Transmission Probability Control Game for Coexisting Random ALOHA Wireless Networks in Unlicensed Bands

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***Abstract*—In this paper, we consider N ad hoc networks, with nodes randomly and uniformly distributed, coexisting and sharing an unlicensed band. The objective of this paper is the optimization of the transmission probabilities in these networks to maximize the throughput in each network. Throughput in each system is limited by the self interference (the interference inflicted by a network on its own nodes) as well as the interference from the other networks. We consider the coexisting wireless networks being independent and random. Each network uses a random access method with its own transmission attempt probability. We show that the throughput in each network depends on the choice of transmission probabilities of all networks. We use the game theory approach to solve the transmission probability control problem. Simulation result indicate that at the Nash equilibrium a closely optimal solution can be obtained. We also consider the fairness and show that the transmission chances of systems are fairly equalized at the NE solutin. This contrasts with the centralized solution in which there is high discrepancy among the transmission probabilities.**

I. Introduction

Spectrum sharing in unlicensed band (e.g., ISM, UNNI, etc.) is considered to be a popular and succeful instance of coexistence of multiple wireless systems. The idea is that a set of networks, e.g., 802.11 networks, bluetooth systems, etc. are allowed to coexist and share the common spectrum. The systems do not cooperate and each selfishly tries to make the most out of the spectrum.

We consider a scenario in which the coexisting networks are randomly and independently distributed ad hoc networks. Random networks and point processes are commonly used to model wireless ad hoc networks with unknown nodes locations. We assume each network uses its own random access method. Throughput in each system is limited by the self interference (the interference inflicted by a network on its own nodes) as well as the interference from the other networks. The statistical modeling of interference has been investigated in both single and coexisting random wireless ad hoc networks [1-3]. We consider a simple Gaussian model in this paper whose parameters depend on the transmission probability [2].

In [4], the coexistence of multiple networks in an unlicensed is considered but neither the distribution of nodes and nor the access method is considered to be random.

In this paper, assuming a gaussian model for interference, we show that the parameters of the distribution of aggregate interference depend on values of transmission probabilities in all of the coexisting networks. We also show that the throughput is a non-trivial function of transmission probabilities. Therefore, the throughput obtained in each network depends on not only on its own choice of transmission probability but also on the transmission probabilities of all other networks. This suggests a game on transmission probabilities. This is unlike other games in wireless communications which are generally on transmission power levels of nodes within a single network (see e.g., [5]). We use the game theory methodology and prove the existence of Nash equilibrium and also propose an iterative method to obtain the NE. We also consider the fairness in terms of the transmission chance among the coexisting system and compare the fairness in a centralized solution with that obtained when the game theory approach is employed.

II. System Model and Notations

Network Model: We consider secondary random networks $\{1, 2, \cdots, N\}$ which are uniformly and independently distributed in a region with densities $\lambda_i,\ 1 \leq i \leq N$. We assume that these networks coexist and compete for the spectrum access right. We denote the distance between an arbitrary node of network i and its nth nearest neighbor in network i and network j as $R_{i,n}$ and $R_{i,j,n}$ respectively. Due to the independence assumption and since no correlation is considered to exist among the networks, we have $R_{i,j,n} \stackrel{d}{=} R_{j,n}$ where $\stackrel{d}{=}$ denotes equality in distribution. These random variables are shown to have Generalized Gamma distribution in [6].

Channel Model: We consider a deterministic distance-dependent path loss as $r^{-\alpha}$, where α is called the path loss exponent. On top of that, a random fading component (x) which is assumed to be i.i.d. across the interferers is considered in the model (i.e., the power is decayed as $x^2 r^{-\alpha}$). We use the notations: $x_{i,j,n}$ and $x_{i,n}$ to denote the fading component along the corresponding internodal links. The second moment of the fading component is assumed to be equal to 1 (i.e., $E\{x^2\} = 1$)

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Access Model: We consider the access method to be slotted ALOHA in each system. The transmission attempt probability in network i is assumed to be p_i.

Interference and Throughput: Assuming P_i is the transmit power level of the secondary network i, we define following variables as the interference power inflicted by each network on its own nodes or on the nodes of the other networks:

$$I_i = \sum_{n=1}^{\infty} P_i.\phi_{i,n}.x_{i,n}^2.R_{i,n}^{-\alpha},\ 1 \leq i \leq N \quad (1)$$

$$I_{i\to j} = \sum_{n=1}^{\infty} P_i.\phi_{i,n}.x_{j,i,n}^2.R_{j,i,n}^{-\alpha},\ 1 \leq j \neq i \leq N \quad (2)$$

where $\phi_{i,n}$ is a Bernoulli random variables with parameter p_i. It is clear from the above definitions that $I_i \stackrel{d}{=} I_{i\to j}$. Assuming a noise power of σ^2, the Signal power to Interference and Noise power Ratio (SINR) at a destination which is R meters away from its transmitter in network i can be written as:

$$SINR_i = \frac{P_i.x_i^2.R^{-\alpha}}{I_i + \sum_{j\neq i} I_{j\to i} + \sigma^2}.\ 1 \leq i \leq N \quad (3)$$

With a half-duplex assumption (a node can either transmit or receive data), the probabilistic throughput in the secondary network i will be

$$\tau_i = p_i(1-p_i)Pr\{SINR_i > \theta_i\},\ 1 \leq i \leq N \quad (4)$$

where θ_i is the $SINR$ threshold for successful detection in system i. τ_i is the probability that the nodes in network i receive successfully and can be interpreted as the probability of the nodes in network i being in the reception mode.

III. Interference Modeling

Interference modeling in random wireless networks is relatively an old problem [7]. The exact distribution of interference in a network with a Poisson field of interferers is an open problem and therefore approximation methods have been proposed in the literature [1-3], [7,8]. The interference characteristic function has been found in AWGN [7] and Rayleigh fading [7] channels. Probability density function of interference can be approximated from its cumulants, e.g., by using Edgeworth or Gram-Charlier series [9]. The other approach is to assume that interference originated from different interferers add up independently and therefore Central Limit Theorem is used to model the interference as a Gaussian RV. The Gaussian assumption simplifies the calculations as only the first two moments are required to completely characterize the interference. In this paper, we follow the second approach and assume a Gaussian model for interference.

For a random Poisson network with density λ, transmit probability of p, transmit power level of P and Rayleigh fading channel with $E\{x^2\} = 1$, the mean and variance of interference is found in [2] as

$$m = \frac{\lambda p d_0^2}{1 - 2/\alpha} \frac{P}{d_0^{\alpha}}, \quad (5)$$

$$v^2 = \frac{2\lambda\pi p d_0^2}{1 - 1/\alpha} \left(\frac{P}{d_0^{\alpha}}\right)^2 \quad (6)$$

where α is the path loss exponent and d_0 is the near field cut-off radius. No other nodes within radius d_0 around destination can transmit in the same time slot.

The distribution of $I_{i\to j}$ (or I_i) can be obtained by replacing the corresponding parameters of network i in (5) and (6). If we denote the mean and variance of $I_{i\to j}$ (or I_i) as (m_i, v_i^2), then, due to the independence assumption of networks, $I_i + \sum_{j\neq i} I_{j\to i}$ will be a Gaussian random variable with parameters $(\sum m_i, \sum v_i^2)$.

It is found in [2] that for $SINR = \frac{Px^2R^{-\alpha}}{I+\sigma^2}$, x Rayleigh random variable with $E\{x^2\} = 1$ and I a Gaussian random variable with parameters (m, v^2), we have

$$Pr\{SINR > \theta\} = exp\left(-\frac{\theta(m+\sigma^2)}{PR^{-\alpha}}\right) exp\left(\frac{\theta^2 v^2}{2P^2R^{-2\alpha}}\right) \times Q\left(\frac{\theta v}{PR^{-\alpha}} - \frac{m}{v}\right) \quad (7)$$

Using this result, $Pr\{SINR_i > \theta_i\}$ can be found by replacing the corresponding parameters. Subsequently, we can find τ_i using (4).

IV. Transmission Probability Control

It can be seen from the definition of interference random variables in (1)-(2), that the parameters of the distribution of I_i or $I_{i\to j}$ depend on the choice of p_i by network i. Also, from (3), the parameters of the distributions of $SINR_i$ and therefore τ_i depend on the choices of transmission probabilities and power levels by all of the networks which we show as $\mathbf{p} = [p_1, p_2, \cdots, p_N]$ and $\mathbf{P} = [P_1, P_2, \cdots, P_N]$. We can explicitly write τ_i as $\tau_i(\mathbf{p}; \mathbf{P}, \theta_i)$. In this paper, we assume that the power level chosen by each network and also the SINR detection threshold are system-defined and fixed. On the other hand, the decisions made by each network on its transmission probability influences not only its own throughput but also the throughput obtained by the other networks. This suggests a game structure as we discuss it in the next section. In a centrally controlled approach, the optimum transmission probability vector, $\mathbf{p}^*$, that maximizes the total throughput is found:

$$\mathbf{p}^* = \underset{\mathbf{p}\in[0,1]^N}{\text{argmax}} \sum_{i=1}^{N} \tau_i(\mathbf{p}; \mathbf{P}, \theta_i) \quad (8)$$

V. Transmission Probability Control Game (TPCG)

When the networks independently make decision on their transmission probability, and assuming that each network makes its decision rationally and behaves strategically [10], the problem has the structure of a strategic game [10]. We call this game the Transmission Probability Control Game (TPCG). In the following subsections, we formulate the problem using the game theory definitions and terminology and show (proove?) that a Nash Equilibrium (NE) esists and propose an iterative optimization algorithm to find the NE.

A. Game Formulation

1) Players: The set of secondary networks $\{1, 2, \cdots, N\}$, sharing the common spectrum can be considered as the palyers of the game. Each network tries to choose a transmission probability which optimizes its throughput. On the other hand, an optimum choice for one network, may deviate the throughput of the other networks from optimality.

2) Actions: Each network can choose a transmission probability within the $[0, 1]$ interval. The action set for network i is therefore the $[0, 1]$ interval (i.e., $p_i \in [0, 1]$). The action space for the game is therefore $[0, 1]^N$. An action profile, $\mathbf{a} \in [0, 1]^N$, is a vector whose ith element is the transmission probability chosen by network i.

3) Utility Function: The utility function for user i can be defined as its throughput.

B. Existenxce of Nash Equilibrium

Not every strategic game has a Nash equilibrium. In the following, we present a well-known theorem by Glicksberg and Fan [11, 12], which states the conditions under which a strategic game has at least one NE.

Theorem 1. *A strategic game has at least one NE if for all* $1 \le i \le N$*,*

- *The set of actions of player i is nonempty compact convex subset of a Euclidean space.*
- *The utility function τ_i is quasi-concave on the actions set of player i and continuous on the action space.*

In the following propositions, we show that the conditions stated above hold and the Glicksberg-Fan theorem can be applied to prove the existence of Nash equilibrium.

Proposition 1. *The set of actions of player i, i.e., $[0, 1]$, is nonempty, compact and convex subset of a Euclidean space.*

Proof: The actions set is by definition nonempty and convex as it is an interval on the real line. Moreover, the set $[0, 1]$ includes the boundary points, i.e., 0 and 1. Therefore, it is bounded and hence compact. ■

Proposition 2. *The utility function τ_i is quasi-concave on the actions set of player i, i.e., $[0, 1]$ and continuous on the action space, i.e., $[0, 1]^N$.*

Proof: Using (5) and (6), we can write

$$m_i = X_i p_i, \text{where, } X_i = \frac{\lambda_i d_0^2}{1 - 2/\alpha} \frac{P_i}{d_0^2}, \tag{9}$$

$$v_i^2 = Y_i p_i, \text{where, } Y_i = \frac{2\lambda_i \pi d_0^2}{1 - 1/\alpha} \left(\frac{P_i}{d_0^\alpha} \right)^2. \tag{10}$$

We also define $Z_i \triangleq \frac{P_i R^{-\alpha}}{\theta_i}$. Using these notations, and considering τ_i as a function of p_i only (i.e., taking the rest of variables as parameter), we can write

$$\tau_i(p_i) = p_i(1 - p_i) \exp\left(-\frac{\sum X_j p_j + \sigma^2}{Z_i} \right) . \exp\left(\frac{\sum Y_j p_j}{2Z_i^2} \right) \times Q\left(\frac{\sum Y_j p_j - Z_i \sum X_j p_j}{Z_i \sqrt{\sum Y_j p_j}} \right). \tag{11}$$

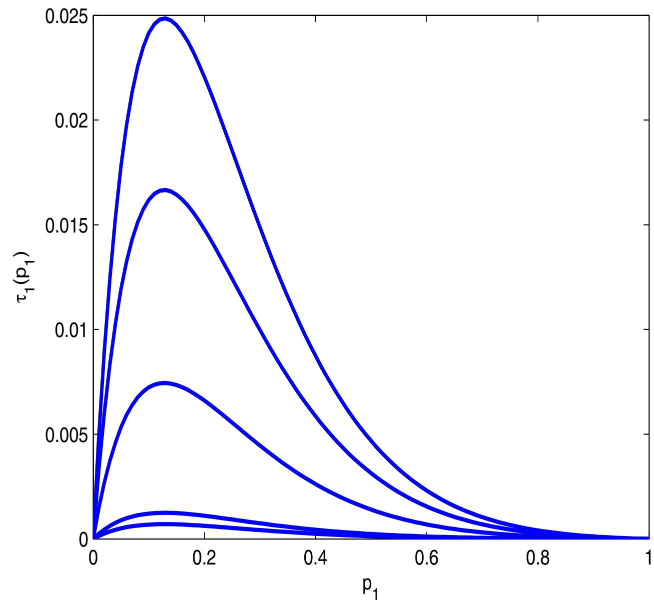

Fig. 1: The unique global maximum property of $\tau_1(p_1)$

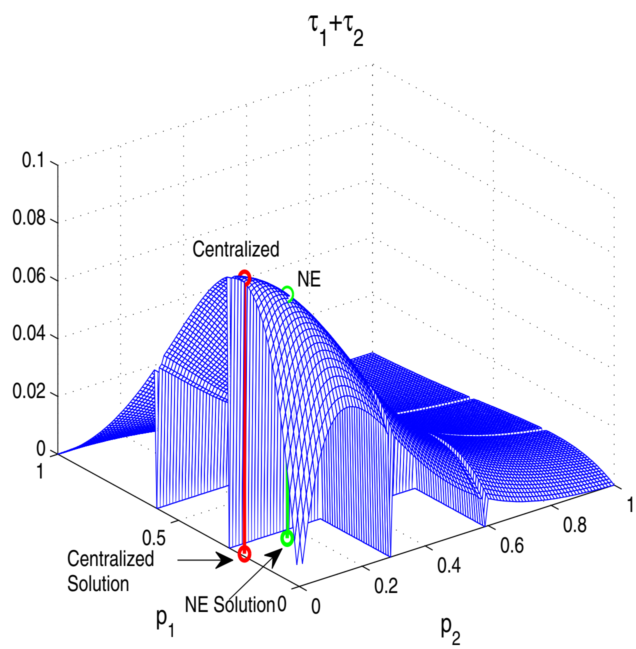

Fig. 2: NE and Centralized Solutions ($\alpha = 4, P_1 = 0.2$mW, $P_2 = 0.1$ mW, $\theta_1 = \theta_2 = 6$ dB, $R = 1$ and $\sigma^2 = 5$ fW)

For practical values of parameters, we can show that the above function has a unique global maximum (See Figure 1). Using the same argument as in the Proposition 1 of [13], we can say that τ_i is quasi-concave on the action space of player i. Moreover, τ_i is continuous on the action space of the game. ■

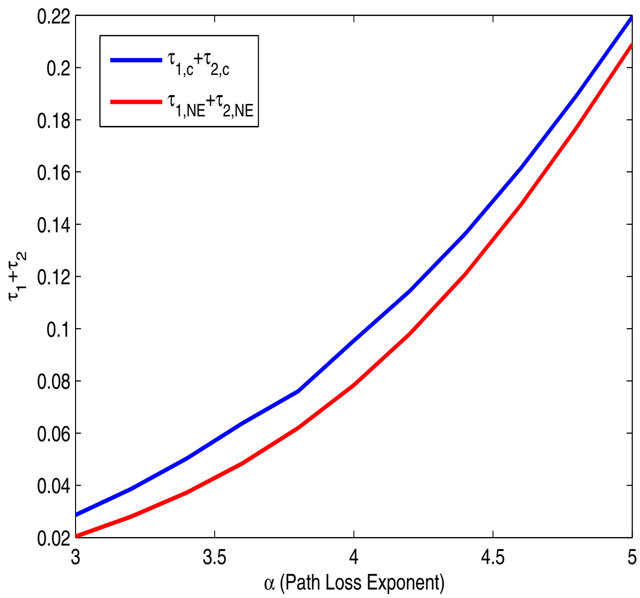

Fig. 3: Sum-throughput for the NE and Centralized Solution versus Path Loss Exponent

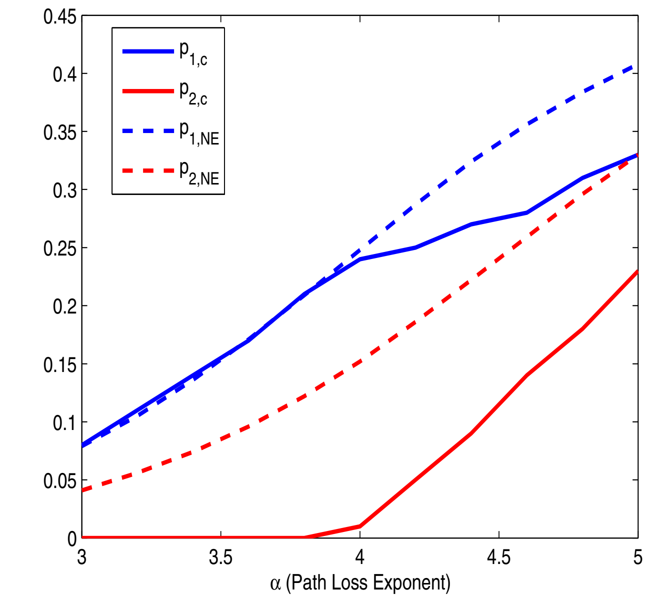

Fig. 4: Transmission probabilities for the NE and Centralized Solution versus Path Loss Exponent

C. Iterative Optimization Algorithm to Find NE

Given the transmission probability of all of the other networks at the NE, i.e., $p_j, j \neq i$, network i needs to solve a scalar optimization and find the action (i.e., transmission probabaility) that maximizes its throughput. This suggest a distributed optimization approach using an iterative alogorithm. In the following, we propose an iterative approach to find the NE. We verify through simulation that the algorithm is insensetive to the starting point which suggests the uniqueness of the NE.

- STEP 1: Set the starting action profile as $\mathbf{p}^s = [p_1^s, p_2^s, \cdots, p_N^s]$.
- STEP 2: Set $\mathbf{p} = \mathbf{p}^s$.
  Find $p_i^* = \underset{0<p_i<1}{\text{argmax}} \; \tau_i(p_i, \mathbf{p}_{-i}; \mathbf{P}, \theta_i), i = 1, 2, \cdots N$. In this equation, $\mathbf{p}_{-i}$, denotes a vector whose elements are $p_j, \; j \neq i$.
- STEP 3: Check for convergence: if there is negligible change in the action profile, then terminate. Otherwise set $\mathbf{p}^s = \mathbf{p}^*$ and return to STEP 2.

VI. Simulation Results

In our simulations, we consider the coexistence of two systems. We consider these networks randomly and independently distributed with parameters $\lambda_1 = 1$ and $\lambda_2 = 2$ respectively. The transmission power levels are assumed to be $P_1 = 100\mu$W and $P_2 = 200\mu$W respectively. The noise power is considered to be $\sigma^2 = 5$fW and SINR threshold for signal detection is assumed to be $\theta = 6$dB.

In Figure 1, we have plotted $\tau_1(p_1)$ versus p_1 for the given values of parameters. As we can observe, $\tau_1(p_1)$ has a unique global maximum. The value of p_2 is chosen to be a sample of a uniform random variable in $[0, 1]$. We have considered multiple realizations of p_2 and as we can see $\tau_1(p_1)$ has a unique global maximum for different values of p_2. By symmetry, $\tau_2(p_2)$ also has a unique global maximum.

In Figure 2, we can see that the centralized solution results in an unfair situation in which the network 2 is assigned a zero transmission chance. This can also be seen from Figure 4 in which $p_{2,c}$ (centralized solution for p_2) is almost zero for path loss exponent less than 4.

The Nash equilibrium (NE) is shown to result in a fair chance of transmission among coexisting networks as the transmission probability for both systems are comparable. This can be seen from Figures 2 and 4. This contrasts with the centralized solution which results in slightly more sum-throughput at the cost of a significant unfairness in the values of transmission probability (see Figure 2).

In Figure 3, we have plotted the sum-throughput for the NE solution along with the centralized solution. The result shows that there is a fairly constant and small gap between the resultant sum-throughput in these two solutions while the NE solution benefits from the fairness property.

VII. Conclusion

In this paper, Game theory methodology is used to analyze the transmission probability control problem among coexisting random ad hoc networks. The throughput in each network depends on not only the transmission probability of that network but also on the transmission probabilities of all of the other networks. The Nash Equilibrium solution is shown to be closely optimal. Moreover, the fairness in system in terms of the chance of transmission significantly increases by using the game theory solution.

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