Statistical Shaping of Interference to Maximize Capacity in Cognitive Random Wireless Networks

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***Abstract*—It is known that in a cognitive random wireless network, the aggregate interference from secondary network is not Gaussian. Using Shannon's bound for the capacity of general additive channels, Gaussian interference/noise is the worst case due to its maximum entropy property which leads to minimum capacity. Therefore, it is favorable to have the distribution of interference as far from Gaussian as possible. In this paper, we seek to deviate the statistics of secondary interference from Gaussianity by maximizing its kurtosis while a regulatory constraint is satisfied to keep the interference power below a threshold. For this purpose, we use power control for statistical shaping of interference. Considering a Gaussian and a non-Gaussian interference zone around primary receiver and assigning a fixed power level for the interfering nodes inside each zone, we show that optimum power levels are found by solving a constrained nonlinear optimization problem. The results show that using optimum power levels, the kurtosis increases significantly compared to the no power control case. Moreover, considering a generalized Gaussian model for interference, we show that the capacity of primary link improves dramatically by using the optimum power levels.**

I. INTRODUCTION

In cognitive radio, the aggregate interference power from the secondary network inflicted on a primary node should not exceed a system-defined threshold. In a cognitive random wireless network, distribution of secondary nodes is according to a spatial point process which is usually assumed to be Poisson Point Process (PPP) due to its ease of mathematical tractability. The aggregate interference power from a Poisson field of interferers is known to be non-Gaussian, although no closed-form result has been found for its distribution [1]. If the interfering nodes use a fixed transmission power level however, the cumulants of interference power can be found in closed form [2]. The cumulants of aggregate secondary interference power has also been found in a cognitive radio set-up where the spectrum sensing is incorporated in the model [3]. The results indicate that the interference power has a skewed distribution. In [4], we show that the aggregate secondary interference power can be modeled as sum of Gaussian and non-Gaussian random variables. Two interference zones are considered around a primary receiver where nodes inside these zones generate the Gaussian and non-Gaussian parts of the interference power.

Recently, considering an Ultra Wideband (UWB) scenario, in which multiuser interference (MUI) is known to be non-Gaussian, it has been shown that by using higher order statistics of interference and designing receivers which are adapted to the non-Gaussian interference, capacity can improve dramatically [5]. Reference [6] considers a UWB antenna design which affects the distribution of interference and thereby improves the capacity. This is expectable and can also be seen from Shannon's capacity bound for the general additive channel [7]

$$W \log_2\left(1+\frac{P}{N_1}\right) \leq C \leq W \log_2\left(\frac{N}{N_1}+\frac{P}{N_1}\right),$$

where P is the received signal power, N_1 is the Entropy power of interference/noise and N is the interference/noise power. Both upper and lower bound of capacity therefore decrease with entropy power. For the case of Gaussian interference/noise, as a result of its maximum entropy property, capacity is minimum and the bound degenerates to equality ($N = N_1$ and $C = \log_2(1+\frac{P}{N})$). We can see that non-Gaussianity of interference can indeed be advantageous to the link capacities.

A common measure used for non-Gaussianity is the kurtosis which is defined as the ratio of fourth cumulant to the square of second cumulant. In [5], the capacity of a discrete memoryless channel, when noise/interference has a generalized Gaussian distribution is obtained numerically. Result shows that for a fixed SNR, capacity is maximized when the kurtosis is maximum and the minimum capacity corresponds to the case that the generalized Gaussian distribution degenerates to Gaussian (i.e., for kurtosis equal to 0).

In this paper, we focus on interference *amplitude*. We show that non-Gaussian and Gaussian interference zones, considered in [4] for interference power, are also valid for interference amplitude. We assign fixed powers level for the interfering nodes inside each zone and obtain the cumulants of interference amplitude in a cognitive radio scenario. While the nodes inside the Gaussian zone influence on the first two cumulants of interference, the nodes inside the non-Gaussian zone affect higher cumulants as well. We seek to maximize the kurtosis of interference by finding the optimum power levels inside each interference zone subject to a regulatory constraint on the maximum interference power and show that optimum

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power levels can be found by solving a constrained nonlinear optimization problem. Our results show that using optimum power levels, the kurtosis increases significantly compared to the no power control case. We consider a generalized Gaussian model for interference and show that the capacity of primary link improves dramatically as a result of increase in kurtosis.

II. System Model and Notations

We focus on a single primary link which is subject to a Poisson field of secondary interferers with density λ. We assume that the primary node is located at the origin and denote the distance from the primary node to its ith nearest secondary neighbor as R_i. The primary receiver sends a beacon and secondary neighbors which can detect it abstain from transmission. The ith nearest secondary neighbor will interfere with the primary if the received beacon's SNR is less than a threshold, i.e., if $\frac{P_b X_i^2}{N R_i^\alpha} < \gamma_0$, where P_b is the beacon power, N is the noise power at the secondary node, α is the path loss exponent and γ_0 is the SNR detection threshold. We assume that $\{X_i\}$ are i.i.d Rayleigh random variables which account for the fading component and $E\{X_i^2\} = 1$.

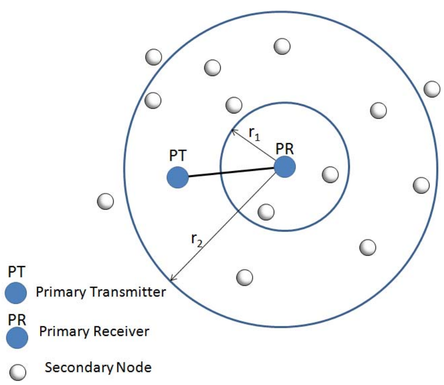

Fig. 1: Primary Link Subject to a Poisson Field of Interferers

We consider two interference zones around the primary receiver: $\{r_1 \le r < r_2\}$ and $\{r \ge r_2\}$, where r is the distance of a point to the primary receiver, and assume that the secondary nodes inside these zones transmit with power levels P_1 and P_2 respectively. No secondary node can transmit if it is located at $r < r_1$, to avoid non-singularity in the interference behavior. In [4], we show that for properly chosen r_1 and r_2, interference power originated from these zones can be very well approximated by non-Gaussian and Gaussian random variables respectively.

In this paper we focus on interference *amplitude*. The received interference signal from ith nearest secondary neighbor is $\frac{\sqrt{P} X_i}{R_i^{\alpha/2}} \cos(\omega_0 t + \varphi_i) \mathbf{1}(\frac{P_b X_i^2}{N R_i^\alpha} < \gamma_0)$ where $\mathbf{1}(.)$ is the indicator function, $\alpha/2$ is the amplitude loss exponent, P is the transmit power level, ω_0 is the carrier frequency and φ_i is a random phase shift which we assume to be uniformly distributed in $[0, 2\pi]$. Assume that the received interference is sampled at time $t = 0$. Using transformation of random variables we can show that $U_i \triangleq \cos(\varphi_i)$ has the following pdf:

$$f_{U_i}(u) = \frac{1}{\pi\sqrt{1-u^2}} \qquad -1 < u < 1.$$

The aggregate interference at the primary node (denoted by I) will be sum of interference originated from interference zones (denoted by I_1 and I_2 respectively), i.e. $I = I_1 + I_2$. By defining

$$S_1 \triangleq \{i : \; r_1 \le R_i < r_2\}$$
$$S_2 \triangleq \{i : R_i \ge r_2\}$$

we have

$$I_j = \sum_{i \in S_j} \frac{\sqrt{P_j} X_i U_i}{R_i^{\alpha/2}} \mathbf{1}(\frac{P_b X_i^2}{N R_i^\alpha} < \gamma_0) \qquad j = 1, 2$$

A regulatory constraint needs to be satisfied on the interference power at the primary receiver which we write as

$$E\{I^2\} \le K \tag{1}$$

This ensures that the secondary interference power at the primary receiver is less than a system-defined threshold.

III. Interference Amplitude Statistics

By using Campbell's theorem [8], the cumulants of I_1 can be found as

$$\kappa_n(I_1) = 2\pi\lambda \int_{-1}^{1} \int_0^\infty \int_{[r_1 r_2] \cap [h(x), \infty)} \left(\frac{\sqrt{P_1} x u}{r^{\alpha/2}} \right)^n r dr \; f_X(x) dx f_U(u) du$$

where $h(x) = \left(\frac{P_b x^2}{\gamma_0 N} \right)^{1/\alpha}$. This integral can be split into two parts as

$$\kappa_n(I_1) = 2\pi\lambda\mu_n \int_0^{x_1} \int_{r_1}^{r_2} P_1^{n/2} r^{1-n\alpha/2} dr x^n f_X(x) dx + 2\pi\lambda\mu_n \int_{x_1}^{x_2} \int_{h(x)}^{r_2} p_1^{n/2} r^{1-n\alpha/2} dr x^n f_X(x) dx$$

where $x_j \triangleq \sqrt{\frac{\gamma_0 N r_j^\alpha}{P_b}}$, $\quad j = 1, 2$ and $\mu_n = \int_{-1}^{1} \frac{u^n du}{\pi\sqrt{1-u^2}}$. After simplification, we will have

$$\kappa_n(I_1) = \frac{2\pi\lambda\mu_n P_1^{n/2}}{2 - \frac{n\alpha}{2}} \left\{ r_2^{2-\frac{n\alpha}{2}} \int_0^{x_2} x^n f_X(x) dx - r_1^{2-\frac{n\alpha}{2}} \int_0^{x_1} x^n f_X(x) dx - \left(\frac{P_b}{\gamma_0 N} \right)^{\frac{4-n\alpha}{2\alpha}} \int_{x_1}^{x_2} x^{4/\alpha} f_X(x) dx \right\} \tag{2}$$

For simplicity, we write $\kappa_n(I_1) = A_n P_1^{n/2}$. By using Campbell's theorem [8], the cumulants of I_2 can also be found as

$$\kappa_n(I_2) = 2\pi\lambda\mu_n \int_0^\infty \int_{[r_2,\infty]\cap[h(x),\infty)} \left(\frac{\sqrt{P_2}x}{r^{\alpha/2}}\right)^n r dr f_X(x)dx$$
$$= 2\pi\lambda\mu_n P_2^{n/2} \left\{\int_0^{x_2}\int_{r_2}^\infty r^{1-n\alpha/2} dr x^n f_X(x)dx + \int_{x_2}^\infty \int_{h(x)}^\infty r^{1-n\alpha/2} dr x^n f_X(x)dx\right\}$$

After simplification, for $2 - n\alpha/2 < 0$

$$\kappa_n(I_2) = \frac{2\pi\lambda\mu_n P_2^{n/2}}{2-\frac{n\alpha}{2}} \left\{-r_2^{2-\frac{n\alpha}{2}} \int_0^{x_2} x^n f_X(x)dx - \left(\frac{P_b}{\gamma_0 N}\right)^{\frac{4-n\alpha}{2\alpha}} \int_{x_2}^\infty x^{4/\alpha} f_X(x)dx\right\} = B_n P_2^{n/2} \quad (3)$$

A_n and B_n are found through (2) and (3) respectively. Since $\mu_1 = 0$ ($\mu_n = 0$ for n an odd number), we can see that $\kappa_1(I) = \kappa_1(I_1) = \kappa_1(I_2) = 0$ and the interference amplitude is zero-mean.

For Rayleigh fading and with $E\{X^2\} = 1$, the pdf of X is $f_X(x) = 2xe^{-x^2},\ x \geq 0$ and for a, b and c real numbers we have [9]

$$\int_0^a x^c f_X(x)dx = \Gamma_l(1 + c/2, a^2)$$
$$\int_a^\infty x^c f_X(x)dx = \Gamma_u(1 + c/2, a^2)$$
$$\int_a^b x^c f_X(x)dx = \Gamma_l(1 + c/2, b^2) - \Gamma_l(1 + c/2, a^2)$$

where $\Gamma_l(.)$ and $\Gamma_u(.)$ are lower and upper incomplete gamma functions respectively. These equalities can be used in (2) and (3) to obtain A_n and B_n respectively.

IV. Optimization Problem

In this section we seek to maximize the kurtosis of secondary interference subject to a constraint on the maximum secondary interference power by allocating optimal power levels, P_1 and P_2, to the interfering nodes in Gaussian and non-Gaussian interference zones. The kurtosis of interference is the ratio of its fourth cumulant to the square of second cumulants, i.e.,

$$kurtosis(I) = \frac{\kappa_4(I)}{\kappa_2^2(I)}$$

In [4], the interference power from Gaussian and Non-Gaussian zones is shown to be independent. Following the same approach, interference amplitudes (i.e., I_1 and I_2) can also be considered independent. Using this independence

$$kurtosis(I) = \frac{\kappa_4(I_1) + \kappa_4(I_2)}{(\kappa_2(I_1) + \kappa_2(I_2))^2} = \frac{A_4 P_1^2 + B_4 P_2^2}{(A_2 P_1 + B_2 P_2)^2} \quad (4)$$

When there is no power control, i.e. when $P_1 = P_2$, the kurtosis does not depend on the transmit power level and we have $kurtosis(I) = \frac{A_4+B_4}{(A_2+B_2)^2}$.

The interference constraint in (1) can be written as

$$E\{I^2\} = \kappa_2(I) + \kappa_1^2(I) = \kappa_2(I_1) + \kappa_1^2(I_1) + \kappa_2(I_2) + \kappa_1^2(I_2) + 2\kappa_1(I_1)\kappa_1(I_2) = (A_2 + A_1^2)P_1 + (B_2 + B_1^2)P_2 \leq K \quad (5)$$

The optimization problem is therefore finding the optimal values of P_1 and P_2 that maximizes the kurtosis of interference in (4) subject to the interference constraint in (5). The interference from the Gaussian zone would have little influence on the kurtosis of I and therefore, intuitively, the optimum value of P_2 is very small. Following constraint can be added to have the Gaussian zone contribute to the interference and thereby have $P_2 > 0$:

$$E\{I_2^2\} \geq \beta K \quad (6)$$

where $0 \leq \beta \leq 1$ accounts for the minimum contribution of the interference from the Gaussian zone in interference power at the primary node. (6) can also be written as

$$\kappa_2(I_2) + \kappa_1^2(I_2) = (B_2 + B_1^2)P_2 \geq \beta K$$

The optimization problem can therefore be written as

$$\text{Maximize } kurtosis(I) \text{ subject to}$$
$$E\{I^2\} \leq K$$
$$E\{I_2^2\} \geq \beta K$$
$$P_1 \geq 0$$
$$P_2 \geq 0$$

This is a nonlinear constrained optimization problem as the objective function is a nonlinear functions of P_1 and P_2.

V. Capacity Analysis

In this section we seek to evaluate the performance of the proposed statistical shaping of interference in terms of primary link capacity. For this purpose we use the formula for the capacity of a discrete memoryless channel with continuous-valued output given in [10]. For BPSK modulation with symbols $a_0 = \sqrt{P}$ and $a_1 = -\sqrt{P}$, the received signal at time slot n, considering an interference limited case (i.e. assuming that noise power can be ignored) is

$$y_n = a_n + I_n,$$

and the capacity, assuming equiprobable symbols, can be found as [10]

$$C = \frac{1}{2}\sum_{i=0}^{1} \int_{-\infty}^{\infty} f_{y|a_i}(y) \log_2\left(\frac{2f(y|a_i)}{f(y|a_0) + f(y|a_1)}\right) dy \quad (7)$$

To obtain $f_{y|a_i}(y)$, pdf of I will be required which we do not have in closed form but only its cumulants (i.e., $\kappa_n(I) = \kappa_n(I_1) + \kappa_n(I_2)$). We consider the generalized

Gaussian distributions to model I. This is a flexible pdf whose parameters depend only on mean, variance and kurtosis [11]. Therefore, the knowledge of K and kurtosis of I is enough to obtain the parameters of $f_{y|a_i}(y)$ (which has the mean a_i). The SIR of the received signal is $\frac{P}{K}$. The capacity can be obtained from (7) by using numerical integration.

VI. Numerical and Simulation Results

In this section we provide numerical and simulation results to verify the effectiveness of the proposed power optimization on the statistical shaping of interference and through that on the capacity of primary link. We consider a single primary link which is subject to a Poisson field of secondary interferers with density $\lambda = .01$. The maximum secondary interference power is assumed to be $K = 100\mu W$ The Gaussian and non-Gaussian zones are determined by setting $r_1 = 1$ and $r_2 = 5$. Unless we consider it as a parameter, α, the path loss exponent is 4. In Figures 2 and 3, we have verified the correctness of our Gaussian and non-Gaussian interference zones assumption through simulation. The result shows that while I_2 is very well approximated by a Gaussian random variable, the distribution of I_1 is far from Gaussian.

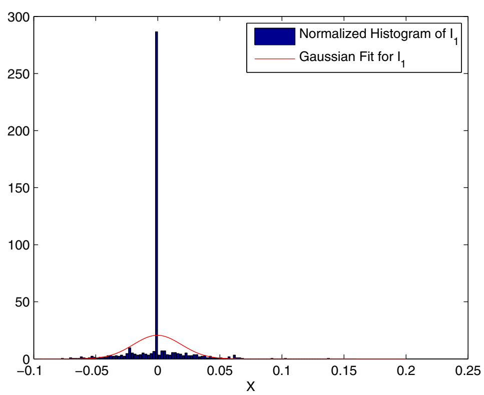

Fig. 2: The Normalized Histogram of I_1 and its Gaussian Fit

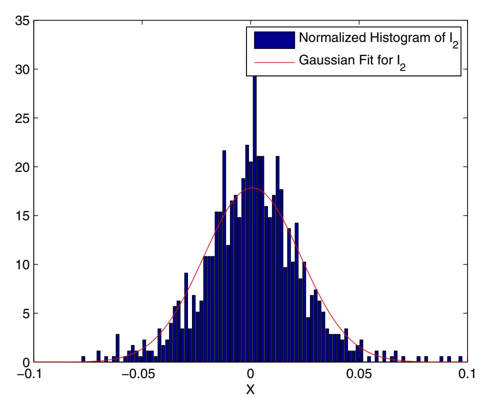

Fig. 3: The Normalized Histogram of I_2 and its Gaussian Fit

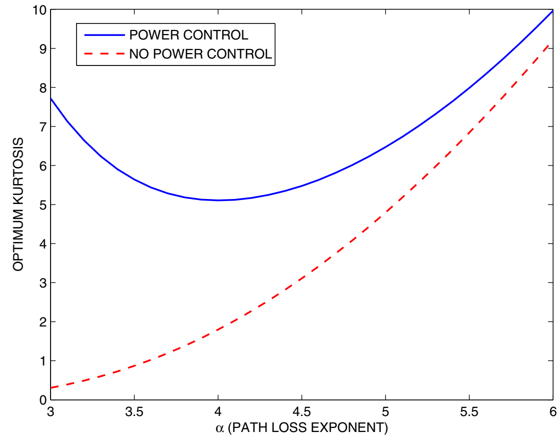

Fig. 4: Kurtosis of I with and without Power Control

In Figure 4, we compare the effect of power control on the kurtosis of I. We use the MATLAB optimization toolbox to solve the optimization problem given in section IV. The result shows that, by using the optimum power levels, a significant gain is obtained for the kurtosis of interference especially for smaller path loss exponents. The resulting gain in capacity is shown in Figure 5 where a generalized Gaussian model is considered for interference and capacity is obtained through numerical integration and by using (7). SIR of the primary link is assumed to be $5dB$. The result shows that there is a direct relation between kurtosis and capacity in this case and as kurtosis increases the channel capacity increases as well.

In Figures 6, we have shown the effect of parameter β on the optimum kurtosis. As we increase β, i.e. the contribution of I_2 in aggregate interference, the optimum kurtosis decreases. This is expectable as I_2 has a distribution which is very close to Gaussian and therefore has little influence on the fourth cumulant of interference. Increasing β means P_1 should decrease to satisfy the interference power constraint. Since I_1 has the dominant effect on the kurtosis of I, decreasing P_1 means that the kurtosis will decrease as well. On the other hand, increasing β would increase the level of optimum P_2. The optimum value for P_2 would be close to 0 when $\beta = 0$, i.e. when no constraint is added for minimum contribution of I_2 in the aggregate interference.

VII. Conclusion

The non-Gaussianity of interference has recently been looked upon as an advantage to communication capacity. A Poisson field of interferers are known to generate non-Gaussian interference. In this paper, we consider a cognitive random wireless scenario in which a primary receiving node is the victim of non-Gaussian interference. Our aim is to shape the statistics of secondary interference in a way that is beneficial to primary link capacity. For this purpose, we use power control and consider two different power levels for the secondary neighbors of the primary receiver and aim to maximize the kurtosis of interference by allocating the optimum power levels. A regulatory constraints which limits the power of secondary interference on a primary receiving node also needs to be satisfied. This is shown to be a nonlinear constrained optimization problem. Our result indicates that by using optimum power levels the kurtosis is maximized. As a consequence, the primary link capacity is also improved compared to the no power control case.

In this paper we do not consider the code design or the receiver structure that achieve the capacity benefit of the non-Gaussian interference and the objective is to illustrate how statistical properties of secondary interference can be changed to improve the capacity of primary links. The optimum receiver structure that is matched to the non-Gaussian secondary interference can be considered as a future work.

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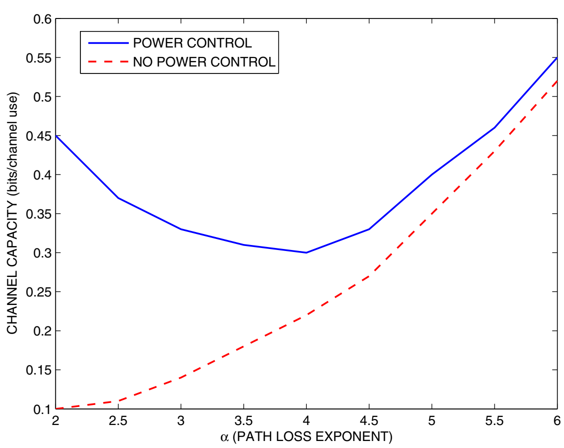

Fig. 5: primary Link Capacity with and without Power Control

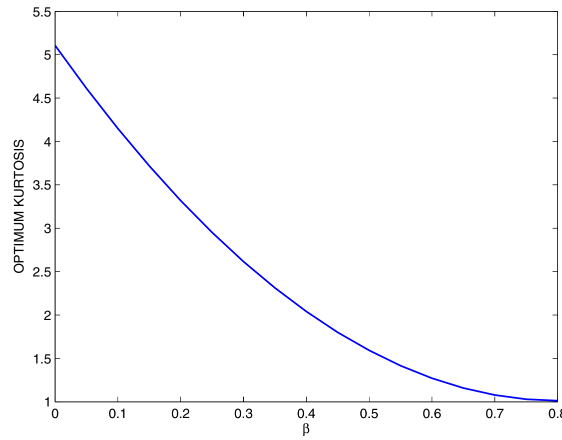

Fig. 6: Effect of β on Optimum Kurtosis

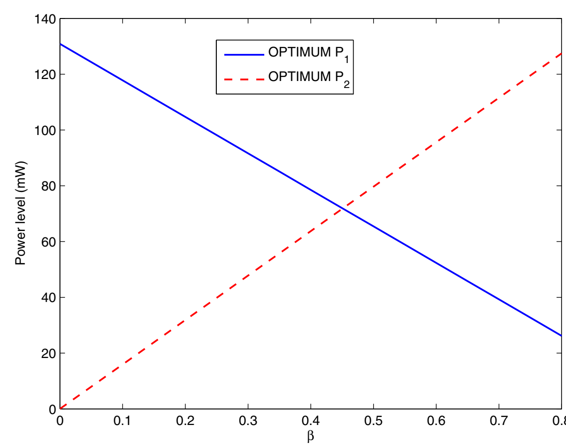

Fig. 7: Optimum Power Levels versus β