2/2 Rectilinear Motion

1. Rectilinear motion is motion such that the velocity vector, \( \vec{v} \), does not change its aligned parallel direction relative to the "s" direction of the coordinate system.
2. The motion of the particle can either be in a straight line or along a curve.
3. The positive "s" direction is in the direction of the velocity vector, \( \vec{v} \).
4. Because of (1), we can treat the vectors, \( \vec{a} \), \( \vec{v} \), and \( s \) as scalars.

\[ \vec{v} \]
Rectilinear Motion – Sec 2/2

1. Constant acceleration \( a = c \)
   a. \( v = v_0 + ct \)
   b. \( v^2 = v_0^2 + 2c(s - s_0) \)
   c. \( s = s_0 + v_0t + \frac{1}{2}ct^2 \)

2. Acceleration as a function of time \( a = f(t) \)

3. Acceleration as a function of velocity \( a = f(v) \)

4. Acceleration as a function of displacement (position) \( a = f(s) \)
The position of a particle is given by \( s = 2t^2 - 40t^2 + 200t - 50 \), where \( s \) is in meters and \( t \) is in seconds.

a. Determine the expression for velocity as a function of time.
b. Determine the expression for acceleration as a function of time.
c. Determine the time at which the velocity is zero.
A particle is traveling along a straight path and experiencing a total deceleration of \( a = 0.7v \) where \( a \) is expressed in ft/s\(^2\) and \( v \) in ft/s. Knowing that at \( t = 0 \) the velocity is 30 ft/s, determine

1. The distance the particle will travel before coming to rest.
2. The time required for the particle to reach a velocity of 15 ft/s.
A rocket sled is used to test the physical behavior of pilot’s during ejection at high speeds. An accelerometer is mounted in the sled and has recorded the following data during a test run.

<table>
<thead>
<tr>
<th>Time (sec)</th>
<th>Accel (ft/sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>8</td>
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<tr>
<td>0.5</td>
<td>16</td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>7.5</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ y = 0.075x^4 - 0.7071x^3 - 1.4258x^2 + 17.941x + 7.6231 \]

\[ R^2 = 0.9974 \]
2/3 Plane Curvilinear Motion

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} \]

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \]

2/4 Rectangular Coordinates \((x - y)\)

\[ \mathbf{r} = xi + yj \]

\[ \mathbf{v} = \mathbf{r} \times \mathbf{v} = xi + yj \]

\[ \mathbf{a} = \mathbf{v} = \mathbf{r} \times \mathbf{a} = xi + yj \]
A horseshoe player releases the shoe at point A with an initial velocity of $v_0 = 30 \text{ ft/s}$.

a. Determine the horizontal distance traveled by the shoe in the air.
b. Determine the time that the shoe is in the air.
c. Determine the maximum vertical height above the ground of the shoe during flight.
2/5 Normal and Tangential Coordinates (n – t)

Figure 2/10
A test car starts from rest on a horizontal circular track of 80-m radius and increases its speed at a uniform rate to reach 100 km/h in 10 seconds. Determine the magnitude $a$ of the total acceleration of the car 8 seconds after the start.

Ans. $a = 6.77 \text{ m/s}^2$

A car rounds a turn of constant curvature between $A$ and $B$ with a steady speed of 45 mi/hr. If an accelerometer were mounted in the car, what magnitude of acceleration would it record between $A$ and $B$?
When the motorcyclist is at A, he increases his speed along the vertical circular path at the rate of \( \dot{v} = 0.04s \) ft/s\(^2\) where \( s \) is in ft. If he starts at \( v_0 = 2 \) ft/s where \( s = 0 \) at A, determine the magnitude of his velocity when he reaches B. Also, what is his initial acceleration?
2/8 Relative Motion (Translating Axes, no Rotation)

1. Fixed coordinate system is one that does not move (Newtonian inertial system).
   a. For earth-bound work, the fixed coordinate system can be considered to be fixed to the earth's surface. (Neglecting the earth's rotation and orbit about the sun)
   b. For earth satellites, the fixed coordinate system can be considered to be a nonrotating system with its origin on the earth's rotational axis (Neglecting the earth's orbit about the sun)
   c. For interplanetary travel, the fixed coordinate system can be considered to be fixed to the sun. (Neglecting the solar system's motion)
2. Absolute displacements, velocities, and accelerations are ones measured from a fixed coordinate system.
3. Relative motion is the motion of an object relative to a moving coordinate system (observer).
4. The moving coordinate system at this point in time is only translating, not rotating.
Problem 2/190

The jet transport $B$ is flying north with a velocity $v_B = 600 \text{ km/h}$ when a smaller aircraft $A$ passes underneath the transport headed in the 60° direction shown. To passengers in $B$, however, $A$ appears to be flying sideways and moving east. Determine the actual velocity of $A$ and the velocity which $A$ appears to have relative to $B$.

At the instant shown, cars $A$ and $B$ are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, $A$ has a decrease in speed of 2 m/s², and $B$ has an increase in speed of 3 m/s². Determine the velocity and acceleration of $B$ with respect to $A$. 
The 50-kg crate is stationary when the force $P$ is applied. Determine the resulting acceleration of the crate if (a) $P = 0$, (b) $P = 150$ N, and (c) $P = 300$ N.

$$\begin{align*}
\mu_s &= 0.20 \\
\mu_k &= 0.15
\end{align*}$$
A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at A.

1. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN.
2. Determine the total force $P$ exerted by the supporting cable on the wheels.
A minivan and trailer are traveling at 20 km/h when the trailer hitch fails and the 250 kg trailer disconnects from the minivan.

1. If the trailer coasts for 45 m before coming to rest, then what is the constant horizontal force $F$ created by rolling friction which causes the trailer to stop. (neglect air drag)

2. Assume that the minivan and trailer were traveling at a constant 20 km/h and the hitch DOES NOT FAIL. Use the rolling resistance force $F$ that you found above to calculate the tensile force acting on the trailer from the hitch.

3. Assume that the minivan and trailer were accelerating at 10,000 km/h$^2$ and the hitch DOES NOT FAIL. Use the rolling resistance force $F$ that you found above to calculate the tensile force acting on the trailer from the hitch.
The small 0.6-kg block slides with a small amount of friction on the circular path of radius 3 m in the vertical plane. If the speed of the block is 5 m/s as it passes point A and 4 m/s as it passes point B, determine the normal force exerted on the block by the surface at each of these two locations.

Ans. \( N_A = 10.89 \text{ N}, N_B = 8.30 \text{ N} \)

A 2-lb slider is propelled upward at A along the fixed curved bar which lies in a vertical plane. If the slider is observed to have a speed of 10 ft/sec as it passes position B, determine (a) the magnitude \( N \) of the force exerted by the fixed rod on the slider and (b) the rate at which the speed of the slider is decreasing. Assume that friction is negligible.
At the instant $\theta = 60^\circ$, the boy's center of mass \( G \) has a downward speed \( v_G = 15 \text{ ft/s} \). Determine the rate of increase in his speed and the tension in each of the two supporting cords of the swing at this instant. The boy has a weight of 60 lb. Neglect his size and the mass of the seat and cords.
The collar has a mass of 5 kg and is confined to move along the smooth circular rod which lies in the horizontal plane. The attached spring has an unstretched length of 200 mm. If, at the instant $\theta = 30^\circ$, the collar has a speed $v = 2 \text{ m/s}$, determine the magnitude of normal force of the rod on the collar and the collar's acceleration.
3/6 Work and Kinetic Energy

Definition of Work (U)

\[ dU = \mathbf{F} \cdot d\mathbf{r} \]

Properties of Work

1. Work is a scalar, but it does possess a sign, + or –
2. Work can be described as the force acting through a displacement.
3. The component of the force parallel to the displacement (\( F_t \)) does work
4. The component of the force perpendicular to the displacement (\( F_n \)) does no work.
5. Positive work occurs when the force component, \( F_t \), is in the same direction as the displacement. For example, pushing a box up a loading ramp.
6. Negative work occurs when the force component, \( F_t \), is in the opposite direction of the displacement. For example, when you are restraining a box from sliding down a ramp.
7. Units of work are (force) times a (distance), such as Nm (joule) or lbft.
8. The symbol \( U_{1,2} \) means the "work done by a force in moving the object from state 1 to state 2."
The 0.5-kg collar C starts from rest at A and slides with negligible friction on the fixed rod in the vertical plane. Determine the velocity \( v \) with which the collar strikes end B when acted upon by the 5-N force, which is constant in direction. Neglect the small dimensions of the collar.

*Ans.* \( v = 2.32 \text{ m/s} \)

**Problem 3/107**

The water-park ride consists of an 800-lb sled which slides from rest down the incline and then into the pool. If the frictional resistance on the incline is \( F_r = 30 \text{ lb} \), and in the pool for a short distance \( F_r = 80 \text{ lb} \), determine how fast the sled is traveling when \( s = 5 \text{ ft} \).
7B. The 1500 kg dragster is traveling at 180 m/s when the engine is shut down and the parachute is released. The parachute drag force as a function of position is shown below (pay attention to the units on the graph). Use the Work-Energy method to calculate the speed of the dragster when it has traveled 400 m from the parachute release position.
7C. A 3.0 lb brick slides down a rough roof ($\mu_k = 0.3$), such that when it is at A it has a velocity of 5 ft/s.

1. Use the Work-Energy method to calculate the velocity of the brick at point B.
2. Calculate the distance $d$ from the wall to where the brick strikes the ground.
3. Use the Work-Energy method to calculate the velocity of the brick when it hits the ground.
Work Done by a Linear Spring

Work done by the spring as the block moves from position $x_1$ to position $x_2$.

\[
U_{s_{1-2}}^{Fs} = \int_{x_1}^{x_2} F_s \cdot dx = \int_{x_1}^{x_2} (-kx) \cdot dx = -\int_{x_1}^{x_2} kx \ dx = \frac{1}{2} k(x_1^2 - x_2^2)
\]

$U_{s_{1-2}}^{Fs} = \frac{1}{2}$ shaded area under the force-displacement curve

Note that the distances $x_1$ and $x_2$ represent the amount of "stretch" that the spring is undergoing from its unstretched or undeformed length. If the position $x_1$ is the position where the spring is "relaxed" or undeformed, then $x_1 = 0$ in the work equation.
Work and Kinetic Energy (cont)

Definition of Power \( P = \frac{dU}{dt} \)

\[
dU = \mathbf{F} \cdot d\mathbf{r}
\]

\[
P = \frac{dU}{dt} = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt} = \mathbf{F} \cdot \mathbf{v}
\]

Properties of Power

1. Time rate of doing work
2. Power is a scalar that is always positive
3. Units of power are \( N \, m/s \) or \( J/s \) which is called a watt. U.S. customary system uses horsepower (hp) to define mechanical power.

Definition of Mechanical Efficiency \( (e_m) \)

The mechanical efficiency is the ratio of the work done by a machine to the work done on the machine, assuming that the machine cannot create energy. All mechanical efficiencies are less than one due to energy loss during operation

\[
e_m = \frac{P_{\text{output}}}{P_{\text{input}}}
\]

Potential Energy

The Work (U) done on an object can be divided into the work done by conservative forces (gravity and springs) plus the work done by other forces.

The Work-Energy equation now becomes

\[
U^{\text{con}}_{1-2} + U^e_{1-2} = T_2 - T_1
\]

Potential Energy of Gravity \( (V_g) \)

\[
V_g = mgh^* \quad \text{where} \, h^* \, \text{is the distance from the center of earth}
\]

However, since we are only interested in the change in the potential energy from state 1 to state 2, we can select a vertical location other than the center of the earth as our reference position or datum for the height, \( h \).
The establishment of an gravitational datum (h = 0) allows to write the work done by the gravity force as

\[ U_{1,2}^g = -(V_{g2} - V_{gl}) = -(mgh_2 - mgh_1) \]

**Potential Energy of a Spring (Ve)**

\[ V_e = \frac{1}{2} k (\Delta x)^2 \]

Where \( \Delta x \) is the amount that the spring has been compressed or stretched from its unloaded length. The amount of work done by a spring is equal to

\[ U_{1,2}^e = -(V_{e2} - V_{e1}) = -\left\{ \frac{1}{2} k (\Delta x_2)^2 - \frac{1}{2} k (\Delta x_1)^2 \right\} \]

**Final Version of the Work-Energy Equation**

\[ U_{1,2}^e - (V_{g2} - V_{gl}) - (V_{e2} - V_{e1}) = T_2 - T_1 \]

Or

\[ U_{1,2}^e + V_{gl} + V_{e1} + T_1 = V_{g2} + V_{e2} + T_2 \]
3/117 The 2-kg collar is released from rest at A and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (a) the velocity \( v \) of the collar as it strikes the spring and (b) the maximum deflection \( x \) of the spring.

An. (a) \( v = 2.56 \) m/s, (b) \( x = 98.9 \) mm

Problem 3/117

3/157 The spring has an unstretched length of 25 in. If the system is released from rest in the position shown, determine the speed \( v \) of the ball (a) when it has dropped a vertical distance of 10 in. and (b) when the rod has rotated 35°.

An. (a) \( v = 3.06 \) ft/sec
(b) \( v = 1.641 \) ft/sec

Problem 3/157
3/8-9 Linear Impulse and Momentum

Concept of Linear Impulse and Momentum

\[ \sum F = \frac{d}{dt}(m \vec{v}) \Rightarrow \sum F \, dt = d(m \vec{v}) \]

\[ \int_0^t \sum F \, dt = \int_0^{\vec{v}_2} d(m \vec{v}) \]

\[ \int_0^t \sum F \, dt = \left( m \vec{v}_2 \right) - \left( m \vec{v}_1 \right) \]

Properties

1. The Impulse-Momentum equation is a vector equation
2. It is easy to apply this concept to multiple bodies that comprise a “system”.

3/193 The 200-kg lunar lander is descending onto the moon’s surface with a velocity of 6 m/s when its retro-engine is fired. If the engine produces a thrust \( T \) for 4 s which varies with the time as shown and then cuts off, calculate the velocity of the lander when \( t = 5 \) s, assuming that it has not yet landed. Gravitational acceleration at the moon’s surface is 1.62 m/s².

\[ \text{Ans. } v = 2.10 \text{ m/s} \]

3/189 Freight car A with a gross weight of 150,000 lb is moving along the horizontal track in a switching yard at 2 mi/hr. Freight car B with a gross weight of 120,000 lb and moving at 3 mi/hr overtakes car A and is coupled to it. Determine (a) the common velocity \( v \) of the two cars as they move together after being coupled and (b) the loss of energy \( |\Delta E| \) due to the impact.

\[ \text{Ans. } (a) v = 2.44 \text{ mi/hr}, (b) |\Delta E| = 2230 \text{ ft-lb} \]
A 0.32 lb baseball traveling at 80 mph rebounds off a bat with a speed of 160 mph. The ball is in contact with the bat for roughly 0.001 sec. The incoming velocity of the ball is horizontal, and the outgoing trajectory forms an angle $\alpha = 30$ degrees with respect to the incoming trajectory.

1. Determine the magnitude of the horizontal impulse imparted to the ball by the bat during the given time interval.
2. Determine the magnitude of the vertical impulse imparted to the ball by the bat during the given time interval.
3. Calculate the magnitude of the average impulsive force in the horizontal direction acting on the ball during this interval.
5/1 Plane Kinematics

<table>
<thead>
<tr>
<th>Type of Rigid-Body Plane Motion</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Rectilinear translation</td>
<td>Rocket test sled</td>
</tr>
<tr>
<td>(b) Curvilinear translation</td>
<td>Parallel-link rotating plate</td>
</tr>
<tr>
<td>(c) Fixed axis rotation</td>
<td>Compound pendulum</td>
</tr>
<tr>
<td>(d) General plane motion</td>
<td>Connecting rod in a reciprocating engine</td>
</tr>
</tbody>
</table>

5/2 Fixed Axis Rotation

Note: All lines on a rigid body undergoing rotation about a fixed point have the same angular displacement ($\beta$), angular velocity ($\omega$) and angular acceleration ($\alpha$).
The right-angle bar rotates clockwise with an angular velocity which is decreasing at the rate of $4 \text{ rad/s}^2$. Write the vector expressions for the velocity and acceleration of point $A$ when $\omega = 2 \text{ rad/s}$. 

![Diagram of the right-angle bar with dimensions: 0.3 m, 0.4 m, and point A.]
Appendix B  Mass Moments of Inertia

\[ I_a = \int r^2 dm \]

\[ I_x = \int (r^2 - z^2) \, dm \]
\[ I_y = \int (r^2 - x^2) \, dm \]
\[ I_z = \int (r^2 - y^2) \, dm \]

Parallel Axis Theorem

\[ I_a = I_{CG} + dz^2 m \]

Where \( I_{CG} \) is the MOI about an axis that is parallel to the “a” axis and that passes through the center of gravity of the body.
\( d \) is the \textbf{perpendicular} distance from the “a” axis to the “CG” axis.
MOI of Thin Plates

\[ I_{xx} = \iint (y^2 + z^2) \, dm \] 
\[ I_{yy} = \iint (x^2 + z^2) \, dm \] 
\[ I_{zz} = \iint (x^2 + y^2) \, dm \] 
\[ I_{xy} = \iint xy \, dm \] 
\[ I_{xz} = \iint xz \, dm \] 
\[ I_{yz} = \iint yz \, dm \]

\[ I_{xx} = \rho t \iint y^2 \, dA \] 
\[ I_{yy} = \rho t \iint x^2 \, dA \] 
\[ I_{zz} = \rho t \iint r^2 \, dA \] 
\[ I_{xy} = \rho t \iint xy \, dA \] 
\[ I_{xz} = \rho t \iint xz \, dA \] 
\[ I_{yz} = \rho t \iint yz \, dA \]

Radius of Gyration

\[ \rho = \sqrt{\frac{I_{xx}}{A}} \] 
\[ \rho = \sqrt{\frac{I_{yy}}{A}} \] 
\[ \rho = \sqrt{\frac{I_{zz}}{A}} \] 
\[ \rho = \sqrt{\frac{I_{xy}}{A}} \] 
\[ \rho = \sqrt{\frac{I_{xz}}{A}} \] 
\[ \rho = \sqrt{\frac{I_{yz}}{A}} \]
\[ (g_1 + g_2) \mu \frac{d}{d t} \frac{d}{d t} = \gamma c c I \]
\[ g \mu \frac{d}{d t} \frac{d}{d t} = \gamma c c I \]
\[ (g_1 + g_2) \mu \omega \theta = \gamma I \]
\[ (g_1 + g_2) \mu \omega \theta = \gamma c I \]
\[ (g_1 + g_2) \mu \omega \theta = \gamma c c I \]

\[ \frac{\mu}{\omega c} = x \]

<table>
<thead>
<tr>
<th>PARALLELEPiped</th>
<th>PRISM</th>
<th>RECTANGLE</th>
</tr>
</thead>
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<tr>
<td>PERPENDICULAR</td>
<td>SEMI-cylinder</td>
<td>Cylindrical</td>
</tr>
<tr>
<td>PERPENDICULAR</td>
<td>Round</td>
<td>Cylindrical</td>
</tr>
</tbody>
</table>

\[ \varepsilon_{nu} \left( \frac{\rho^2}{\rho t} - \frac{1}{\rho} \right) = \gamma I \]
\[ \varepsilon_{nu} = \gamma I \]
\[ \varepsilon_{nu} = \gamma c I \]
\[ \varepsilon_{nu} = \gamma c c I \]
\[ \varepsilon_{nu} = \gamma c c c I \]
\[ \frac{\mu}{\omega c} = x \]

<table>
<thead>
<tr>
<th>CYLINDER</th>
<th>HEMISPHERE</th>
<th>CYLINDRICAL SHELL</th>
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</thead>
<tbody>
<tr>
<td>CYLINDER</td>
<td>CYLINDRICAL SHELL</td>
<td>CYLINDRICAL SHELL</td>
</tr>
</tbody>
</table>

**TABLE D4: PROPERTIES OF HOMOGENEOUS SOLIDS**

(m = mass of body shown)
Compute the moment of inertia of the mallet about the O-O axis. The mass of the head is 0.8 kg, and the mass of the handle is 0.5 kg.

Ans. $I_{OO} = 0.0671 \text{ kg} \cdot \text{m}^2$
The “L” shaped body below is made up of two cylindrical rods that have a radius of 0.05 meters. The rod material has a mass of .25 kg per foot. Calculate the mass moment of inertia for this body about an z-axis running through point O.
Find the moment of inertias about the $x$, $y$ and $z$ axes for the structure shown below.
Chapter 6

Section 6/1 and 6/2 Plane Motion (2D) of Rigid Bodies

A. General Plane Motion with respect to the Center of Mass (G)

Basic Equations of Motion

\[
\Sigma F = ma_G \\
\Sigma M_G = I_G \alpha 
\]

Where \( a_G \) = the acceleration of the mass center (G)
\( I_G \) = the mass moment of inertia about the z axis through the mass center (G).
\( \Sigma M_G \) = the summation of moments about the mass center (G)

Free-Body Diagram  Kinetic Diagram

Basic Equations of Motion (Moments summed about a point other than G)

\[
\Sigma F = ma_G \\
\Sigma M_P = I_G \alpha + ma_G d 
\]

Where \( a_G \) = the acceleration of the mass center (G)
\( I_G \) = the mass moment of inertia about the z axis through the mass center (G).
\( \Sigma M_P \) = the summation of moments about point P
\( d \) = perpendicular distance from the \( ma_G \) vector and point P

Free-Body Diagram  Kinetic Diagram
6/3 Rigid Body Translation (special case of General Motion) 
\( \alpha = 0 \) and \( \omega = 0 \)

Basic Equations of Motion

\[ \Sigma F = ma_G \]
\[ \Sigma M_G = I_G \alpha = 0 \quad \text{or} \quad \Sigma M_P = I_G \alpha + ma_Gd = ma_Gd \]

Where \( a_G \) = the acceleration of the mass center (G)
\( I_G \) = the mass moment of inertia about the z axis through the mass center (G).
\( \Sigma M_G \) = the summation of moments about the mass center (G)
\( \Sigma M_P \) = the summation of moments about point P

(a) Rectilinear Translation
\( (\alpha = 0, \omega = 0) \)

(b) Curvilinear Translation
\( (\alpha = 0, \omega = 0) \)
6/2 A passenger car of an overhead monorail system is driven by one of its two small wheels A or B. Select the one for which the car can be given the greater acceleration without slipping the driving wheel and compute the maximum acceleration if the effective coefficient of friction is limited to 0.25 between the wheels and the rail. Neglect the small mass of the wheels.

The block A and attached rod have a combined mass of 60 kg and are confined to move along the 60° guide under the action of the 800-N applied force. The uniform horizontal rod has a mass of 20 kg and is welded to the block at B. Friction in the guide is negligible. Compute the bending moment \( M \) exerted by the weld on the rod at B.
6/4 Fixed-Axis Rotation (special case of General Motion)

Basic Equations of Motion

\[ \Sigma F = ma_G \]

\[ \Sigma M_G = I_G \alpha \quad \text{or} \quad \Sigma M_P = I_G \alpha + ma_Gd \quad \text{or} \quad \Sigma M_o = I_o \alpha \]

Where

- \( a_G \) = the acceleration of the mass center (G)
- \( I_G \) = the mass moment of inertia about the z axis through the mass center (G).
- \( I_o \) = the mass moment of inertia about the z axis through the fixed rotation point (o)
- \( \Sigma M_G \) = the summation of moments about the mass center (G)
- \( \Sigma M_P \) = the summation of moments about point P
- \( \Sigma M_o \) = the summation of moments about the fixed rotation point (o)

The pendulum has a mass of 7.5 kg with center of mass at G and has a radius of gyration about the pivot O of 295 mm. If the pendulum is released from \( \theta = 0 \), determine the total force supported by the bearing at the instant when \( \theta = 60^\circ \). Friction in the bearing is negligible.
Determine the angular acceleration and the force on the bearing at \( O \) for (a) the narrow ring of mass \( m \) and (b) the flat circular disk of mass \( m \) immediately after each is released from rest in the vertical plane with \( OC \) horizontal.

<table>
<thead>
<tr>
<th>BODY</th>
<th>MASS CENTER</th>
<th>MASS MOMENTS OF INERTIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular Cylindrical Shell</td>
<td>-</td>
<td>( I_{xx} = \frac{1}{3}mr^2 + \frac{1}{12}ml^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{y'y'} = \frac{1}{2}mr^2 + \frac{1}{3}ml^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{zz} = ml^2 )</td>
</tr>
<tr>
<td>Circular Cylinder</td>
<td>-</td>
<td>( I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{y'y'} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( I_{zz} = \frac{1}{3}mr^2 )</td>
</tr>
</tbody>
</table>