Cooperative Spectrum Sharing for a Primary Network with Capacity Constraint

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Abstract—**In this paper, we propose a cooperative framework for spectrum sharing between a secondary random ad hoc network and a primary network with capacity constraint. While the interference from secondary network must satisfy a power constraint, there is no stipulation on its higher order statistics. Considering a generalized Gaussian model for interference, we show that the capacity of a primary link can be improved by increasing the kurtosis of interference. We show that by using power control, the secondary network can increase the kurtosis of its interference and thereby can help the primary links to maintain their minimum capacity constraint. Secondary network is allowed to share the spectrum for a fraction of time proportional to the capacity gain obtained by the primary link. Simulation results show that by cooperation from the secondary network, both secondary and primary networks can achieve their goals in accessing the spectrum and satisfying the capacity constraint.**

I. INTRODUCTION

In a cognitive wireless network, a secondary unlicensed network can coexist and share the spectrum with a primary licensed network provided that the aggregate interference power originated from the secondary network does not exceed a predefined threshold [1]. This constraint, however, is not concerned with the exact statistics of interference.

The aggregate interference power from a Poisson field of interferers is known to be non-Gaussian, although no closed-form result has been found for its distribution [2]. If the interfering nodes use a fixed transmission power level however, the cumulants of interference power can be found in closed form [3]. The cumulants of aggregate secondary interference power has also been found when the spectrum sensing is incorporated in the model [4]. The results indicate that the interference power has a skewed distribution. In [5], we show that the aggregate secondary interference power can be modeled as sum of Gaussian and non-Gaussian random variables.

Recently, considering an Ultra Wideband (UWB) scenario, in which multiuser interference (MUI) is known to be non-Gaussian, it has been shown that by using higher order statistics of interference and designing receivers which are adapted to the non-Gaussian interference, capacity can improve dramatically [6]. Assuming a generalized Gaussian distribution, the results in [6] show that for a fixed SNR, capacity is maximized when the kurtosis is maximum and the minimum capacity corresponds to the case that the generalized Gaussian distribution degenerates to Gaussian. Reference [7] considers a UWB antenna design which affects the distribution of interference and thereby improves the capacity.

Cooperation from secondary network has recently been considered in literature. In [8], the authors consider leasing the spectrum for a portion of time to a secondary ad hoc network in return for the cooperation of the secondary network in the form of distributed space-time coding. Reference [9] uses a similar model with the difference that the primary's obtained revenue and the secondary's payment are also incorporated in the model. In [10], the secondary network assists the primary nodes by sending feedback and helping them to use beam nulling to minimize their interference at the primary receivers.

In this paper, we propose a framework, in which the secondary network cooperates with the primary network by changing its statistical properties in a way that improves the capacity of primary links. We show that the kurtosis of interference from secondary network can be increased when the secondary nodes use power control and assign optimal power levels by solving a nonlinear optimization problem. Using a generalized Gaussian model for secondary interference, we obtain numerical results that show the capacity improves considerably when the interference kurtosis is maximized. Proportional to the capacity gain obtained by a primary link, the spectrum is taken over by the secondary network for a fraction of time. We show that using this cooperative strategy, both secondary and primary networks can achieve their goals in accessing the spectrum and satisfying the capacity constraint.

II. SYSTEM MODEL AND NOTATIONS

We consider the uplink of a primary cellular network where the primary Base Station (BS) is located at the center of a circular cell with radius R (see Figure 1). A secondary ad hoc network coexists and shares the spectrum with the primary network. The nodes in the secondary network are distributed according to a Poisson Point Process with density λ. We assume that the primary BS is located at the origin and consider the distance to its ith closest secondary neighbor in the cell as r_i. The secondary network can coexist with the primary link given that its aggregate interference power is less than a system-defined threshold K. This constraint affects the statistics of secondary interference only up to the second order, i.e., mean and variance. The primary network, therefore, is not interested in the exact statistics of secondary network interference as long as its power constraint is satisfied.

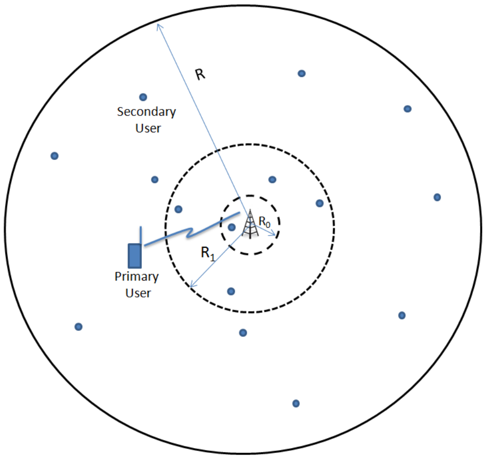

Fig. 1: System Model

A. Primary Capacity Constraint

We consider primary link is subject to a minimum capacity constraint of C_0. This constraint can be due to multimedia traffic (e.g. User Created Content (UCC) traffic in the uplink of 3G/4G applications [11]) which require a minimum or fixed capacity.

B. Access Model

We divide time into frames of duration T and assume that the primary link experiences block Rayleigh fading h, with $E\{h^2\} = 1$, which is fixed during each frame. The received power level at the primary receiver from the primary transmitter is P_P and the target capacity for the primary link is considered C_0 which is found with the assumptions that Signal to Interference Ratio (SIR) at the primary receiver is $\frac{P_P}{K}$[1]. The instantaneous SIR of the primary link is on the other hand $\frac{P_P h^2}{K}$ which leads to a fluctuating capacity.

We are interested to see how the secondary nodes can help the primary link to maintain its target capacity by cooperation with the primary. The kind of cooperation we consider in this paper is by modifying the interference statistics, while keeping the interference power constraint satisfied, in a way that improves the primary link's capacity. In return for this cooperation, the secondary network is allowed to access the spectrum for a fraction of time proportional to the capacity gain obtained by the primary link. The secondary network is informed by the primary BS that for what fraction of time (if any) the spectrum will be assigned to it in compensation for its cooperation and the secondary network has the choice to cooperate or not cooperate. In section IV, we discuss how primary link capacity is influenced by secondary interference statistics.

C. Interference Amplitude

We focus on the interference *amplitude*. The received interference signal from ith nearest secondary neighbor is $\frac{\sqrt{P X_i}}{r_i^{\alpha/2}} a_i \cos(\omega_0 t + \varphi_i)$ where $\alpha/2$ is the amplitude loss exponent[2], P is the transmit power level, ω_0 is the carrier frequency, φ_i is a random phase shift which we assume to be uniformly distributed in $[0, 2\pi]$, X_i is the Rayleigh fading with $E\{X_i^2\} = 1$ and a_i is the information symbol. Without loss of generality, we consider a BPSK modulation with equiprobable symbols 1 and -1. Assuming that the received interference is sampled at time $t = 0$, the received interference amplitude from ith nearest secondary neighbor is $\frac{\sqrt{P X_i}}{r_i^{\alpha/2}} a_i U_i$ where $U_i \triangleq \cos(\varphi_i)$. Using transformation of random variables we can show that U_i has the following pdf:

$$f_{U_i}(u) = \frac{1}{\pi\sqrt{1-u^2}} \qquad -1 < u < 1.$$

D. Interference Zones

In [5], we show that the interference power from a Poisson wireless network comprises of a non-Gaussian component (mainly from the interfering nodes close to the receiver) and a Gaussian component (from the rest of the interfering nodes) and that the two components can be assumed independent. Accordingly, we divide a cell into three regions. $\{r < R_0\}$ is the prohibition region where no secondary node is allowed to transmit. This rule is enforced to avoid singularity in interference behavior for close distances [4]. The other two regions are $\{R_0 \le r < R_1\}$ and $\{R_1 \le r < R\}$. We denote the interference originated from these regions as I_1 and I_2 respectively. For a large enough R_1, I_1 and I_2 can be assumed independent [5].

III. SECONDARY INTERFERENCE STATISTICS

The aggregate interference received at the primary receiver will be: $I = I_1 + I_2$. By defining

$$S_1 \triangleq \{i : \ R_0 \le r_i < R_1\}$$
$$S_2 \triangleq \{i : R_1 < r_i \le R\}$$

we have

$$I_j = \sum_{i \in S_j} \frac{\sqrt{P_i} X_i U_i a_i}{r_i^{\alpha/2}} \qquad j = 1, 2$$

The regulatory constraint on the interference power at the primary receiver is

$$E\{I^2\} \le K \tag{1}$$

We consider two cases when the secondary network employs power control and the nodes inside the two interference regions employ different power levels P_1 and P_2 and the case that

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[1] We assume negligible multiple access interference from primary network and ignore its effect in finding the SIR at the primary receiver.

[2] α is the power loss exponent which takes value between 2 and 6 depending on the propagation environment.

there is no power control and the nodes inside the two regions use the same power level. We show that using power control enables the secondary network to control the value of its kurtosis. We later show that increasing the kurtosis of interference leads to the improvement in primary link's capacity. Therefore, power control for secondary network, not just for satisfying the interference power constraint but also by changing the interference statistical properties, is a means for secondary network to cooperate with and help the primary link.

A. *Power Control Scenario*

We first consider a more general case when the secondary nodes employs power control. By using Campbell's theorem [12], the cumulants of $I_1 = \sum_{i\in S_1} \frac{\sqrt{P_1}X_iU_ia_i}{r_i^{\alpha/2}}$ can be found as

$$\kappa_n(I_1) = 2\pi\lambda E\{a_i^n\}\int_{-1}^{1}\int_{0}^{\infty}\int_{R_0}^{R_1}\left(\frac{\sqrt{P_1}xu}{r^{\alpha/2}}\right)^n r dr f_X(x)dx \times f_U(u)du$$

After simplification, we have $\kappa_n(I_1) = A_nP_1^{n/2}$ where

$$A_n = 2\pi\lambda E\{U_i^n\}E\{a_i^n\}E\{X_i^n\}\left(\frac{R_1^{2-n\alpha/2} - R_0^{2-n\alpha/2}}{2-n\alpha/2}\right) \tag{2}$$

For Rayleigh fading with $E\{X_i^2\} = 1$, we have $E\{X_i^n\} = \Gamma(1+n/2)$. Also, we have $E\{a_i^n\} = \begin{cases} 1 & \text{even } n \\ 0 & \text{odd } n \end{cases}$ and

$$\begin{aligned} E\{U_i^n\} &= \int_{-1}^{1}\frac{u^n\ du}{\pi\sqrt{1-u^2}} \\ &= \frac{1-(-1)^{n+1}}{\pi(n+1)}\,{}_2F_1\left(\frac{1}{2},\frac{n+1}{2};\frac{n+3}{2};1\right) \end{aligned}$$

where ${}_2F_1(.)$ is the hypergeometric function [13].

Similarly, we have $\kappa_n(I_2) = B_nP_2^{n/2}$ and B_n can be found from (2) with R_0 and R_1 replaced by R_1 and R respectively. Using the independence of I_1 and I_2, we have $\kappa_n(I) = \kappa_n(I_1) + \kappa_n(I_2)$. Note that since $\kappa_1(I_1) = 0$ and $\kappa_1(I_2) = 0$, we also have $\kappa_1(I) = 0$ and the interference is zero mean.

The kurtosis of interference in this case will be

$$\text{Kurtosis}(I) = \frac{\kappa_4(I)}{\kappa_2(I)^2} = \frac{A_4P_1^2 + B_4P_2^2}{(A_2P_1 + B_2P_2)^2} \tag{3}$$

B. *No Power Control Scenario*

In the no power control scenario, the secondary nodes use the same power level and we have $P_1 = P_2$. In this case, using (3)

$$\text{Kurtosis}(I) = \frac{A_4 + B_4}{(A_2 + B_2)^2}$$

The kurtosis is therefore a fixed value and independent of the choice of the power level. The power level should be chosen such that the interference power constraint is satisfied.

C. *Interference Power Constraint*

The interference power constraint given in (1) can be written as

$$\begin{aligned} E\{I^2\} &= \kappa_2(I) + \kappa_1^2(I) \\ &= \kappa_2(I_1) + \kappa_1^2(I_1) + \kappa_2(I_2) + \kappa_1^2(I_2) + 2\kappa_1(I_1)\kappa_1(I_2) \\ &= (A_2 + A_1^2)P_1 + (B_2 + B_1^2)P_2 \\ &\leq K \end{aligned} \tag{4}$$

For the no power control scenario with $P_1 = P_2 = P$, using (4), we need to have

$$P \leq \frac{K}{A_2 + A_1^2 + B_2 + B_1^2}$$

IV. CAPACITY MAXIMIZATION PROBLEM

In this section we consider the effect of secondary interference statistics on the capacity of primary link. The classic assumption for interference is taking it as a Gaussian random variable and using the well-known capacity of Gaussian interference channels. This assumption, however, is the most conservative one due to the maximum entropy property of Gaussian random variables which leads to minimum capacity. This can be seen from Shannon's capacity bound for the general additive channel [14]

$$W\log_2\left(1 + \frac{P}{N_1}\right) \leq C \leq W\log_2\left(\frac{N}{N_1} + \frac{P}{N_1}\right),$$

where P is the received signal power, N_1 is the entropy power of interference/noise and N is the interference/noise power. To find entropy power, complete statistics of interference is required. However, for a given N, we always have $N_1 \leq N$ for a general probability distribution where the equality holds for Gaussian distribution. This is known as the maximum entropy property of Gaussian distribution. On the other hand, both upper and lower bound of capacity decrease with entropy power. For the case of Gaussian interference/noise, capacity is minimum and the bound degenerates to equality ($N = N_1$ and $C = \log_2(1 + \frac{P}{N})$). We can see that non-Gaussianity of interference can indeed be advantageous to the link capacities.

A common measure used for non-Gaussianity is the kurtosis which is defined as the ratio of fourth cumulant to the square of second cumulant. In the next section, we model the interference using a generalized Gaussian model. This is a flexible model whose parameters depend on mean, variance and kurtosis [15]. For kurtosis equal to 0, the model degenerates to Gaussian. We find the capacity of primary link assuming that interference follows a generalized Gaussian model and show that maximizing the capacity is equivalent to maximizing the kurtosis of interference for a given variance.

A. *Capacity with Generalized Gaussian Interference*

Assume that the primary link uses BPSK modulation. Given the fading value of h, the received signal at time slot n will be $y_n = I_n + b_n$ where $b_n \in \{s_0 = \sqrt{P_P}h, s_1 = -\sqrt{P_P}h\}$ and the link SIR is $\frac{P_Ph^2}{K}$. Using the generalized Gaussian assumption for interference, the conditional pdf of the received signal at time slot n is

$$f_{y_n|b_n}(y) = \frac{\beta}{2\gamma\Gamma(1/\beta)} e^{-\left(\frac{|y-b_n|}{\gamma}\right)^{\beta}}$$

where $\beta > 0$ is the shape parameter and $\gamma > 0$ is the scale parameter [13]. The variance and kurtosis of generalized Gaussian distribution are given by

$$\begin{aligned}\text{variance} &= \frac{\gamma^2\Gamma(3/\beta)}{\Gamma(1/\beta)},\\ \text{Kurtosis} &= \frac{\Gamma(5/\beta)\Gamma(1/\beta)}{\Gamma(3/\beta)^2}\end{aligned}$$

Therefore, by having the kurtosis and the variance (which equals K), we can have the parameters γ and β and thereby find the above conditional pdf. Using the formula for the capacity of a discrete memoryless channel with continuous valued output given in [16], we can find the capacity of primary link as follows:

$$C = \frac{1}{2}\sum_{i=0}^{1}\int_{-\infty}^{\infty} f_{y|s_i}(y)\log_2\left(\frac{2f(y|s_i)}{f(y|s_0)+f(y|s_1)}\right)dy \quad (5)$$

In section VI, we show that for a given SIR (i.e., for a given h and K), the capacity is an increasing function of kurtosis. In the following subsection, we find how power control can be used to improve the kurtosis.

B. Kurtosis Maximization Problem

We seek to maximize the kurtosis of secondary interference subject to a constraint on the maximum secondary interference power by allocating optimal power levels, P_1 and P_2, to the nodes inside interference zones. The kurtosis of interference and interference power constraint are given in (3) and (4) respectively. The optimization problem can be written as

$$\begin{aligned}&\text{Maximize } kurtosis(I) \text{ subject to}\\ &E\{I^2\} \le K\\ &P_1, P_2 \ge 0\end{aligned}$$

This is a nonlinear constrained optimization problem as the objective function is a nonlinear function of P_1 and P_2. We use the MATLAB optimization toolbox to solve this problem and find the maximum kurtosis and the optimal power levels.

V. COOPERATIVE SPECTRUM SHARING

The primary link is subject to a capacity constraint C_0 which is obtained at SIR= $SIR_0 = \frac{P_P}{K}$ and with the no power control assumption from the secondary network, in which case the kurtosis of interference equals $\frac{A_4+B_4}{(A_2+B_2)^2}$. Due to the block fading assumption however, the SIR at each frame is $\frac{P_P h^2}{K}$ which leads to a time-varying capacity C_n.

We assume that at every frame, the primary BS determines for what fraction of frame time the spectrum will be given for the exclusive use of secondary in case of its cooperation. Based on that, the secondary network has the choice to cooperate or not to cooperate[3]. In case of cooperation, it uses power control so that its interference has maximum kurtosis. We denote the resulting capacity as C_o and define the gain in capacity as $g = max\{0, \frac{C_o - C_0}{C_0}\}$. In reward for this cooperation, the secondary network is allowed to acquire the spectrum completely for its own use for a portion δ of a frame duration where

$$\delta = \frac{g}{g+1}$$

In case $\delta = g = 0$, the secondary network may decide not to cooperate. In case the secondary network decides to cooperate, there will be a win-win situation where a primary link is aided by the secondary network to maintain its minimum capacity constraint and the secondary network can use the spectrum for a portion of time depending on how much extra capacity (i.e., above the target capacity) it can provide for the primary link.

VI. NUMERICAL AND SIMULATION RESULTS

In this section we provide numerical and simulation results to verify the effectiveness of the proposed cooperative framework. We consider a single primary link which is subject to a Poisson field of secondary interferers with density $\lambda = .01$. The maximum secondary interference power is assumed to be $K = 100\mu W$. We consider $R_0 = 1$m, $R_1 = 10$m, $R = 1000$m and α, the path loss exponent is 3. The received power level of primary BS is assumed to be 1mW which leads to a target SIR of 10 dB.

In Figure 2, we have numerically evaluated the integral in (5) and have plotted the capacity versus kurtosis for different values of SIR. The results show that for a given SIR (i.e., for a given h and K), the capacity is an increasing function of kurtosis. This is an indication that if the secondary network has a way to increase the kurtosis of interference, while keeping the maximum interference power constraint of K satisfied, it can help to improve the capacity of primary link. For example, the target capacity (obtained at $SIR_0 = 10$ dB with minimum kurtosis equal to 0.0121 which corresponds to the no power control case) can be maintained at $SIR = 5$ dB if the kurtosis can increase to 3.5. At the optimum kurtosis which is 3.6338, the obtained capacity at $SIR = 5$ dB is even larger than the target capacity obtained at $SIR_0 = 10$ dB and with no power control (i.e., at minimum kurtosis).

Figure 3, shows the capacity with power control (i.e., with optimum kurtosis of 3.6338) and with no power control (i.e., at minimum kurtosis). The result shows that while at the case of no power control, the primary link can not maintain its capacity for a large portion of time, the capacity can increase considerably using power control and increasing the kurtosis. In Figure 4, the corresponding values of δ, i.e. the portion of time that spectrum can be used by the secondary network is shown, The result show that except time slots 4 and 15 where due to deep fading, the primary capacity constraint can not be maintained even if the secondary network cooperates, the secondary network can access the spectrum for a some fraction of time during the other time slots.

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[3] The decision to cooperate or not to cooperate, can be either jointly made by the secondary nodes or by a centralized authority in the secondary network.

VII. CONCLUSION

In this paper we consider a primary network with a constraint on capacity. Due to fading, a primary link experiences fluctuation in capacity and therefore can not maintain its minimum capacity constraint. On the other hand, a secondary random ad hoc network coexists and is interested to share the spectrum with the primary network. First, we show that the capacity of primary link, not only depends on the interference power from secondary network, but also depends on its higher order statistics. Particularly, we show that using a generalized Gaussian model for interference, capacity of primary link improves if the kurtosis of interference can be increased. Based on this finding, we propose a cooperative framework in which the secondary network optimizes its kurtosis using power control and the primary network lets the secondary network use the spectrum exclusively for a fraction of time which is proportional to its capacity gain. Our results show that by cooperation from the secondary network, both secondary and primary networks can achieve their goals in accessing the spectrum and satisfying the capacity constraint.

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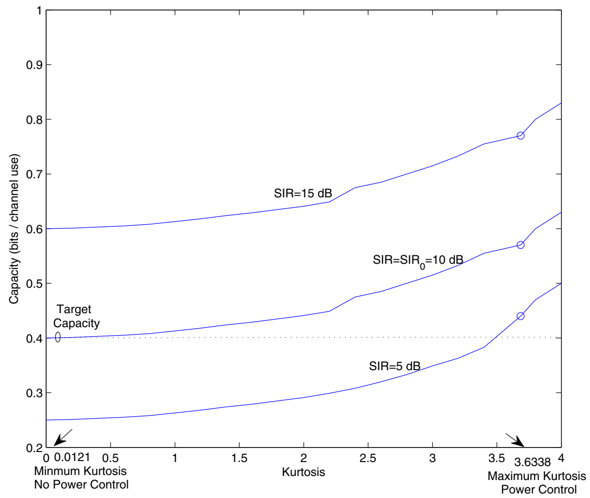

Fig. 2: Capacity vs Kurtosis

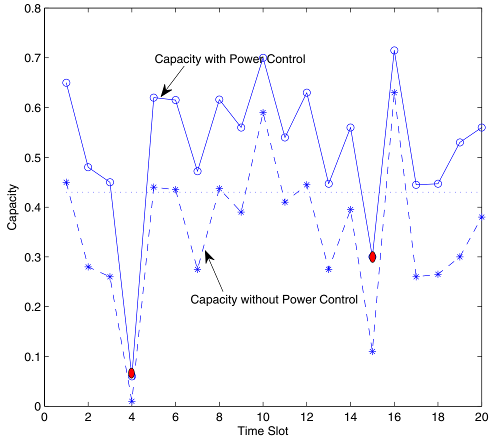

Fig. 3: Capacity with and without Power Control

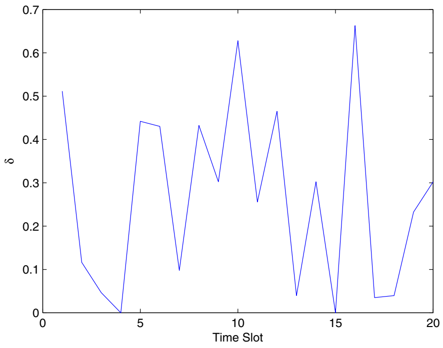

Fig. 4: Fraction of Frame Time for Secondary Network’s Exclusive Access (δ)