1. A spring-powered mortar is used to launch 10-lb packages of fireworks into the air. The package starts from rest with the spring compressed to a length of 6 inch. The unstretched length of the spring is 30 inch and the spring constant is \( k = 1300 \) lb/ft. The friction coefficient between the package and the tube wall is \( \mu_k = 0.2 \).
   a. Calculate the friction force acting on the package as it moves along the tube.
   b. Use Work-Energy to calculate the velocity of the package after it has traveled 24 inches along the tube (spring length is 30 inches).

\[ \begin{align*}
\sum F_y &= 0 \\
-w\sin 30^\circ + N &= 0 \\
N &= w\sin 30^\circ = 0.5 W \\
N &= 51 lb \\
F_m &= \mu_k N = 0.2N \\
F_m &= 1.01 lb
\end{align*} \]

\[ \begin{align*}
T_1' - T_2 &= V_{g1} + V_{e1} + T_1 = V_{g2} + V_{e2} + T_2 \\
T_1' &= -F_m (2 	ext{ ft}) = -21 lb \text{ ft} \\
V_{g1} &= mgh_1 = 0 \\
V_{e1} &= \frac{1}{2}k (Ax_1)^2 = \frac{1}{2} (1300) (2 \text{ ft})^2 = 2600 \text{ lb ft} \\
T_1 &= 0 \\
V_{g2} &= mg h_2 = mg (2 \sin 60^\circ) = 10 (2) (\sin 60^\circ) = 17.32 \text{ lb ft} \\
V_{e2} &= \frac{1}{2} k (Ax_2)^2 = 0 \\
T_2 &= \frac{1}{2} m \nu_2^2
\end{align*} \]

\[ -2 + 0 + 2600 + 0 = 17.32 + 0 + \frac{1}{2} m \nu_2^2 \]

\[ 2580.7 = \frac{1}{2} (10 \frac{\text{lb}}{2 \text{ ft}}) \nu_2^2 \]

\[ \nu_2 = 129 \text{ ft/s} \]
2. Two cars are traveling in a parking lot when they encounter a patch of "black ice" which reduces the road friction to zero. The cars are shown from above in the figure below just prior to their collision. During the collision, they get locked together and move as a single unit. Car A weighs 1500 lb and car B weighs 2200 lb. Using the coordinate system shown on the figure, calculate their velocity components after the collision. Write the velocity of the connected cars after the collision in terms of $i$ and $j$ unit vectors.

\[ \vec{v}_B, 12 \text{ ft/s} \]

\[ \vec{v}_A, 20 \text{ ft/s} \]

\[ \text{Treat the 2 cars as a single system} \]

\[ \text{Impulse - Momentum} \]

\[ (m \vec{v})_{1} + \sum F \Delta t = (m \vec{v})_{2} \]

\[ \text{Let} \quad (m \vec{v})_{1} = (m \vec{v})_{2} \]

\[ (m \vec{v})_{1} = (m_A \vec{v}_A) + (m_B \vec{v}_B) \]

\[ (m_A \vec{v}_A) = \frac{1500}{g} (20 \text{mm} \cdot 40^\circ \hat{i} + 20 \text{mm} \cdot 40^\circ \hat{j}) \]

\[ (m_B \vec{v}_B) = \frac{2200}{g} (12 \text{mm} \cdot 10^\circ \cdot \hat{i} + 12 \text{mm} \cdot 10^\circ \cdot \hat{j}) \]

\[ (m_B \vec{v}_B) = \frac{2200}{g} (-11.82 \hat{i} - 2.08 \hat{j}) \]

\[ (m \vec{v})_{1} = \frac{1500 + 2200}{g} \]

\[ \text{and} \quad (m \vec{v})_{2} = (\frac{1500 + 2200}{g}) \vec{v}_2 \]

\[ \frac{1500 (12.86 \hat{i} + 15.3 \hat{j}) + 2200 (-11.82 \hat{i} - 2.08 \hat{j})}{g} = (\frac{3700}{g}) \vec{v}_2 \]

\[ 19290 \hat{i} + 22950 \hat{j} + 26004 \hat{i} - 4576 \hat{j} = 3700 \vec{v}_2 \]

\[ -6714 \hat{i} + 18374 \hat{j} = 3700 \vec{v}_2 \]

\[ -1.81 \hat{i} + 4.96 \hat{j} = \vec{v}_2 \quad (\text{ft/s}) \]
3. A worker is pushing a door open by applying a horizontal force of 60 lb at point C. The door has a weight of 200 lb and a center of mass at G. The door rides on a frictionless rail using two rollers at A and B. The door does not contact the ground.
   a. Draw a FBD of the door
   b. Draw a kinetic diagram of the door
   c. Calculate the acceleration of the door
   d. Calculate the forces applied to the rollers by the rail at points A and B.

\[ \sum F_x = m a_x = m a_0 \]
\[ 60 \text{lb} = m a_x \]
\[ a_{0x} = \frac{60 \text{ft/s}^2}{200} = 0.3 \text{ft/s}^2 \]
\[ \sum F_y = m a_y = 0 \]
\[ R_A + R_B - W = 0 \]
\[ \sum M_A = I_{0x} + m a_0 d \]
\[ 60 \text{lb}(9 \text{ft}) - 200 \text{lb}(6 \text{ft}) + R_B(12 \text{ft}) = m a_0(7 \text{ft}) \]
\[ 540 - 1200 + 12 R_B = \frac{200}{32.2} (9.66) (7) \]
\[ R_B = 90 \text{lb} \]
\[ R_A = 110 \text{lb} \]
4. The uniform 100 kg rod is being pulled by the cord that passes over a frictionless peg (pulley) at point C. The rod is pinned to the ground at point A and has a counterclockwise angular velocity of $\omega = 5 \text{ rad/s}$ when $\theta = 0$. The cable load is $P = 200 \text{ N}$

a. Draw a FBD of the rod when $\theta = 0$ (remember to include the weight of the rod).

b. Draw a kinetic diagram of the rod.

c. Calculate the angular acceleration of the rod and express this acceleration as a vector for $\theta = 0$.

d. Calculate the normal and tangential components of the acceleration vector for the mass center $G$ of the rod for $\theta = 0$.

\[ \sum M_A = I_A \alpha \]

\[ I_A = \frac{1}{3} m_1 l^2 = \frac{1}{3} (100 \text{ kg})(4 \text{ m})^2 = 533 \text{ kgm}^2 \]

\[ 200 \text{ N}(\sin 45^\circ)(3) = 533 \alpha \]

\[ \alpha = 0.796 \text{ rad/s}^2 \]

\[ \sum F_n = m a_n \]

\[ A_n - W + P \cos 45^\circ = m a_n = m r \omega^2 \]

\[ A_n = W - P \cos 45^\circ + m r \omega^2 \]

\[ A_n = (100)(9.81) - 200 \cos 45^\circ + 100(2)(5) \]

\[ A_n = 5840 \text{ N} \]

\[ \sum F_t = m a_t \]

\[ A_t + P \sin 45^\circ = mr \alpha \]

\[ A_t = mr \alpha - P \sin 45^\circ \]

\[ A_t = (100)(1.592) - 200 \sin 45^\circ \]

\[ A_t = 17.7 \text{ N} \]
5. A body is made up of two, slender rods welded together into an "T" shape. Rod AB has a mass of 10 slugs and rod CD has a mass of 8 slugs. Calculate the mass moment of inertia of this body about an axis perpendicular to the page passing through point A.

\[ I_A = I_A^0 + I_A^2 \]

\[ I_A^0 = \frac{1}{3} m_1 l_1^2 = \frac{1}{3} (10)(2)^2 = 13.3 \text{ slug ft}^2 \]

\[ I_A^2 = I_{xc}^2 + d^2 m_2 \]

\[ = \frac{1}{12} m_2 l_2^2 + d^2 m_2 \]

\[ = \frac{1}{12} (8)(1.8)^2 + (2)(8) \]

\[ = 2.16 + 32 \]

\[ I_A = 34.16 \text{ slug ft}^2 \]

\[ I_A = 13.3 + 34.16 \]

\[ I_A = 47.5 \text{ slug ft}^2 \]