A STATISTICAL MODEL OF ELASTO-PLASTIC ASPERITY CONTACT OF ROUGH SURFACES

Robert L. Jackson (Member, ASME)
Itzhak Green (Fellow, ASME)
George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology

ABSTRACT

In this work, the results of a new finite element analysis of an elasto-plastic sphere in contact with a rigid flat are used to statistically model an entire surface of asperities in contact. The individual asperity contact model used accounts for a varying hardness effect that has recently been documented. The contact between real surfaces with known material and surface properties, such as the elastic modulus, yield strength, and roughness are modeled. The asperity is modeled as an elastic-perfectly plastic material. The model produces predictions for contact area, contact force, and surface separation. The results of this model are compared to other existing models of asperity contact. The models compared are shown to agree in some cases but disagree in others. Significant limitations of the statistical models (including the Greenwood and Williamson model) are also identified.

NOMENCLATURE

\( A \) = area of contact
\( A_n \) = nominal contact area
\( B \) = material dependant exponent
\( C \) = critical yield stress coefficient
\( D \) = contact area factor
\( d \) = separation of mean asperity height
\( E \) = elastic modulus
\( H \) = hardness
\( h \) = separation of mean surface height
\( K \) = hardness factor
\( N \) = total number of asperities
\( P \) = contact force
\( R \) = radius of hemispherical asperity
\( S_y \) = yield strength
\( a \) = radius of the area of contact
\( e_y \) = uniaxial yield strain, \( S_y/E \)
\( y_s \) = distance between the mean asperity height and the mean surface height
\( z \) = height of asperity measured from the mean of asperity heights
\( \eta \) = area density of asperities
\( \sigma \) = standard deviation of surface heights
\( \sigma_a \) = standard deviation of asperity heights
\( \phi \) = distribution function of asperity heights
\( \psi \) = plasticity index
\( \omega \) = interference between hemisphere and surface
\( \nu \) = Poisson’s ratio

Subscripts
\( E \) = elastic regime
\( c \) = critical value at onset of plastic deformation

Superscripts
\( * \) = normalized by \( \sigma \)

INTRODUCTION

Since in reality all engineering surfaces are rough to some degree, the modeling of the contact between these rough surfaces is very important. Modeling the contact between rough surfaces leads to an improved understanding of the friction, wear, thermal and electrical conductance between surfaces. When loading presses two rough surfaces together, only the peaks or asperities on the surface will be in contact. Thus, the asperities or peaks of the surfaces often carry very high loads. These high loads will often cause yielding in the surface material and thus purely elastic contact models of rough surfaces are not always adequate.

One of the earliest models of elastic asperity contact is that of Greenwood and Williamson [1]. This (GW) model uses the solution of the contact of an elastic hemisphere and a rigid flat plane, otherwise known as the Hertz contact solution, to stochastically model an entire contacting surface of asperities with a postulated Gaussian height distribution. The GW model also assumes that the asperities do not interfere with adjacent asperities and that the bulk material below the asperities does not deform. Supplementing the GW model, many elasto-plastic asperity models have been devised. The Appendix provides a
summary of these models. Although these previous models have proven useful, they contain clear pitfalls which may be detrimental to their validity. The present work attempts to provide a more accurate model and highlight the limitations of the statistical models.

While in the elastic regime, as the load or interference increases, the stresses within the hemisphere also increase. These stresses eventually cause the material within the hemisphere to yield. The interference at this initial point of yielding is known as the critical interference, \( \omega_c \). The resent work by Jackson and Green [2] derives this critical interference analytically using the von Mises yield criterion. The resulting equation is:

\[
\omega_c = \left( \frac{\pi \cdot C \cdot S_y}{2E'} \right)^2 R \tag{1}
\]

where \( C \) is

\[
C = 1.295 \exp(0.736\nu) \tag{2}
\]

The Poisson’s ratio, \( \nu \), to be used in Eq. (2) is that of the material which yields first. For \( \nu = 0.32 \), as is used in this work, Eq. (2) results in \( C = 1.639 \). By solving for the critical interference independently of the hardness Eq. (1) improves upon previously derived Eq. (A16).

The critical force, \( P_c \), is then calculated from the critical interference, \( \omega_c \), by substituting Eq. (1) into Eq. (A11). The resulting critical contact force at initial yielding is:

\[
P_c = 4 \left( \frac{R}{E'} \right)^2 \left( \frac{C}{2} \pi \cdot S_y \right)^3 \tag{3}
\]

Similarly, the critical contact area is calculated from Eq. (A10) and given by:

\[
A_c = \pi \left( \frac{CS_y R}{2E'} \right)^2 \tag{4}
\]

Notice that Eqs. (1-4) are all independent of the hardness, which has recently been shown not to be a constant with respect to \( S_y \) (see Jackson and Green [2], and Mesarovic and Fleck [3]). This is a notable improvement compared to previous formulations.

**Elasto-Plastic Hemispherical Contact Models**

The current work focuses on using the single sphere or asperity results of Jackson and Green (JG) [2], which is the most recent of all contact models, in a statistical model of a rough surface. In their work a finite element analysis is performed that produced different results than the similar Kogut and Etsion (KE) model [4]. The JG model accounts for geometry and material effects which are not accounted for in the KE model. Most notable of these effects is that the predicted hardness is not a material constant as suggested by Tabor [5] and many others since, rather it changes with the evolving contact geometry and the material properties as proven by Jackson and Green [2]. Moreover, the work in [2] used a mesh that is orders of magnitude finer that that in [4] which was mandated by mesh convergence. The work in [2] models deformation surpassing \( \omega = 110 \) (the limit of KE), and likewise models five different material strengths, \( S_y \), that showed a markedly different behavior in the transition from elasto-plastic to fully plastic deformation. While there are regions where the KE and JG models provide similar results, they differ significantly in others. The present work investigates these differences as they are manifested in a statistical model.

At \( 0 \leq \omega / \omega_c \leq 1.9 \) the current single asperity model (i.e., the JG FEM results) effectively coincides with the Hertz contact solution (Eq. (A10) and Eq. (A11)). At interferences larger than this a new formulation generated by Jackson and Green [2] is used as the current single asperity model:

For \( \omega \geq 1.9 \omega_c \)

\[
\bar{A}_{cJG} = \pi R \omega \left( \frac{\omega}{1.9 \omega_c} \right)^B \tag{5}
\]

\[
\bar{p}_{cJG} = \frac{4H_G}{CS_y} \left[ 1 - \exp \left( - \frac{1}{4} \left( \frac{\omega}{\omega_c} \right)^{5/2} \right) \right] \left( \frac{\omega}{\omega_c} \right)^{3/2}
\tag{6}
\]

where

\[
B = 0.14 \exp(23 \cdot e_y) \tag{7}
\]

\[
e_y = \frac{S_y}{E'} \tag{8}
\]

\[
H_G = 2.84 \left[ 1 - \exp \left( -0.82 \left( \frac{\omega}{\omega_c} \right)^{0.7} \right) \right] \left( \frac{\omega}{1.9 \omega_c} \right)^{-0.7} \tag{9}
\]

Statistically, Eq. (5) differs from the FEM data generated by Jackson and Green [2] by an average of 1.3% and a maximum of 4.3%, while Eq. (6) differs by an average error of 0.94% and a maximum of 3.5%. This is true for the entire range of deformation, elastic to fully plastic.

**Statistical Model**

This work uses a Gaussian distribution for the asperity height distribution that is given as:

\[
\phi = (2\pi)^{-1/2} \left( \frac{\sigma}{\sigma'} \right) \exp \left[ -0.5 \left( \frac{z}{\sigma'} \right)^2 \right] \tag{10}
\]

where \( \sigma \) is the standard deviation of the surface heights, and \( \sigma' \) is the standard deviation of the asperity heights. These values also describe the roughness of the surfaces. McCool [6] relates these values by the following:

\[
\sigma^2 = \sigma'^2 + \frac{3.717 \times 10^{-4}}{\eta^2 R^2} \tag{11}
\]
Greenwood and Williamson [1] also define a plasticity index as:

$$\psi = \sqrt{\frac{\sigma_s}{\omega_c}}$$  \(12\)

The plasticity index relates the critical interference and the roughness of the surface to the plastic deformation of the surface. A higher plasticity index indicates a surface whose asperities are more likely to yield. Asperities are thus more likely to deform plastically on rougher surfaces with lower critical interference values. Greenwood and Williamson [1] suggest that for real surfaces the plasticity index can range from \(\psi=0.1\) to \(\psi=100\). This range will be analyzed in this work by holding the surface roughness constant and varying the material yield strength, which differs from previous approaches that usually vary the surface roughness.

Limitations of the Statistical Model

The outlined statistical model is only valid when the individual asperity contact models are also valid. Most current asperity contact models assume that the deformations are relatively small and limited to the asperity tip. The largest deformations considered thus far are those in Jackson and Green [2], where even those are given only up to \(a/R=0.41\). Thus, during the integrations of Eq.(A8) and Eq.(A9), \(a/R\) should remain smaller than that value.

From Eq. (5), the radius of contact can be written as:

$$a = \sqrt{D\omega R}$$  \(13\)

where

$$D = \left\{ \begin{array}{ll}
1 & 0 \leq \omega / \omega_c \leq 1.9 \\
\left( \frac{\omega}{1.9\omega_c} \right)^B & \omega / \omega_c \geq 1.9
\end{array} \right.$$  \(14\)

and \(B\) is found from Eq. (7). Thus the equation for \(a/R\) can be written as:

$$\frac{a}{R} = \sqrt{D\omega \sigma R}$$  \(15\)

The normalized interference (\(\omega^* = \omega^\prime\sigma\)) is then substituted into Eq. (15) yielding

$$\frac{a}{R} = \sqrt{D\omega^* \sigma R}$$  \(16\)

From this point forward, units of length will be normalized by \(\sigma\) and designated by a star superscript. This analysis uses a minimal value \(D=1\), which sets Eq. (16) equal to the Hertz elastic solution. Also, \(\omega^*=1\) is used because at this value a large number of the asperities on the rough surface are clearly in contact. These values are conservative in that for most cases both \(D\) and \(\omega^*\) are usually larger than one.

A sampling of the experimental values reported by Nuri and Halling [7] and implemented by Chang et. al. [8] and Zhao et. al. [9], are presented in Table 1, along with the resulting values of \(R/\sigma\) and \(a/R\), using \(D=1\) and \(\omega^*=1\). The resulting values of \(a/R\) indicate that very large deformations are being modeled by those using the experimental values of Nuri and Harding [7]. Even for sample one, the contact radius is approximately 10% of the asperity radius (\(a/R=0.097\)). Assuming Nuri and Halling’s data is realistic, these results put into question the validity of the statistical model used by Chang et. al. [8], Zhao et. al. [9], and even originally by Greenwood and Williamson [1]. In reality, the values of \(a/R\) will be larger than those calculated in Table 1 because both \(D\) and \(\omega^*\) can assume values (sometimes significantly) larger than one. The values in Table 1 also suggest that many real rough surfaces may undergo extreme deformations during asperity contact and that the bulk material below the asperities would likewise deform significantly (a condition that is not considered in any of the existing single asperity contact models).

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>(\sigma (\mu \text{N}))</th>
<th>(R (\mu \text{m}))</th>
<th>(\sigma R)</th>
<th>((a / R)^4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.16</td>
<td>16.81</td>
<td>0.00952</td>
<td>0.097</td>
</tr>
<tr>
<td>2</td>
<td>1.35</td>
<td>7.14</td>
<td>0.189</td>
<td>0.43</td>
</tr>
<tr>
<td>3</td>
<td>3.25</td>
<td>6.12</td>
<td>0.531</td>
<td>0.72</td>
</tr>
</tbody>
</table>

1. Based on Eq. (16) and assuming \(D=1\), and \(\omega^*=1\).

Table 3: Plasticity indices and corresponding yield strengths.

<table>
<thead>
<tr>
<th>(\psi)</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>10.0</th>
<th>40.0</th>
<th>70.0</th>
<th>100.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_c^*)</td>
<td>3.98x10^0</td>
<td>9.94x10^-1</td>
<td>2.49x10^-1</td>
<td>9.94x10^-3</td>
<td>6.21x10^-4</td>
<td>2.03x10^-4</td>
<td>9.94x10^-5</td>
</tr>
<tr>
<td>(S_y (\text{GPa}))</td>
<td>11.6</td>
<td>5.79</td>
<td>2.89</td>
<td>0.579</td>
<td>0.145</td>
<td>0.0827</td>
<td>0.0579</td>
</tr>
</tbody>
</table>

Table 2: Material and Surface Properties Implemented in Analysis.

| \(E = 200 \text{ GPa}\) |
| \(\nu = 0.32\) |
| \(R = 2.0 \mu \text{m}\) |
| \(\sigma = 9.0 \text{ nm}\) |
| \(\eta = 100.0 \times 10^{11} \text{ m}^2\) |
It is clearly evident, that great care should be taken when using the statistical model first used by Greenwood and Williamson [1], and all subsequent models. Otherwise, the models may be calculating the contact area and contact force for deformations outside of their intended range. These calculations could produce meaningless or misleading results. The R and σ values used in the current analysis produce acceptable values for a/R that are less than the maximum value of 0.41.

RESULTS AND DISCUSSION

This analysis uses the surface and material properties found in Table 2. Note the dependence between the plasticity index, ψ, and the yield strength, S_y, through Eqs. (1) and (12). In this case, the plasticity index is varied over the range shown in Table 3. The critical interference, \( \omega^* \), is then calculated for each ψ by using Eqs. (11, 12). Then the corresponding yield strength, S_y, is calculated from the critical interferences using Eq. (1). That value of S_y is now used in Eqs (1-9). Equations (A8) and (A9) are then numerically integrated using each of the asperity contact models outlined above for \( \overline{P} \) and \( \overline{A} \). The integrals are evaluated using Gauss-Legendre quadrature.

Figure 1: Comparison of numerically and analytically produced results for the CEB model.

The numerically evaluated CEB model is compared to the analytical solution of the CEB model provided by Green [10] in Fig. 1. For each solution, the contact area ratio (\( A/A_n \)) is plotted as a function of the dimensionless load (\( P/(E_A_n) \)). First, this plot verifies that the numerically evaluated integrals produce nearly identical results as Green’s solution for large plasticity indices (\( \psi=4.0 \)). Second, there is a significant amount of error between Green’s solution and the numerical results at small plasticity indices (\( \psi=0.5 \)). Thus, when the Hertz elastic solution is dominant, numerical techniques should be used to solve the CEB model (by definition this solution is identical to the GW model as shown in Fig. 1). Although when Eq. (A14) and Eq. (A15) are dominant, Green’s solution provides accurate values. This makes sense because Green solves exactly the integrals (Eq. (A8) and Eq. (A9)) of elasto-plastic portion of the CEB model (Eq. (A14) and Eq. (A15)) and only approximates the elastic portion (Eq. (A10) and Eq. (A11)).

Figure 2: Contact area versus load for various values of the plasticity index.

Figure 2 shows the resulting contact area ratios (\( A/A_n \)) versus the dimensionless load (\( P/(E_A_n) \)) for different plasticity indices. As expected, the contact area increases with the load. The plot also indicates that an increase in the plasticity index results in larger contact areas at the same loads. When \( \psi=0.5 \), all the models converge to the GW model and are dominated by the Hertz elastic solution. As the plasticity index increases, so do the differences between the models. At \( \psi=10 \) the CEB differs largely from the GW model, while the KE and current model still differ relatively little. For higher plasticity indices the CEB model always has a larger contact area than the other models. Once \( \psi=40 \) is reached, slight differences appear between the KE and current model. Finally at \( \psi=100 \) it is clear that at the same load the contact area predicted by the current model is larger than the KE model. This is because the KE model’s contact area is limited by Abbott and Firestone’s truncation model [11] at large interferences.

Figure 3: Comparison of predicted contact areas.
Next, the contact area ratio for each model is plotted as a function of the plasticity index, while $h$ is held constant at a value of 1.0 (see Fig. 3). At low $\psi$, all the models follow the GW model, before any significant plastic deformation occurs. The CEB model clearly increases too quickly with $\psi$. Once again, the KE and CEB models are clearly limited by assuming Abbott and Firestone’s truncation model [11] at large plasticity indices. However, the current model and KE model follow closely initially, but then the current model continues past both the CEB and KE models. As reported in Jackson and Green [2], the truncation model is invalid, and that is clearly evident in Fig. 3.

The dimensionless load is also plotted as a function of the plasticity index in Fig. 4. All the models again begin at the GW model when $\psi=0.5$. However, the CEB immediately increases past the GW model. This is physically not possible since the GW model is elastic and is thus the limiting case. Both the KE and the current model slowly decrease from the GW model as the plasticity index is increased. At $\psi=10$ the KE and current model differ by only 1.7%, but at $\psi=100$ this difference increases to 23%.

Overall though, and especially at plasticity indices less than ten, the two models agree very well due to an averaging effect of the integrals in Eq. (A8) and Eq. (A9). Thus, even though the individual asperity contact results of the KE and current model differ at some interferences, the integration averages out these differences.

CONCLUSIONS

The KE model and the current model are found to be interchangeable at plasticity indices less than ten. However, on a single asperity scale, it has been proven in Jackson and Green [2] that the current model is a more complete model. This is especially true when the models are used to predict large deformations. The CEB model is also proven invalid since at some surface separations it predicts a higher load carrying capacity for surfaces deforming elasto-plastically than for those deforming only elastically (GW model). The contact area predicted by the KE and CEB models are also limited by the Abbot and Firestone [11] truncation model, which Jackson and Green [2] have proven to be unfounded.

It is also shown that the statistical models originally used by Greenwood and Williamson and subsequently used by Chang et. al. [8], and Zhao et. al. [9], among others, may not be valid for certain sets of surface parameters as indicated by Eq. (16). Great care should thus be taken when implementing Eq. (A8) and Eq. (A9) for surfaces having large value for $\sigma R$. This also suggests that the contact of rough surfaces will likely result in a large number of asperities plastically deforming at their tip and into the bulk material.

APPENDIX: EXISTING CONTACT MODELS

Chang et al. [8] developed a plastic contact model (CEB) that supplemented the Greenwood and Williamson [1] elastic contact model. Notably, the CEB model contains approximations: (1) the shape of the contact area is not accurately captured, (2) the volume assumed to be conserved during plastic deformation is set arbitrarily, and (3) that outside this volume an asperity remains undeformed (although beneath the plastically deformed region the asperity is bound to deform also elastically). Since the transition from the elastic regime (GW) to the plastic regime (CEB) is abrupt, Zhao et al. [9] proposed a mathematical (polynomial) template to allow a “smooth transition” between the two regimes. Because eventually any contact model accumulates statistically the contribution of all asperity contact points, the integration process tends to diminish the deviations between the various models (suggesting dominance by the statistics rather than by the models). All aforementioned models apply to static conditions.

The original CEB work calculated the various integrals numerically because of the perceived complexity confederated by the Gaussian distribution. To bypass such cumbersome numerical integrations the Gaussian distribution has commonly been replaced with simplified exponential distribution functions to allow for closed-form solutions (see GW [1], Etsion and Front [12], Polycarpou and Etsion [13], Hess and Soom [14 & 15], Liu et al. [16]). Recently, Green [10] solved analytically the integrals for the CEB model using the complete Gaussian height distribution. In the current work the uncompromised Gaussian distribution is used, and the integrals are evaluated numerically.

As mentioned above, rough surfaces can be modeled as a collection of individual asperities of various heights. These asperities are then categorized by a few statistical parameters describing the surface. First, the GW model assumes that all asperities have the same radius of curvature, $R$. Then, the distance between the surfaces can be described in two ways: (1) the distance between the mean of the surface heights, $h$, and (2) the distance between the mean of the surface asperities or peaks, $d$. These values are related by

$$h = d + y,$$  \hspace{1cm} (A1)
The value of $y_s$ is derived by Front [17] and given as:

$$y_s = \frac{0.045944}{\eta R} \quad (A2)$$

where $\eta$ is the area density of the asperities.

When the surfaces are pressed together, some of the asperities will interfere a distance $\omega$ with the opposing surface. Since the surfaces cannot penetrate each other, $\omega$ is also the distance each asperity compresses perpendicular to the surfaces. The interference is defined as:

$$\omega = z - d \quad (A3)$$

where the height of each asperity is defined by a distance, $z$, from the mean asperity height. The heights of the asperities are also assumed to have a statistical distribution function, $f(z)$. The nominal contact area, $A_n$, is the area of the surface upon which the asperities in contact are scattered. Thus, the number of asperities on the contacting surface can be found by multiplying the nominal surface area by the area density of the asperities:

$$N = \eta A_n \quad (A4)$$

Then, the total number of asperities in contact is defined as:

$$N_c = \eta A_n \int_{d}^{\omega} f(z) dz \quad (A5)$$

The individual asperity contact area, $A$, and force, $P$, are functions of each asperity’s interference, $\omega$. Thus, the contribution of all asperities of a height $z$ to the total contact area and total contact force can be calculated as:

$$A'(z) = \eta A_n \bar{A}(z - d) \phi(z) \quad (A6)$$
$$P'(z) = \eta A_n \bar{P}(z - d) \phi(z) \quad (A7)$$

Then, the total area of contact and total contact force between the surfaces is found by simply integrating Eq. (A6) and Eq. (A7) over the entire range of asperity contact:

$$A(d) = \eta A_n \int_{d}^{\omega} \bar{A}(z - d) \phi(z) dz \quad (A8)$$
$$P(d) = \eta A_n \int_{d}^{\omega} \bar{P}(z - d) \phi(z) dz \quad (A9)$$

The GW model then assumes that the hemispherical asperities deform elastically and are defined by the Hertz elastic solution [18]. The Hertzian solution provides closed-form expressions to the deformations and stresses of two spheres in a purely elastic contact. The two spheres may have different radii and different elastic properties. However, the closed-form solutions render an equivalent case where a single elastic sphere, having an equivalent elastic modulus, $E'$, and an equivalent radius, $R$, is in contact with a rigid flat (see Eqs. (A10-A13) that follow). The interference, $\omega$, can be described as the distance from a point not deformed in the sphere to the rigid surface. The Hertz solution assumes that the interference is small enough such that the geometry does not change significantly. The solution also approximates the sphere surface as a parabolic curve with an equivalent radius of curvature at its tip. The resulting equations for contact radius and force from the Hertz solution are:

$$\bar{A}_e = \pi R \omega \quad (A10)$$
$$\bar{P}_e = \frac{4}{3} E' \sqrt{R} (\omega)^{3/2} \quad (A11)$$

where

$$\frac{1}{E'} = \frac{1}{E_1} + \frac{1}{E_2} \quad (A12)$$
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad (A13)$$

and $E_1, E_2, R_1, R_2$ are the elastic properties and radii of spheres 1 and 2, respectively.

Instead of the Hertzian elastic solution, models which account for elasto-plastic deformation of an asperity can be used in Eq. (A8) and Eq. (A9). A representation of these elasto-plastic models is outlined below.

First, Chang et al. [8] (CEB model) approximated elasto-plastic contact by modeling a plastically deformed portion of a hemisphere using volume conservation. The CEB model assumptions are discussed above, namely: (1) that the hemisphere deformation is localized to near its tip, (2) the hemisphere behaves elastically below the critical interference, $\omega_c$, and fully plastically above that value, and (3) the volume of the plastically deformed hemisphere is conserved. Using these assumptions the following approximations for contact area and force in the elastic-plastic range ($\omega < \omega_c$) are analytically derived as

$$\bar{A}_{CEB} = \pi R \omega (2 - \omega_c / \omega) \quad (A14)$$
$$\bar{P}_{CEB} = \pi R \omega (2 - \omega_c / \omega) KH \quad (A15)$$

where $K$ is the hardness factor given by $K = 0.454 + 0.41 \nu$. Also, the critical interference used in the CEB model, is given by:

$$\omega_c = \left( \frac{\pi KH}{2 E'} \right)^{2} R \quad (A16)$$

where the hardness is assumed to be $H=2.8 S_y$. From an engineering perspective the corresponding values given by Eq. (1) and Eq. (A16) are very close. However, the CEB model is limited to this fixed relationship between the hardness and the yield strength, and the model also contains a discontinuity at $\omega_c$.

If the plastic deformation covers the entire area of contact, it is said that a fully-plastic condition is reached. Abbott and Firestone [11] theorized that under fully plastic conditions the area of contact of an asperity pressed against a rigid flat can be approximately calculated by truncating the asperity tips as the
rigid flat translates an interference, $\omega$. For a hemisphere, this approximated fully plastic area is given by:

$$A_{AF} = 2\pi R \omega$$

(A17)

which predicts larger contact areas than Eq. (A10). Using Eq. (A17), the contact force of the hemispherical asperity is simply the contact area multiplied by the average contact pressure, which in this case is the hardness, since the contact is assumed to be fully plastic. The approximated fully plastic contact force is thus:

$$P_{AF} = 2\pi R \omega H$$

(A18)

Since plastic deformation of the asperity will increase the area of contact, the truncation model produces the proper trend to some degree. However, the FEM results of Jackson and Green [2] show that this model is unjustifiable, since the contact area can become larger than that predicted by Eq. (A17). Nevertheless, most elasto-plastic asperity contact models incorrectly assume this model once the contact becomes fully plastic.

Kogut and Etsion [4] also performed a FEM analysis of the case of an elastic-perfectly plastic sphere in contact with a rigid flat. Their work gives a very detailed analysis of the stress distribution in the contact region, and empirical expressions are provided for the contact area and the contact force. These are given in a piece-wise form:

For $1 \leq \omega / \omega_c \leq 6$

$$P_{KE} = P_c \cdot 1.03 \left( \frac{\omega}{\omega_c} \right)^{1.425}$$

(A19)

$$A_{KE} = A_c \cdot 0.93 \left( \frac{\omega}{\omega_c} \right)^{1.136}$$

(A20)

For $6 \leq \omega / \omega_c \leq 110$

$$P_{KE} = P_c \cdot 1.40 \left( \frac{\omega}{\omega_c} \right)^{1.263}$$

(A21)

$$A_{KE} = A_c \cdot 0.94 \left( \frac{\omega}{\omega_c} \right)^{1.146}$$

(A22)

These equations are discontinuous at $\omega / \omega_c=1$ (in comparison to the Hertz solution) and at $\omega / \omega_c=6$. Also they describe the deformation only up to $\omega / \omega_c = 110$, at which point the fully plastic Abbott and Firestone model [11] is assumed. The KE model also assumes the value of $H$ to be fixed at 2.85.

References