Gauss elimination method using elementary row operations (EROs)

- Interchange any two rows \( r_i \leftrightarrow r_j \)
- Multiply any row with a non-zero constant “m”
  \[ r_i \leftarrow m \cdot r_i \]
- Add or subtract multiple of a row to another row
  \[ r_i \leftarrow m \cdot r_k \pm r_j \]

**Example of elementary row operations**

LAE Problem: \[ 2x + y = 4 \] \( ---(1) \)
\[ 8x + 2y = 12 \] \( ---(2) \)
Matrix problem: \[ [A] = \begin{bmatrix} 2 & 1 \\ 8 & 2 \end{bmatrix} \]
\[ [x] = \begin{bmatrix} x \\ y \end{bmatrix} \]
\[ [b] = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \]

EROs summary

- Interchange any two rows \( r_i \leftrightarrow r_j \)
- Multiply any row with a non-zero constant “k”
  \[ r_i \leftarrow k \cdot r_i \]
- Add or subtract multiple of a row to another row
  \[ r_i \leftarrow m \cdot r_k \pm r_j \]

**Example of EROs**

**Gauss Elimination (central idea)**

- Complete a series elimination steps using EROs (elementary row operations) to reduce the A matrix into an upper triangular form
- Do backward substitution to solve the \( Ax = b \) problem for the upper triangular system

Upper triangular matrix problem is simple to solve:

\[ \begin{bmatrix} 4 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \end{bmatrix} \]
\[ 4x + 2y + 3z = 10 \]
\[ 3y + z = 7 \]
\[ 2z = 2 \]
We can easily do backward substitution to evaluate \( x,y,z \) at \( 3,2,1 \)

**Example for ERO-3**

LAE Problem: \[ 2x + y = 4 \] \( ---(1) \)
\[ 8x + 2y = 12 \] \( ---(2) \)
Matrix problem: \[ [A] = \begin{bmatrix} 2 & 1 \\ 8 & 2 \end{bmatrix} \]
\[ [x] = \begin{bmatrix} x \\ y \end{bmatrix} \]
\[ [b] = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \]

EROs-3 \[ \begin{bmatrix} r2 \leftarrow 0.25 \cdot r2 - r1 \end{bmatrix} \]
Multiply eqn. (2) with 0.25 and subtract from (1)
\[ 2x + y = 4 \]
\[ -0.5y = -1 \]
\[ [A] = \begin{bmatrix} 2 & 1 \\ 0 & -0.5 \end{bmatrix} \]
\[ [x] = \begin{bmatrix} x \\ y \end{bmatrix} \]
\[ [b] = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \]

Note, ERO operations never altered the final solution. If you check, solution to \( x \& y \) are 1, 2 for all the matrix problems presented above

**Gauss Elimination Steps**

1) Write the augmented matrix
2) Select the first column
3) Interchange rows (which is an ERO) to get the largest number (absolute value) on the top row at location \((1,1)\). This number is known as the first “pivot”
4) Perform a series of EROs to eliminate all non-zero numbers in the column under the pivot to zero
5) Move to the next column and select the second pivot for location \((2,2)\)