### 12-21.

A particle is moving along a straight line such that its acceleration is defined as \( a = (4s^2) \text{ m/s}^2 \), where \( s \) is in meters. If \( v = -100 \text{ m/s} \) when \( s = 10 \text{ m} \) and \( t = 0 \), determine the particle's velocity as a function of position.

\[
\begin{align*}
\int_{-100}^{v} v \, dv &= \int_{10}^{s} 4s^2 \, ds \\
\frac{1}{2} v^2 \bigg|_{-100}^{10} &= 4 \cdot \frac{1}{3} s^3 \bigg|_{10}^{10} \\
\frac{1}{2} (v^2 - (-100)^2) &= \frac{4}{3} (s^3 - (10)^3) \\
v &= -(2.667s^3 + 7333.3) \text{ m/s} \quad \text{Ans}
\end{align*}
\]

### 12-22.

A particle is moving along a straight line such that its acceleration is defined as \( a = (-2v) \text{ m/s}^2 \), where \( v \) is in meters per second. If \( v = 20 \text{ m/s} \) when \( s = 0 \) and \( t = 0 \), determine the particle's position, velocity, and acceleration as functions of time.

\[
\begin{align*}
a &= -2v \\
\frac{dv}{dt} &= -2v \\
\int_{10}^{v} \frac{dv}{v} &= \int_{10}^{s} -2 \, dt \\
\ln \frac{v}{20} &= -2t
\end{align*}
\]
12-26. A sandbag is dropped from a balloon which is ascending vertically at a constant speed of 6 m/s. If the bag is released with the same upward velocity of 6 m/s when \( t = 0 \) and hits the ground when \( t = 8 \) s, determine the speed of the bag as it hits the ground and the altitude of the balloon at this instant.

\[
\begin{align*}
(\uparrow) \quad s &= s_0 + v_0 t + \frac{1}{2} a t^2 \\
&= 0 + 6(8) + \frac{1}{2} (9.81)(8)^2 \\
&= 265.92 \text{ m}
\end{align*}
\]

During \( t = 8 \) s, the balloon rises

\[
\begin{align*}
\Delta h &= vt = 6(8) = 48 \text{ m}
\end{align*}
\]

Altitude = \( h + \Delta h = 265.92 + 48 = 314 \) m \quad \text{Ans}

\[
\begin{align*}
(\downarrow) \quad v &= v_0 + a t \\
&= -6 + 9.81(8) = 72.5 \text{ m/s} \quad \text{Ans}
\end{align*}
\]

12-27. A particle is moving along a straight line such that when it is at the origin it has a velocity of 4 m/s. If it begins to decelerate at the rate of \( a = -1.5 \sqrt{v} \) m/s², where \( v \) is in m/s, determine the distance it travels before it stops.

\[
\begin{align*}
a &= \frac{dv}{dt} = -1.5 \sqrt{v} \\
\int_4 v^{-\frac{1}{2}} \, dv &= \int_0^{1.5} \, dt \\
2v^{\frac{1}{2}}|_4 &= -1.5 \Delta t_0 \\
2(\sqrt{1} - 2) &= -1.5 t \\
v &= (2 - 0.75t^2) \text{ m/s} \quad (1)
\end{align*}
\]

\[
\begin{align*}
\int_0^v dt &= \int_0^{1.5} (2 - 0.75t^2) \, dt = \int_0^{1.5} (4 - 3t + 0.5625t^2) \, dt \\
s &= 4(1.5) - 1.5(1.5^2) + 0.1875(1.5^3) \quad (2)
\end{align*}
\]

From Eq. (1), the particle will stop when

\[
0 = (2 - 0.75t^2)
\]

\[
t = 2.667 \text{ s}
\]

\[
s_{t=2.667} = 4(2.667) - 1.5(2.667)^2 + 0.1875(2.667)^3 = 3.56 \text{ m} \quad \text{Ans}
\]
12-38. A two-stage missile is fired vertically from rest with the acceleration shown. In 15 s the first stage A burns out and the second stage B ignites. Plot the $v-t$ and $s-t$ graphs which describe the motion of the second stage for $0 \leq t \leq 20$ s.

Since $v = \int a \, dt$, the constant lines of the $a-t$ graph become sloping lines for the $v-t$ graph.

The numerical values for each point are calculated from the total area under the $a-t$ graph to the point.

At $t = 15$ s, $v = (18)(15) = 270$ m/s
At $t = 20$ s, $v = 270 + (25)(20 - 15) = 395$ m/s

Since $s = \int v \, dt$, the sloping lines of the $v-t$ graph become parabolic curves for the $s-t$ graph.

The numerical values for each point are calculated from the total area under the $v-t$ graph to the point.

At $t = 15$ s, $s = \frac{1}{2}(15)(270) = 2025$ m
At $t = 20$ s, $s = 2025 + 270(20 - 15) + \frac{1}{2}(395 - 270)(20 - 15) = 3687.5$ m = 3.69 km

Also:

$0 \leq t \leq 15$:

$$a = 18$$
$$v = v_0 + a_t = 0 + 18t$$
$$s = s_0 + v_0 t + \frac{1}{2} a_t t^2 = 0 + 0 + 9t^2$$

At $t = 15$:
$$v = 18(15) = 270$$
$$s = 9(15)^2 = 2025$$

$15 \leq t \leq 20$:

$$a = 25$$
$$v = v_0 + a_t = 270 + 25(t - 15)$$
$$s = s_0 + v_0 t + \frac{1}{2} a_t t^2 = 2025 + 270(t - 15) + \frac{1}{2} (25)(t - 15)^2$$

When $t = 20$:
$$v = 395 \text{ m/s}$$
$$s = 3687.5 \text{ m} = 3.69 \text{ km}$$
12-61. The \( a-s \) graph for a train traveling along a straight track is given for the first 400 m of its motion. Plot the \( v-s \) graph. \( v = 0 \) at \( s = 0 \).

\[
0 \leq s \leq 200: \quad a = \frac{1}{100} s
\]

\[
a \, ds = v \, dv
\]

\[
\int_0^s \frac{1}{100} \, ds = \int_0^v \, dv
\]

\[
\frac{1}{200} v^2 = \frac{1}{2} v^2
\]

\[
v = 0.1 s
\]

At \( s = 200 \), \( v = 20 \) m/s

\[
200 \leq s \leq 400: \quad a = \frac{1}{2}
\]

\[
a \, ds = v \, dv
\]

\[
\int_{200}^s 2 \, ds = \int_{20}^v \, dv
\]

\[
2(s - 200) = \frac{1}{2} (v^2 - 400)
\]

\[
v^2 = 4s - 400
\]

At \( s = 400 \) m, \( v = \sqrt{4(400) - 400} = 34.6 \) m/s
12-51. A missile starting from rest travels along a straight track and for 10 s has an acceleration as shown. Draw the \( v-t \) graph that describes the motion and find the distance traveled in 10 s.

For \( t \leq 5 \text{ s} \),

\[
a = 6t
\]

\[
dv = a \, dt
\]

\[
\int_0^t dv = \int_0^t 6t \, dt
\]

\[
v = 3t^2
\]

When \( t = 5 \text{ s} \),

\[
v = 75 \text{ m/s}
\]

For \( 5 < t < 10 \text{ s} \),

\[
a = 2t + 20
\]

\[
dv = a \, dt
\]

\[
\int_{7.5}^t dv = \int_{7.5}^t (2t + 20) \, dt
\]

\[
v = 75 = t^2 + 20t - 125
\]

\[
v = t^2 + 20t - 50
\]

When \( t = 10 \text{ s} \),

\[
v = 250 \text{ m/s}
\]

Distance at \( t = 5 \text{ s} \):

\[
ds = v \, dt
\]

\[
\int_0^5 ds = \int_0^5 3t^2 \, dt
\]

\[
s = (5)^3 = 125 \text{ m}
\]

Distance at \( t = 10 \text{ s} \):

\[
ds = v \, dv
\]

\[
\int_{125}^{10} ds = \int_{125}^{10} (t^2 + 20t - 50) \, dt
\]

\[
s - 125 = \frac{1}{3} t^3 + 10r^2 - 50r \bigg|_{125}^{10}
\]

\[
s = 917 \text{ m} \quad \text{Ans}
\]
12-43. A car starting from rest moves along a straight track with an acceleration as shown. Determine the time \( t \) for the car to reach a speed of 50 m/s and construct the \( v-t \) graph that describes the motion until the time \( t \).

\[
\begin{align*}
\text{For } 0 \leq t \leq 10 \text{ s,} \\
a &= \frac{8}{10} t \\
dv &= a \, dt \\
\int_0^t dv &= \int_0^t \frac{8}{10} t \, dt \\
v &= \frac{8}{20} t^2
\end{align*}
\]

At \( t = 10 \) s,
\[
v = \frac{8}{20} (10)^2 = 40 \text{ m/s}
\]

For \( t > 10 \) s,
\[
a = 8 \\
dv &= a \, dt \\
\int_{10}^t dv &= \int_{10}^t 8 \, dt \\
v &= 40 + 8t - 80 \\
v &= 8t - 40 \quad \text{When } v = 50 \text{ m/s}
\]

\[
t = \frac{50 + 40}{8} = 11.25 \text{ s} \quad \text{Ans}
\]