma$ value for a given consolidation stress.
To perform this comparison, the equivalent field CRRM=7.5,σ′,α=0 for each of the soils was calculated, as previously discussed, and mapped to an equivalent (N1)60cs through the corresponding liquefaction triggering curve from each of the methods (Youd et al. 2001 and Idriss and Boulanger 2008). The equivalent (N1)60cs value was then used to determine Kσ values (through an equivalent D_R) using the equations for each of the two methods.

The data for clean sands and sands with silt were first sorted into bins of different CRRM=7.5,σ′,α=0 ranges (Figures 6-8) by Montgomery et al. (2012). The data indicates a general trend of Kσ becoming more strongly dependent on σ′,vc as CRRM=7.5,σ′,α=0 increases, as expected. This is shown by the data for both clean sands tested in triaxial or torsional shear devices (Figure 6a-c), clean sands tested in simple shear devices (Figure 7a-b) and sands with silt tested in triaxial devices (Figure 8a-c). This trend agrees with the expected pattern of Kσ curves which become more strongly dependent on σ′,vc as the density of the soil increases, where the CRRM=7.5,σ′,α=0 values are used here as a proxy for the soil density. These results are shown for an assumed K0 of 0.5, whereas the use of K0 = 1.0 has the effect of shifting the Kσ curves lower with respect to the data points.

The Kσ data for clean sands tested in triaxial and torsional shear were separated into three bins with CRRM=7.5,σ′,α=0 values of 0.07 to 0.11 (Figure 6a), 0.14 to 0.17 (Figure 6b), and 0.22 to 0.25 (Figure 6c) adjusted to a field K0 of 0.5. The design relationships from

Figure 6. Kσ database for clean sands tested in triaxial or torsional shear devices, binned according to CRR at 1 atm, compared to a curve from each relationship for a CRR at the center of each bin.

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Youd et al. (2001) and Idriss and Boulanger (2008) are also shown in these figures for a DR that corresponds to a CRR$_{M=7.5,\sigma'_{vc}=1,\alpha=0}$ value at the middle of each bin's range (i.e., CRR$_{M=7.5,\sigma'_{vc}=1,\alpha=0}$ values of 0.09, 0.155, and 0.235 for K$_0$ of 0.5). The Idriss and Boulanger (2008) relationship is in reasonable overall agreement with the data, whereas the Youd et al. (2001) relationship is generally conservative relative to the data.

![Figure 7](image1.png)

**Figure 7.** K$_\sigma$ database for clean sands tested in simple shear device, binned by CRR at 1 atm, compared to a curve from each relationship for a CRR at the center of each bin.

![Figure 8](image2.png)

**Figure 8.** K$_\sigma$ database for sand with varying levels of silt, binned by CRR at 1 atm, compared to a curve from each relationship for a CRR at the center of each bin.
The test data from clean sands tested in simple shear was separated into bins of approximately equal size and similarly compared to the design relationships (Figure 7). The $K_\sigma$ values for these data points fall well above the values predicted by the relationships suggested by Youd et al. (2001) and Idriss and Boulanger (2008), although the values are much closer to the latter. These tests were all from water pluviated specimens of Fraser River sand and do not exhibit the same scatter for a similar range of cyclic strength seen in the triaxial and torsional shear bins.

The test data for sands with varying levels of silty fines, all from cyclic triaxial tests, were similarly compared to the design relationships, with the results shown in Figure 8. Much of the data falls in a narrower range of $\text{CRR}_M=7.5, \sigma'_v=1, \alpha=0$ values with a large number of the specimens having a $\text{CRR}_M=7.5, \sigma'_v=1, \alpha=0$ close to 0.14. Two specimens had a significantly higher $\text{CRR}_M=7.5, \sigma'_v=1, \alpha=0$ than the rest of the points and were placed in a separate bin (Figure 8c). A similar trend of lower $K_\sigma$ values with increasing $\text{CRR}_M=7.5, \sigma'_v=1, \alpha=0$ can be seen between the three bins. The data for the looser specimens, (Figure 8a-b) generally fall between the two relationships, while the two points in the higher bin (Figure 8c) agree better with Youd et al. (2001).

For sands or silty sands tested at less than 1 atm, the database indicates that the cap on $K_\sigma$ of 1.1 recommended by Idriss and Boulanger (2008) fits well with the clean sand data. A single data point within the silty sand category would suggest that a higher cap on $K_\sigma$ may be warranted, but this specimen was prepared through moist tamping which can apply a significant preconsolidation stresses and lead to an over-estimation of $K_\sigma$ at low stresses (Montgomery et al. 2012).

The more conservative nature of the $K_\sigma$ relationship recommended by Youd et al. (2001), at least for clean sands, may be attributed to the fact it appears to have been guided, at least in part, by its comparison to the original Seed and Harder (1990) database. The original Seed and Harder (1990) database included the data from clayey sands (Figure 5c) and well-compacted sands with varying levels of silt and clay (Figure 5d), which together comprised the majority of their lowest data points (relative to their recommended design curve). The parameters recommended by Youd et al. (2001) produced curves that reasonably fit the Seed and Harder (1990) database.

The current database suggests that sands with nonplastic fines may have $K_\sigma$ curves with a stronger dependence on $\sigma'_v$ (i.e. lower $K_\sigma$ values) than clean sands, but this result is complicated by the fact that the majority of the silty sand studies involved testing of undisturbed samples. Test programs using undisturbed samples are, as discussed previously, most susceptible to the effects of varying $D_R$, OCR, and cementation across the range of imposed consolidation stresses. The net effect of these factors, along with the influence of sample disturbance, is difficult to quantify, and thus it is suggested that laboratory testing of silty sands under more controlled conditions may be required to determine the effect that fines content may have on $K_\sigma$ relationships.

The fines content in sand can also be expected to affect the overburden correction factor for penetration resistances ($C_N$), which is equally important for the assessment of cyclic strengths at high overburden stresses. Boulanger and Idriss (2004) showed that increasing

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**Overburden Correction Factors**

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the compressibility of sand can be expected produce $K_\sigma$ curves with a stronger dependence on $\sigma'_{ve}$ (i.e. lower $K_\sigma$ values at $\sigma'_{ve} > 1$ atm) and $C_N$ curves with a weaker dependence on $\sigma'_{ve}$ (i.e. higher $C_N$ values at $\sigma'_{ve} > 1$ atm), which together have compensating effects on the final estimate of cyclic strength. The field-based $C_N$ relationship derived for the silty and clayey sands in the foundation of Perris Dam showed $C_N$ curves with a lesser dependence on $\sigma'_{ve}$ (i.e. higher $C_N$ values at $\sigma'_{ve} > 1$ atm) than is commonly assumed for clean sands (Wehling and Rennie 2008, Idriss and Boulanger 2010). The combined roles and the potentially compensating nature of the $K_\sigma$ and $C_N$ relationships were recently illustrated by application of different design relationships to the data from Duncan Dam (Boulanger and Idriss 2012).

In view of the above observations and the uncertainties in the $K_\sigma$ database, it is suggested that the relationships by Idriss and Boulanger (2008) continue to provide a reasonable basis for evaluating liquefaction effects in clean sands. For silty sands, the laboratory test results currently available indicate that intermediate values between the Youd et al. (2001) and the Idriss and Boulanger (2008) relationships may give the best estimates for $K_\sigma$. However, the existing $C_N$ relationships were derived primarily for clean sands and are likely conservative for silty sands, based on recent work at Perris Dam and other projects. Thus, it may be that a $K_\sigma$ relationship that is slightly unconservative for silty sands may be compensated for by using a $C_N$ relationship that is conservative for silty sands.

Therefore, the use of the current Idriss and Boulanger (2008) $K_\sigma$ and $C_N$ relationships may be adequate for both clean and silty sands due to these compensating effects. It is recommended that if $K_\sigma$ values for silty sands are selected on the basis of intermediate values from the two relationships, as is indicated by currently available data, then extra attention should be given to evaluating whether the selected $C_N$ curve is appropriate for the soil of interest with the idea of avoiding excessive conservatism.

**EXAMPLE FOR CLEAN SAND**

The effect of different liquefaction procedures on the estimation of CRR at large depths is illustrated by the following example for clean sand as summarized in Table 1. In this example, it is assumed that SPT $N_{60}$ values of 30 and 60 are measured in clean sand at a depth where the vertical effective stress is 800 kPa. Values for $C_N$, $(N_{1})_{60}$, $\text{CRR}_{M=7.5, \sigma'_{ve}=1, \alpha=0}$, $K_\sigma$, and $\text{CRR}_{M=7.5, \sigma'_{ve}, \alpha=0}$ are then computed using both the Youd et al. (2001) and Idriss and Boulanger (2008) procedures, as listed in Table 1.

**Table 1. Comparison of liquefaction triggering procedures for clean sand at high $\sigma'_{ve}$.**

<table>
<thead>
<tr>
<th>$\sigma'_{ve}$ (kPa)</th>
<th>$N_{60}$</th>
<th>$C_N$</th>
<th>$(N_{1})_{60}$</th>
<th>$\text{CRR}<em>{M=7.5, \sigma'</em>{ve}=1, \alpha=0}$</th>
<th>$K_\sigma$</th>
<th>$\text{CRR}<em>{M=7.5, \sigma'</em>{ve}, \alpha=0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Youd et al. (2001)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>30</td>
<td>0.36</td>
<td>10.7</td>
<td>0.119</td>
<td>0.61</td>
<td>0.072</td>
</tr>
<tr>
<td>800</td>
<td>60</td>
<td>0.36</td>
<td>21.4</td>
<td>0.233</td>
<td>0.49</td>
<td>0.115</td>
</tr>
<tr>
<td>Idriss and Boulanger (2008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>30</td>
<td>0.32</td>
<td>9.7</td>
<td>0.116</td>
<td>0.81</td>
<td>0.094</td>
</tr>
<tr>
<td>800</td>
<td>60</td>
<td>0.45</td>
<td>27.1</td>
<td>0.351</td>
<td>0.63</td>
<td>0.222</td>
</tr>
</tbody>
</table>
For an N$_{60}$ value of 30, the resulting CRR$_{M=7.5,\sigma_v'=\sigma=0}$ values were 0.072 and 0.094 by the Youd et al. (2001) and Idriss and Boulanger (2008) procedures, respectively. The 30% greater CRR obtained from the Idriss and Boulanger (2008) procedure is attributed to the combined effects of: (1) its C$_N$ relationship producing a 9% smaller (N$_1$)$_{60}$, and (2) its K$_\sigma$ relationship producing a 33% greater K$_\sigma$ value. The liquefaction triggering correlation by Idriss and Boulanger (2008) is only about 4% higher for this range of (N$_1$)$_{60}$ values (Figure 1) and thus the primary difference in CRR is due to the K$_\sigma$ relationships.

For an N$_{60}$ value of 60, the resulting CRR$_{M=7.5,\sigma_v'=\sigma=0}$ values were 0.115 and 0.222 by the Youd et al. (2001) and Idriss and Boulanger (2008) procedures, respectively. The 93% greater CRR obtained from the Idriss and Boulanger (2008) procedure is attributed to the combined effects of: (1) its C$_N$ relationship producing a 26% greater (N$_1$)$_{60}$, and (2) its K$_\sigma$ relationship producing a 29% greater K$_\sigma$ value. The liquefaction triggering correlations for these two procedures are again almost the same for this range of (N$_1$)$_{60}$ values, (Figure 1) and thus the differences in the C$_N$ and K$_\sigma$ relationships are the major contributors to the difference in cyclic strengths.

There are cases in practice where the cyclic strength of cohesionless soils beneath large embankment dams (e.g., point A in the alluvial foundation stratum shown in Figure 9) needs to be estimated based solely on data obtained from the same stratum but at the toe of the dam (point B in Figure 9). This situation is complicated by the fact that construction of the dam can be expected to change the D$_R$, OCR, cementation, and aging-induced fabric of the soils beneath the dam. If the stratum is believed to be geologically the same across the dam footprint, it is sometimes assumed that the (N$_1$)$_{60}$ values in the stratum beneath the dam would be the same as at the toe. This assumption may be in significant error if the changes in D$_R$, OCR, cementation, and aging-induced fabric are significant. An alternative approach used at Duncan Dam (Pillai and Byrne 1994) was to obtain frozen samples from the toe area and then test the samples under confining stresses representative of those under the dam. In this approach, the test specimens experience changes in D$_R$, OCR, cementation and fabric that are comparable to those expected to have occurred beneath the dam in the field, and thus the laboratory-measured strengths were expected to be representative of the in-situ values. The K$_\sigma$ relationships discussed in this report: (1) are only intended to account for the effect of confining stress on cyclic

![Figure 9. The liquefaction resistance of foundation strata beneath large embankment dams will be affected by the stresses imposed by the dam and changes in soil properties caused by consolidation to higher stresses.](image)
strength with all other properties held constant and (2) do not account for how increasing confining stresses may also alter the in-situ soil properties.

**SUMMARY AND CONCLUSIONS**

The cyclic strength of sandy soils depends on both relative density (D_r) and effective consolidation stress (\(\sigma'_{vc}\)), which together reflect the state of the soil. The density and strength of soil in-situ is often estimated using correlations to SPT and CPT penetration resistances to avoid the effects of sample disturbance. Changes in in-situ consolidation stress will affect both the penetration resistance and the cyclic strength of the soil. The potential for liquefaction is often evaluated by normalizing the penetration resistance to a reference \(\sigma'_{vc}\) (commonly 1 atm) for a soil at the same D_r using an overburden correction factor for penetration resistance (C_N). The resulting normalized penetration resistance (e.g., an (N_1)_60cs value) is then correlated to the cyclic strength at the same reference stress level (e.g., a CRR_M=7.5,\(\sigma'_{vc}=1,\alpha=0\) value) based on analyses of liquefaction case histories which generally correspond to consolidation stresses less than 1.5 atm. The CRR_M=7.5,\(\sigma'_{vc}=1,\alpha=0\) value obtained from such a correlation is subsequently extrapolated to the in-situ stress conditions for the same soil at the same D_r using an overburden stress correction factor for liquefaction resistance (K_{\sigma}).

The K_{\sigma} factor is intended to account for the effects of consolidation stress on the cyclic strength of a cohesionless soil with all other properties held constant (Seed 1983). However, the interpretation of K_{\sigma} from laboratory test data can be inadvertently affected by the changes in specimen properties which may occur as the confining stress is increased. Examples of these effects were presented in Montgomery et al. (2012) to illustrate the potential magnitude of the resulting errors in the interpreted K_{\sigma} values. The increase in specimen D_r that may occur as the consolidation stress is increased can contribute to K_{\sigma} curve that is too weakly dependent on \(\sigma'_{vc}\) (higher K_{\sigma} values at stresses greater than 1 atm) if this effect is not accounted for. The variation in specimen overconsolidation ratio that may occur when compacted or undisturbed field samples (i.e., with their associated large preconsolidation stresses) are tested at different laboratory stresses can contribute to K_{\sigma} curves with an overly strong dependence on \(\sigma'_{vc}\) if this effect is not properly accounted for. The progressive destruction of cementation or aging-induced fabric that may occur when undisturbed field samples are tested at increasing levels of consolidation stress can also contribute to K_{\sigma} curves with an overly strong dependence on \(\sigma'_{vc}\) if not accounted for.

A summary from an updated K_{\sigma} database by Montgomery et al. (2012) was presented which includes details on the major factors affecting cyclic strengths and the interpretation of K_{\sigma} values. Data points from the categories of clayey sands (fines contents of 23% to 28% and plasticity indices of 20 to 25) and well-compacted sands with varying levels of silt and clay (fines contents of 7% to 35%) were eliminated from further consideration in the evaluation of K_{\sigma} relationships based on the presence of significant amounts of plastic fines and the adverse effects of significant compaction-induced preconsolidation stresses (and hence varying OCR values across the range of
imposed consolidation stresses). The presence of these same data points in the Seed and Harder (1990) database appear to have contributed to the development and recommendation of $K_\sigma$ curves with an overly strong dependence on $\sigma'_{vc}$ in past studies.

Two current $K_\sigma$ relationships that account for the dependence of $K_\sigma$ on $D_R$ were evaluated against the updated database. The relationship by Hynes and Olsen (1999) in combination with the parameters recommended by Youd et al. (2001) was found to be conservative by with respect to the data for clean sands, and to be less conservative for sands with fines contents between 7% and 35%. The relationship by Idriss and Boulanger (2008) was found to provide a reasonably good fit to the data for clean sands, and to be slightly unconservative for sands with fines contents between 7% and 35%.

It is suggested that the relationships by Idriss and Boulanger (2008) continue to provide a reasonable basis for evaluating liquefaction effects in clean sands. For silty sands, the laboratory test results currently available indicate that intermediate values between the Youd et al. (2001) and the Idriss and Boulanger (2008) relationships may give the best estimates for $K_\sigma$. The $K_\sigma$ and $C_N$ factors are not independent, however, because they are both influenced by the same soil properties (often in opposing ways). The existing $C_N$ relationships used in practice were derived primarily for clean sands and are likely conservative for silty sands, based on recent work at Perris Dam and other projects. Therefore, the use of the current Idriss and Boulanger (2008) $K_\sigma$ and $C_N$ relationships may be adequate for both clean and silty sands due to the compensating effects of the $K_\sigma$ and $C_N$ factors. It is recommended that if $K_\sigma$ values for silty sands are selected on the basis of intermediate values from the two relationships, extra attention should be given to evaluating whether the selected $C_N$ curve is appropriate for the soil of interest.

The effect of different liquefaction procedures on the estimation of CRR at large depths was illustrated by an example for clean sands. The results illustrate how differences in the $C_N$ and $K_\sigma$ relationships can both contribute to significant differences in the final estimates of cyclic strength.

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