1. Problem description

Figure 1 shows a group of slabs subjected to uniformly distributed loading. The cracking pattern of these slabs is examined and compared to the fracture line (or yield line) estimated from the classical yield line theory.

C = Column  B = Beam

Fig. 1 Reinforced concrete slabs under uniformly distributed loads
2. Material properties

The compressive strength of the culvert concrete is 28 MPa, while the yield stress of the reinforcement was 422 MPa.

The applied element method follows a discrete crack approach, in which, the material is represented by a group of springs located at the surfaces of the element. The springs represent the axial and shear behavior of the material as shown in Fig. 2. Figure 3 shows the constitutive models adopted in the ELS. As for modeling of concrete under compression, Maekawa compression model\(^2\), as shown in Fig. 3(a), is adopted. In this model, the initial Young's modulus, the fracture parameter, representing the extent of the internal damage of concrete, and the compressive plastic strain are introduced to define the envelope for compressive stresses and compressive strains. Therefore unloading and reloading can be conveniently described. The tangent modulus is calculated according to the strain at the spring location.

After peak stresses, spring stiffness is assumed as a minimum value to avoid negative stiffness. This results in difference between calculated stress and stress corresponds to the spring strain. These residual stresses are redistributed by applying the redistributed force values in the reverse direction in the next loading step.

For concrete springs subjected to tension, spring stiffness is assumed as the initial stiffness till reaching the cracking point. After cracking, stiffness of springs subjected to tension is set to be zero. The residual stresses are then redistributed in the next loading step by applying the redistributed force values in the reverse direction.

For concrete springs, the relationship between shear stress and shear strain is assumed to remain linear till the cracking of concrete. Then, the shear stresses drop down as shown in Fig. 3(b). The level of drop of shear stresses depends on the aggregate interlock and friction at the crack surface.

For reinforcement springs, the model presented by Ristic et. al.\(^3\) is used and it is shown in Fig. 3(c). The tangent stiffness of reinforcement is calculated based on the strain from the reinforcement spring, loading status (either loading or unloading) and the previous history of steel spring which controls the Bauschinger's effect. The main advantage of this model is that it can consider easily the effects of partial unloading and Bauschinger's effect without any additional complications to the analysis. The rupture strain of reinforcement is defined in the ELS. For more details about material models used, refer to Tagel-Din\(^4\).

The concrete is assumed to crack when the major principal stress reaches the tensile strength of concrete. The 3D state of stresses at each spring location and the major principal stress is calculated as shown in Fig. 4. After cracking, there are mainly two ways to consider the crack if the crack is not coinciding on the element surface:

1) to split the element into two elements and to generate new springs among the crack surface
2) to leave the element as it is and to redistribute stresses resulting from cracking

The first method is generally more accurate but it is very complicated and time consuming specially when talking about large problems and progressive collapse problems. The other method is not accurate but it gives reasonable results. If the shear cracks would govern the behavior, it would be advisable to reduce the element size in order to get cracks close to reality.
Fig. 2 Simulation of material as springs in a discrete crack

(a) concrete under axial stresses
(b) concrete under shear stresses
(c) Reinforcement under axial stresses

Fig. 3. Constitutive models of concrete and reinforcement
Fig. 4. Cracking criterion of concrete

Splitting Elements
(Complicated but accurate)

Redistributing stresses at element edge
(Simple but not accurate for shear transfer problems)

Fig. 5. Behavior of concrete elements after cracking
3. Results

Figure 6 illustrates the fracture (yield) line predicted by both classical yield line theory and ELS. As can be seen from Fig. 6, the fracture line predicted by the ELS is very similar to the classical yield line theory.
Fig. 6 Fracture line (Yield line) by yield line theory and ELS
4. Conclusions
Based on the analytical results, it can be concluded that the ELS can successfully predict the fracture (yield) line of reinforced concrete slabs under different boundary conditions.

5. References

2) Ristic, D., Yamada, Y., and Iemura, H. (1986), “Stress-strain based modeling of hysteretic structures under earthquake induced bending and varying axial loads”, Research report No. 86-ST-01, School of Civil Engineering, Kyoto University, Kyoto, Japan