Delta-Sigma Modulation in Direct Digital Frequency Synthesis

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ABSTRACT: This paper presents comparisons of various ΔΣ modulations in direct digital frequency synthesis. ΔΣ modulators such as MASH, feedforward, feedback and error feedback have been implemented in both phase and frequency domains in a CMOS DDS. The DDS prototype is fabricated in a 0.35μm CMOS technology with core area of 1.7x2.1 mm^2 and total 75 mA current. Measured DDS output demonstrates that frequency domain ΔΣ modulation achieves better SFDR and SINAD than phase domain ΔΣ modulation.

Keywords: *DDS, ΔΣ modulation, DAC, frequency synthesis.*

1. INTRODUCTION

As illustrated in Fig.1, a conventional direct digital synthesizer (DDS) consists of a numerically controlled oscillator (NCO) and a digital-to-analog converter (DAC). The NCO further includes a phase accumulator and a lookup table that transforms digital phase information to digital amplitude information. The DDS output spectrum contains spurious components mainly due to phase word truncation before the ROM lookup table. The periodic truncated phase error sequence causes spurs in its output spectrum. Although ΔΣ modulators have been implemented in both frequency [1] and phase [2] domains in DDS to reduce the spurs, their noise shaping effects and design tradeoffs haven't been thoroughly compared. In this paper, for the first time, we compare and implement eight different ΔΣ modulators including MASH, feedforward, feedback and error feedback in both frequency and phase domains in a CMOS DDS chip. The noise shaping effects, in-band SFDR, SINAD, operation speeds, and stability are compared according to measured results.

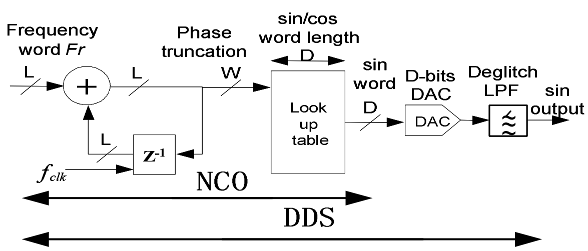


Fig. 1. A conventional direct digital frequency synthesizer.

2. PHASE DOMAIN ΔΣ MODULATION

In an ideal DDS without phase truncation ($L = W$) and with infinite amplitude precision, the output sequence of the NCO is given by

$$S(n) = \sin\left(2\pi \frac{Fr}{2^L} n\right) \qquad (1)$$

where F_r is the input frequency control word with L bits and W is the length of phase word to address the accumulator.

In order to reduce the ROM size, one can either truncate the phase accumulator output from L bits to W bits. With a phase word truncation, we define B to be the number of bits truncated such that L-W=B. The output of the DDS becomes

$$S_t(n) = \sin\left(2\pi \frac{2^B}{2^L}\left[\frac{Fr}{2^B} n\right]\right) \qquad (2)$$

where the operator [] represents truncation to integer values. Equation (2) can alternatively be expressed as

$$S_t(n) = \sin\left(\frac{2\pi}{2^L}\left[Fr \cdot n - \varepsilon_p(n)\right]\right) \qquad (3)$$

where $\varepsilon_p(n)$ represents the phase error sequence. Applying trigonometric identities, Eq. (3) can be rewritten as

$$S_t(n) = \sin\left(2\pi \frac{Fr \cdot n}{2^L}\right)\cos\left(2\pi \frac{\varepsilon_p(n)}{2^L}\right) - \cos\left(2\pi \frac{Fr \cdot n}{2^L}\right)\sin\left(2\pi \frac{\varepsilon_p(n)}{2^L}\right) \qquad (4)$$

Assuming $\varepsilon_p(n) << 2^L$, we get

$$S_t(n) = \sin\left(2\pi \frac{Fr \cdot n}{2^L}\right) - 2\pi \frac{\varepsilon_p(n)}{2^L} \cdot \cos\left(2\pi \frac{Fr \cdot n}{2^L}\right) \qquad (5)$$

Therefore, the output spectrum of the DDS is composed of a sine wave at the desired output frequency corrupted by the cosine modulated harmonics of the phase error $\varepsilon_p(n)$. The periodic sequence $\varepsilon_p(n)$ can be expressed as Fourier series. Thus, the conventional DDS with phase truncation ends up with spurs at different places in its output spectrum.

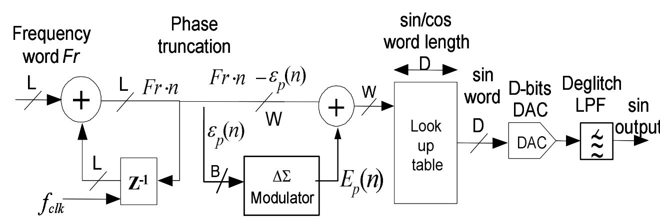


Fig. 2. Phase domain ΔΣ modulation in DDS.

In order to reduce spurs due to phase truncation, we apply ΔΣ modulation in phase domain, as shown in Fig. 2. The phase word $Fr \cdot n$ after the accumulator is truncated into $Fr \cdot n - \varepsilon_p(n)$ (W bits) and $\varepsilon_p(n)$ (B bits). The phase error $\varepsilon_p(n)$ is fed into a ΔΣ modulator. The modulator's output $E_p(n)$ is expressed as a single-bit or multi-bit word and added back to the truncated phase.

Using a linear model for delta-sigma modulators, we get

$$E_p(z) = \varepsilon_p(z) + Q(z)(1 - z^{-1})^k = \varepsilon_p(z) + Noise(z) \qquad (6)$$

where $Q(z)$ is the quantization noise from the quantizer inside the modulator and k is the order of the modulator. We can rewrite (6) as

$$E_p(n) = \varepsilon_p(n) + Noise(n) \qquad (7)$$

where the modulated quantization noise, $Noise(n)$, is the inverse Z-transform of $Q(z)(1-z^{-1})^k$.

The DDS output is thus given by

$$S_t(n) = \sin\left(\frac{2\pi}{2^L}\left[Fr\cdot n - \varepsilon_p(n) + Ep(n)\right]\right) \\ = \sin\left(\frac{2\pi}{2^L}\left(Fr\cdot n + Noise(n)\right)\right) \quad (8)$$

Assuming $Noise(n) << 2^L$, we obtain

$$S_t(n) = \sin\left(2\pi\frac{Fr\cdot n}{2^L}\right) - 2\pi\frac{Noise(n)}{2^L}\cdot\cos\left(2\pi\frac{Fr\cdot n}{2^L}\right) \quad (9)$$

In equation (9), the modulated quantization *Noise(n)* takes place of the original phase error $\varepsilon_p(n)$. Thus, spurs introduced by the phase truncation are reduced, and modulated quantization noise with high frequency shaping shows up in the spectrum. ΔΣ modulators with different noise transfer functions thus lead to different DDS output spectra.

3. Frequency Domain ΔΣ Modulation

ΔΣ modulation can also be implemented in the frequency domain of a DDS. For frequency word truncation, the frequency word *Fr* is truncated from *L* bits to *W* bits before the phase accumulator. The discarded frequency bits *B=L-W,* and the output sequence of the DDS ROM become

$$S(n) = \sin\left(2\pi\frac{Fr}{2^L}n\right) = \sin\left(2\pi\frac{(Fr/2^B)}{2^W}\cdot n\right) \\ = \sin\left(2^{1-W}\pi\left(\left[\frac{Fr}{2^B}\right]\cdot n + \left(Fr - \left[\frac{Fr}{2^B}\right]\cdot 2^B\right)\cdot n \Big/ \left(2^B\right)\right)\right) \\ = \sin\left(2\pi\left(\left[\frac{Fr}{2^B}\right]n + pe\cdot n\right)\Big/ 2^W\right) \quad (10)$$

where the operator [] represents truncation to an integer and the frequency error due to frequency word truncation is

$$\left(\left(Fr - \left[\frac{Fr}{2^B}\right]\cdot 2^B\right)\Big/ 2^B\right) = pe \quad (11)$$

As shown in (10), the phase accumulator size can be reduced to *W* bits if the input frequency word *Fr* is truncated to $[Fr/2^B]$.

The frequency word truncation will also cause a phase error *(pe·n)* which is periodic in nature and thus leads to spurs at the DDS output. If the input word $Fr \le 2^B$, then $[Fr/2^B]=0$, there will be no DDS output due to frequency word truncation. However, to avoid losing frequency information, the constant frequency error *pe* needs to be modulated and added back to the accumulator.

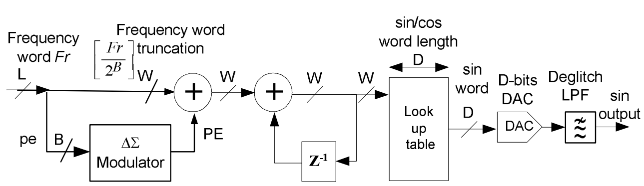


Fig. 3 Proposed DDS with frequency domain ΔΣ modulation.

As shown in Fig. 3, the modulated frequency error *pe* is added back to the truncated *Fr* that is represented by $[Fr/2^B]$. Note that in Fig. 3, the ΔΣ modulator's input *pe* is constant, which benefits the modulator design with improved stability, input range and speed. The noise shaped frequency error is thus a series of numbers and is given by

$$PE(z) = pe(z) + Q(z)(1 - z^{-3}) = pe(z) + Noise(z) \quad (12)$$

where *Q(z)* is the quantization noise introduced by the ΔΣ modulator.

With the frequency domain ΔΣ modulation, the DDS output is given by

$$S_t(n) = \sin 2\pi\left(\left[\frac{Fr}{2^B}\right]n + PE(n)\cdot n\right)\Big/ 2^W \\ = \sin 2\pi\left(\left[\frac{Fr}{2^B}\right]n + pe\cdot n + Noise(n)\cdot n\right)\Big/ 2^W \\ \approx \sin\left(2\pi\frac{Fr\cdot n}{2^L}\right) - \cos\left(2\pi\frac{Fr\cdot n}{2^L}\right)\cdot\left(2\pi\frac{Noise(n)\cdot n}{2^W}\right) \quad (13)$$

Thus, the DDS output spectrum is composed of a sine wave at the desired output frequency and a cosine wave that is modulated by the quantization noise shaped by the ΔΣ modulator. Based on the linear model of a ΔΣ modulator, the periodic phase error due to frequency word truncation is reduced. Instead, the modulated quantization noise from the modulator occurs at the DDS output.

4. Comparison of Measured NCO Output Spectra for Various ΔΣ Modulators

Although various ΔΣ modulators in both the phase domain and frequency domain can move the spurs and quantization noise to a high frequency band, their performances on noise shaping are different. To compare various ΔΣ modulator performances, we consider factors such as the modulator topology, the order of the modulator, the modulator input, the in-band spurious tones, the number of quantizer bits, the modulator speed and area, etc. We first implemented an NCO with several types of ΔΣ modulators in both frequency and phase domain in FPGA as shown in Fig. 4. The NCO output is captured into a PC for analysis. First we analyze the output characteristics such as the spurious-free-dynamic-range (SFDR), defined as the ratio between the fundamental signal and the highest spurs and the signal-to-noise-and-distortion ratio (SINAD) shown in (14).

$$SINAD = 20\cdot\log\left(\frac{Signal}{SUM\left(Noise + Harmonics\right)}\right) \quad (14)$$

The oversampling ratio (OSR) of the ΔΣ modulator is chosen as 64 and the band of interest is from zero to 1/64 of the clock frequency.

The measured in-band SINAD and SFDR of the NCO output with various ΔΣ modulators are given in Fig. 5. The measurements show that without ΔΣ modulation, direct phase truncation has a low SFDR and SINAD. With ΔΣ modulation, in-band SFDR and SINAD increase. In phase domain modulation, the phase error *pe* (*L-W* bits LSBs) has different repeating periods with respect to different Fr, which leads to a varying the input to the modulator. In contrast, frequency domain modulators have constant dc input. As a result, the frequency domain ΔΣ modulation has higher SFDR and SINAD than phase domain. Although using a high-order ΔΣ modulator results in sharper noise shaping effect, it suffers from degraded SFDR and SINAD.

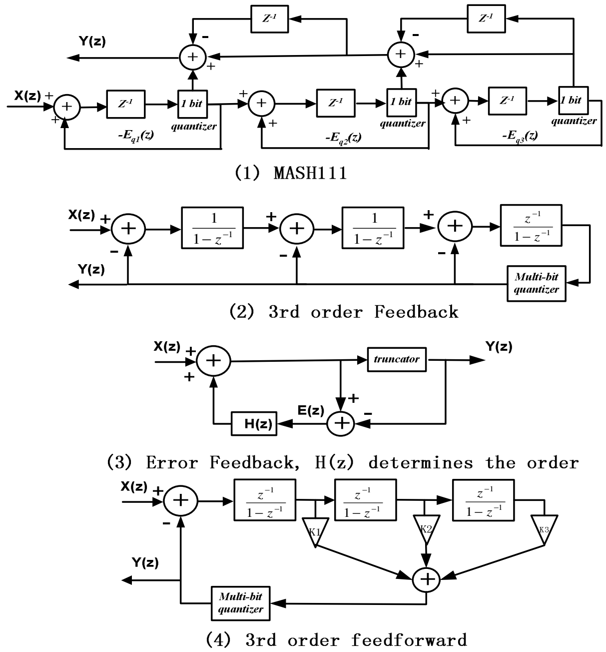


Fig. 4. Four types ΔΣ modulators implemented in a DDS and NCO for comparison.

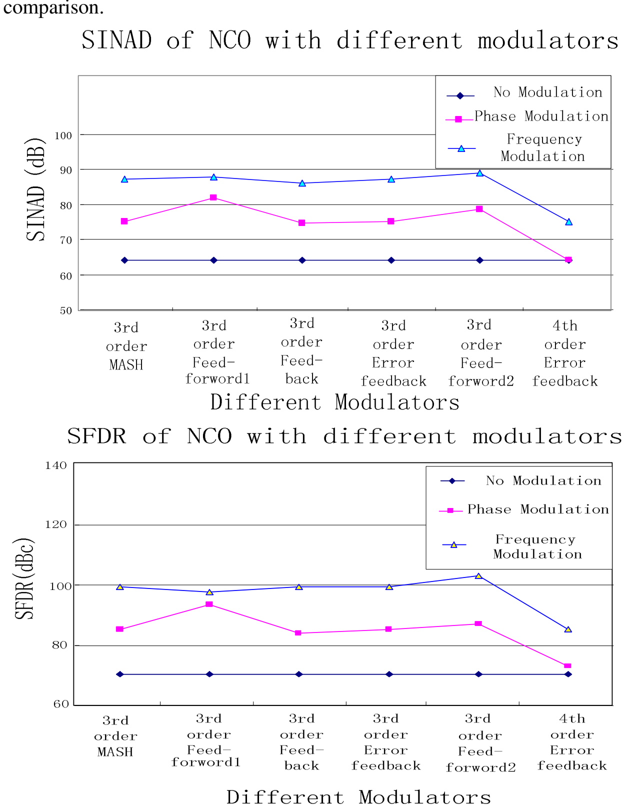


Fig. 5. Measured in-band SINAD and SFDR of the NCO output with various ΔΣ modulators in frequency and phase domains.

Since the modulated quantization noise dominates the DDS output spectrum, its noise transfer function *He(Z)* can greatly affect the DDS output. We implemented four types of ΔΣ modulators, i.e., MASH, feedforward, feedback and error feedback as shown in Fig.4. MASH, feedback and error feedback type ΔΣ modulators have the same noise transfer function of $He(Z)=(1-Z^{1})^{k}$, where k is the order of the modulator. MASH-111 has three 1-bit quantizers, while the feedback type modulator has a multi-bit quantizer and the error feedback ΔΣ modulator has one single-bit quantizer. A feedforward type ΔΣ modulator is first presented in the frequency domain in [1]. It's a 2nd order feedforward type with a multi-bit quantizer. The modulator's order can be increased by adding more accumulators in cascade and different noise transfer functions can be obtained by varying the feedforward coefficients k_1, k_2, k_3. We propose a *feedforward2* ΔΣ modulator with coefficients of k_1=2.2, k_2=1.92, k_3=0.72.

Fig. 6 compares the noise transfer function of MASH, *feedforward1* [3], and the proposed *feedforward2* modulator. It's clear that MASH has much sharper noise shaping effect in-band. But it has high out-of-band noise that requires a high-order LPF for noise rejection. The *feedforward2* modulator has lower in-band noise compared with existing *feedforward1* modulator and has flat out-of-band noise compared with MASH type. Fig. 7 gives the measured NCO output spectrum for two types of ΔΣ modulators. The measured spectra of the two ΔΣ modulators are almost identical with the modulated noise. Multi-bit quantizer inside the modulator can make feedback signal match input signal more accurately and make the quantization noise more random, which better fits the liner model for ΔΣ modulators.

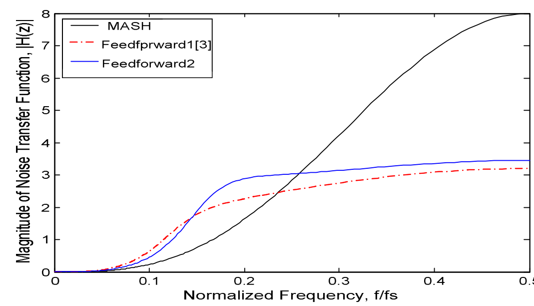
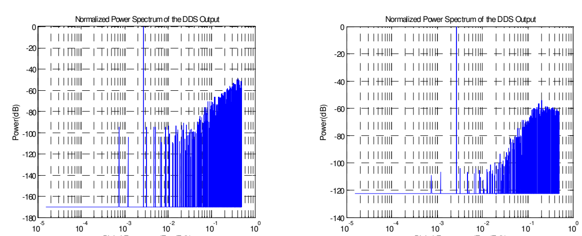


Fig. 6 simulated noise transfer function of MASH and feedforward ΣΔ modulators.


(a) NCO output spectrum with 3rd order MASH type delta-sigma modulation in frequency domain(1-bit quantizer).

(b) NCO output spectrum with frequency domain 3rd order *feedforward2* ΔΣ modulation (multi-bit quantizer).

Fig. 7 Comparison of Measured NCO output spectrum with different noise shaping effects.

Although feedforward modulator can increase in-band SINAD by a few dB, it has drawback of instability. In contrast, a MASH type modulator is good for its high speed, sharp slope and full input dynamic range, and it is always stable. The drawback of a MASH modulator lies on its fixed number of output bits. The error feedback has the same noise transfer function as that of MASH except for its lower speed. However, error feedback modulator can flexibly choose the number of output bits. Feedback modulator also has the same noise transfer function as MASH and it also has an advantage of a multi-bit quantizer, but it has stability problem. The proposed *feedforward2* ΔΣ modulator is good for both in-band and out-band performances, but its implementation requires more hardware and its speed is lower.

5. IMPLEMENTATION OF DDS WITH VARIOUS ΔΣ MODULATORS IN 0.35μM CMOS TECHNOLOGY

To compare the DDS performance with various ΔΣ modulations, we designed ΔΣ modulators including MASH1-1-1, 3rd order feedforward, feedback and error feedback ΔΣ modulators and error feedback ΔΣ modulator as shown in Fig. 4 in both frequency and phase domains. Different ΔΣ modulators can be selected individually while other ΔΣ modulators are turned off.

The proposed DDS with frequency domain and phase domain ΔΣ modulator was implemented in 0.35µm CMOS technology with two poly and four metal layers. A 16-bit accumulator is designed, and 8 phase bits are used for addressing the look-up ROM. The 12-bit current steering DAC is integrated to convert the ROM output to an analog signal. For 12-bit amplitude resolution in a conventional DDS without a ΔΣ modulator, at least 12 phase bits should be used, which requires a look-up ROM with 2^{12}x12 bits. The use of a ΔΣ noise shaper effectively reduces the required number of phase bits. Thus, we use only 8 phase bits to address the ROM, which reduces the ROM size by a factor of 2^4 or 16 times compared to that of a conventional DDS without a ΔΣ modulator.

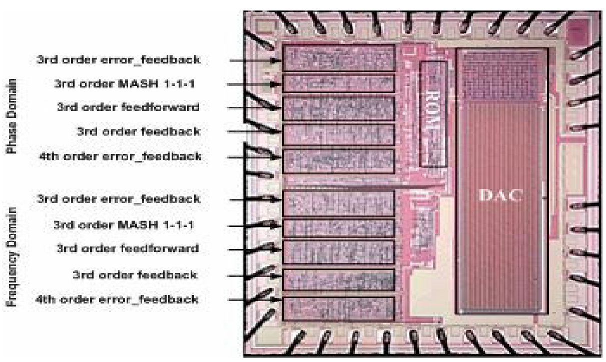
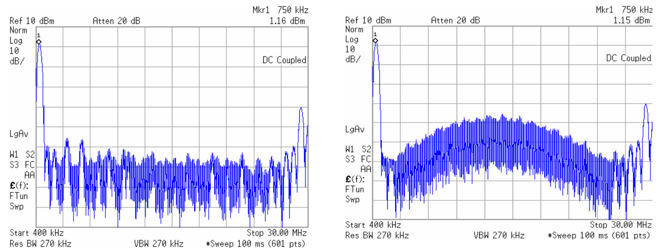


Fig. 8 Die photo of the CMOS DDS prototype chip with various ΔΣ modulators in frequency and phase domain.

The die photo of the fabricated CMOS ΔΣ DDS prototype chip is shown in Fig. 8. The total die area is 2x2.5mm^2, in which the DDS active core area is 1.7 x 2.1mm^2 including the DAC, and the rest of the die area is used for pads and ESD diodes. The 16-bit phase accumulator and ten ΔΣ modulators occupy 0.7x2.1mm^2 die area. The 2^8x12-bit ROM occupies only 0.1x0.8mm^2, which would be 16 times larger without the ΔΣ noise shaper. In a conventional DDS, the ROM normally takes the majority of the die area, whereas the ROM takes only a small portion of the total area in this DDS implementation, which clearly demonstrates the advantage of using ΔΣ noise shaping in DDS designs. The measured DDS output spectra with and without ΔΣ modulators are shown in Fig. 9. It's clear that the in-band spurs shown in Fig. 9(a) are reduced in Fig. 9(b).



(a) without ΔΣ modulator

(b) with frequency domain 3rd order MASH ΔΣ modulator

Fig. 9 Comparison of the measured output spectra for (a) conventional DDS without ΔΣ modulation and (b) proposed DDS with frequency domain ΔΣ modulation, f_o=750KHz, F_{clk}=30MHz.

Table 1. Performance comparison of ΔΣ modulators in frequency and phase domains of DDS. (In-band SFDR and SINAD are measured from NCO)

Frequency domain	High freq noise	No.of output bits	In-band SFDR (dBc)	In-band SINAD (dB)	Stability	Area (mm^2)	Speed (MHz)
3rd order MASH 111	poor	3	99.5	87.23	absolutely stable	0.112	180
3rd order Feedback	fair	≥ 3	99.4	86.11	fair	0.126	140
3rd order Feedforward	good	≥ 3	103	89.03	poor	0.161	150
3rd order EF	poor	≥ 3	99.5	87.23	good	0.147	140
4th order EF	poor	≥ 4	85.4	75.15	good	0.148	160
Phase domain							
3rd order MASH 111	poor	3	85.4	75.15	absolutely stable	0.112	180
3rd order Feedback	fair	≥ 3	84	74.57	fair	0.126	140
3rd order Feedforward	good	≥ 3	87	78.59	poor	0.161	150
3rd order EF	fair	≥ 3	85.4	75.15	good	0.147	140
4th order EF	poor	≥ 4	73	64.10	good	0.148	160

CONCLUSIONS

We have implemented various ΔΣ modulators in both frequency and phase domains in a CMOS DDS chip. The measured data demonstrates that frequency domain modulations have better SFDR and SINAD than their phase domain counterparts. We have also compare factors such as input range, quantizer bits, speed and stability for different type of ΔΣ modulators(table 1). Mash 1-1-1 ΔΣ modulator in frequency domain provides a good noise shaping means for DDS application with optimal speed, stability and input dynamic range. Feedforward ΔΣ modulator can provide both good in-band noise shaping and flat high frequency performance. Error-feedback takes advantage in its flexible output bit numbers.

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