

Interference Modeling and Avoidance in Spectrum Underlay Cognitive Wireless Networks

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Abstract—In spectrum underlay cognitive wireless networks, secondary nodes need to limit their aggregate interference on the primary receiving nodes. The trends for interference modeling has been either indiscriminate use of Central Limit Theorem to model the aggregate interference as a Gaussian random variable or by application of the Campbell's Theorem and approximating the probability density function of interference from its cumulants (e.g., by using Edgeworth or Gram-Charlier series). In the latter case, the theorem can be applied only when the interfering nodes have the same power level. In this paper, we deviate from the previous trends of interference modeling in following ways: 1) We allow the secondary neighbors of a primary node to have arbitrary power levels. 2) We split the set of interfering neighbors of a primary node into non-Gaussian (close neighbors) and Gaussian (far neighbors) interferers. For the case of Log-normal fading, we show that an accurate model for interference is sum of a Normal and a Log-normal random variables. We proceed to obtain an upper bound for the complementary cumulative distribution function of interference and show its tightness through simulation. Simulations results confirm the accuracy of the proposed model. Finally, we propose adjustable interference avoidance strategies and show that interference constraint is satisfied using these strategies.

I. INTRODUCTION

Cognitive radio is a novel approach for better utilization of the underutilized radio spectrum [1]. To this end, unlicensed secondary wireless devices are envisioned to coexist with the primary network. In spectrum underlay cognitive radio, power constraints are imposed on the secondary devices so that their collective operation does not pose any problem for the detection of primary signals. Secondary users are tolerated so long as the aggregate interference originated from them is below the noise floor of primary receivers [2].

Due to the randomness in propagation channel and the geographic dispersion of secondary nodes, the aggregate interference from the secondary nodes has a statistical nature. The exact distribution of interference in a network with a Poisson field of interferers is an open problem and therefore approximation methods have been proposed in the literature [3], [4], [8]–[10]. The interference characteristic function has been found in AWGN [8] and Rayleigh fading [9] channels. The results in [8] and [9] can be more directly found through the application of Campbell's Theorem (see for example [4] and [10]). The other indiscriminate but mathematically attractive approach is to assume that interference originated from different interferers add up independently and therefore

CLT is used to model the interference as a Gaussian RV which simplifies the calculations. Nevertheless, this is shown to be a conservative assumption and interference is in fact a positively skewed and right-tailed RV [4].

The degree of freedom considered for the secondary network to control its aggregated interference, is usually assumed to be the common transmit power level among all the secondary neighbors. Using this assumption, [4] uses Campbell's Theorem (see [5]) to find closed-form expressions for cumulants of the aggregate interference and proposes different approaches to approximate its probability density function (pdf).

In this paper, we show that for a properly chosen large N , the interference originated from the N nearest secondary interfering neighbors of a primary node is non-Gaussian (for the case of Log-normal fading, we show that this is accurately approximated by another Log-normal RV), while CLT can be applied to model the total interference from the rest of the interferers as a Gaussian RV. The non-Gaussian part leads to the distribution of interference being right-tailed and positively skewed. This observation is also reported in [4] using the cumulants method. We obtain an upper bound for the Complementary Cumulative Distribution Function (CCDF) of interference. The secondary neighbors of a primary node are allowed to have arbitrary power levels, by tuning of which, the parameters of the model can be adjusted. Using the CCDF bound, interference avoidance strategies are proposed to satisfy the interference constraint.

II. SYSTEM MODEL AND NOTATIONS

Network Model: The primary and secondary nodes are randomly distributed in a region. We consider the model developed in [3] based on a spatial bivariate Poisson point process. We focus on a single primary node located at the origin. Conditioned on the primary node, the secondary nodes will then be distributed with intensity λ_c .

Internodal Distances: We denote the distance between the primary node and its n th nearest secondary neighbor as $R_{PS,n}$. In [3], it is found that the expected value of $R_{PS,n}^\alpha$, for α a real number and when primary and secondary nodes are highly correlated will be

$$E\{R_{PS,n}^\alpha\} = \frac{\Gamma(n + \alpha/2)}{\lambda^{\alpha/2}(n-1)!}, \quad n \geq -\lfloor \alpha/2 \rfloor. \quad (1)$$

where $\lambda = \lambda_c \pi$ and $\lfloor x \rfloor$ is, by definition, the largest integer, smaller than or equal to x . For $n < -\lfloor \alpha/2 \rfloor$, defining $f_n(x) = x^{n+\alpha/2-1}$, and for M large enough, we have extended the results,

$$E\{R_{PS,n}^\alpha\} \approx \frac{1}{\lambda^{\alpha/2}(n-1)!} \sum_{i=1}^M w_i f_n(x_i), \quad n < -\lfloor \alpha/2 \rfloor, \quad (2)$$

where x_i and w_i are the abscissas and the weights for Gauss-Laguerre quadrature of order M^1 [6]. The details of derivations are not presented due to space limitation (see [7]).

Interference: Defining $P_{S,l}$ to be the transmit power level of l th nearest secondary neighbor and I_P the interference originated from the nodes of the primary network only, the aggregate interference at the primary node is

$$I_t = I_P + \sum_{l=1}^{\infty} \xi_l P_{S,l} R_{PS,l}^\alpha \quad (3)$$

where ξ_l is a random fading component and α is a negative real number ($-\alpha$ is the path loss exponent in distance-dependent path loss model).

Interference Constraint: The interference constraint at the primary node can be written as:

$$Pr\{I_P + \sum_{l=1}^{\infty} \xi_l P_{S,l} R_{PS,l}^\alpha > I_{P,max}\} \leq \epsilon \quad (4)$$

where $(I_{P,max}, \epsilon)$ are system-defined values. (3) can also be written as

$$P_{exc} = Pr\left\{\sum_{l=1}^{\infty} \xi_l P_{S,l} R_{PS,l}^\alpha > \eta\right\} \leq \epsilon \quad (5)$$

where η is the maximum aggregated interference that secondary network can inflict on a primary node. P_{exc} is the probability of excess interference at the primary node.

III. GAUSSIAN AND NON-GAUSSIAN INTERFERERS

Let us define $I \triangleq \sum_{i=1}^{\infty} \xi_i P_{S,i} y_i^{\alpha/2}$, where $y_i = R_{PS,i}^2$. For a Poisson process with parameter λ_c , from corollary 2 in [12], y_1 and $y_i - y_{i-1}, i > 1$ are i.i.d exponential random variables with parameter $\lambda (= \pi \lambda_c)$ and therefore y_i is an Erlang random variable with mean $\frac{i}{\lambda}$. y_i is sum of i i.i.d exponential random variables. Using the law of large numbers, for large i , $y_i \rightarrow \frac{i}{\lambda}$ almost surely². So, for large N ,

$$I \rightarrow \sum_{i=1}^N \xi_i P_{S,i} R_{PS,i}^\alpha + \sum_{i=N+1}^{\infty} \xi_i P_{S,i} \left(\frac{i}{\lambda}\right)^{\alpha/2}. \quad (6)$$

By defining $I_1 = \sum_{i=1}^N \xi_i P_{S,i} R_{PS,i}^\alpha$ and $I_2 = \sum_{i=N+1}^{\infty} \xi_i P_{S,i} \left(\frac{i}{\lambda}\right)^{\alpha/2}$, we can invoke CLT to approximate

¹Note that, for $\alpha > -6$, which is typically the case in wireless communications, in the worst case, the distances to the nearest two neighbors are found using this approximation.

²This can also be seen from $\frac{\sqrt{\text{Var}\{y_i\}}}{E\{y_i\}} = \frac{1}{\sqrt{i}} \rightarrow 0$ for large i (using (1)). So, for large i , y_i acts like a constant and can be replaced by its mean.

I_2 as a Gaussian random variable as it is a linear combination of infinite independent random variables (i.e. $I_2 \rightarrow \tilde{I}_2 \sim \mathcal{N}(\mu_2, \sigma_2)$). Similar observation has also been reported in [11] in which it is shown that the non-Gaussian part of interference can be suppressed by including a large guard zone (in which no interferer can be present) around each node.

The mean and variance of \tilde{I}_2 can be readily found as

$$\mu_2 = E\{\tilde{I}_2\} = \frac{E\{\xi\}}{\lambda^{\alpha/2}} \sum_{i=N+1}^{\infty} P_{S,i} i^{\alpha/2}, \quad (7)$$

$$\sigma_2^2 = \text{var}\{\tilde{I}_2\} = \frac{\text{var}\{\xi\}}{\lambda^\alpha} \sum_{i=N+1}^{\infty} P_{S,i}^2 i^\alpha. \quad (8)$$

I_1 is a linear combination of a finite number of random variables $(\xi_i R_{PS,i}^\alpha)$ with unknown distribution and there is no straightforward way to obtain or approximate its pdf for a general fading case. It is shown in [13] that linear combination of Log-normal random variables of the form $\sum_{i=1}^N A_i \xi_i$, where A_i 's are positive and independent random variables can be approximated by another Log-normal random variable. In the summation for I_1 , $\{R_{PS,i}\}$ are not independent. Our simulation results show that Log-normal is still a good approximation (see Figure 1). So, we can write $I_1 \sim \mathcal{LN}(\mu_1, \sigma_1)$. μ_1 and σ_1^2 are related to $m_1 = E\{I_1\}$ and $s_1^2 = \text{var}\{I_1\}$ [14] as

$$\mu_1 = \ln \left(\frac{m_1^2}{\sqrt{m_1^2 + s_1^2}} \right), \quad (9)$$

$$\sigma_1^2 = \ln \left(\left(\frac{s_1}{m_1} \right)^2 + 1 \right). \quad (10)$$

m_1 and s_1^2 can be readily found as

$$m_1 = E\{\xi\} \sum_{i=1}^N P_{S,i} E\{R_{PS,i}^\alpha\}, \quad (11)$$

$$s_1^2 = \sum_{i=1}^N \sum_{j=1}^N P_{S,i} P_{S,j} \text{cov}\{\xi_i R_{PS,i}^\alpha, \xi_j R_{PS,j}^\alpha\}. \quad (12)$$

To simplify the calculations, we assume that the first N nearest neighbors transmit a fixed power level (i.e. $P_{S,i} = p_1, 1 \leq i \leq N$). In [7], we find m_1 , a lower bound ($s_{1,l}^2$) and an upper bound ($s_{1,u}^2$) for s_1^2 as follows:

$$m_1 = p_1 E\{\xi\} \sum_{i=1}^N E\{R_{PS,i}^\alpha\}. \quad (13)$$

$$\frac{s_{1,l}^2}{p_1^2} = E^2\{\xi\} \sum_{i=1}^N \left\{ \sum_{j=1}^{i-1} \frac{[i(j+1)]^{\alpha/2}}{\lambda^\alpha} + \sum_{j=i+1}^N \frac{[j(i+1)]^{\alpha/2}}{\lambda^\alpha} \right\} + C, \quad (14)$$

$$\frac{s_{1,u}^2}{p_1^2} = C + E^2\{\xi\} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \sqrt{E\{R_{PS,i}^{2\alpha}\} E\{R_{PS,j}^{2\alpha}\}} \quad (15)$$

where

$$C = E\{\xi^2\} \sum_{i=1}^N E\{R_{PS,i}^{2\alpha}\} - E^2\{\xi\} \left\{ \sum_{i=1}^N E\{R_{PS,i}^\alpha\} \right\}^2. \quad (16)$$

Replacing $s_{1,l}^2$ in (9) and $s_{1,u}^2$ in (10), we can find $(\mu_{1,u}, \sigma_{1,u}^2)$, upper bounds for μ_1 and σ_1 . The lower bounds $(\mu_{1,l}, \sigma_{1,l}^2)$ can also be found by replacing $s_{1,u}^2$ in (9) and $s_{1,l}^2$ in (10).

Now we proceed to find the CCDF of Interference. We have found $I \rightarrow I_1 + \tilde{I}_2$, with $I_1 \sim \mathcal{LN}(\mu_1, \sigma_1)$, and $\tilde{I}_2 \sim \mathcal{N}(\mu_2, \sigma_2)$ and $I_1 \perp \tilde{I}_2$. Using these distributions, we have from (5)

$$\begin{aligned} P_{exc} &\approx \int_0^\infty Pr\{I_1 + \tilde{I}_2 > \eta | I_1 = x\} f_{I_1}(x) dx \\ &= \int_0^\infty Pr\{\tilde{I}_2 > \eta - x\} f_{I_1}(x) dx \\ &= \int_0^\infty Q\left(\frac{\eta - x - \mu_2}{\sigma_2}\right) \frac{1}{\sqrt{2\pi}\sigma_1 x} e^{-\frac{(\ln x - \mu_1)}{2\sigma_1^2}} dx \quad (17) \end{aligned}$$

Using integration by substitution in (17) with $u = \frac{\ln x - \mu_1}{\sqrt{2}\sigma_1}$, after simplification, we will have

$$\begin{aligned} P_{exc} &\approx \int_{-\infty}^\infty \frac{1}{\sqrt{\pi}} Q\left(\frac{\eta - \mu_2 - e^{\mu_1 + \sqrt{2}\sigma_1 u}}{\sigma_2}\right) e^{-u^2} du \\ &\approx \sum_{i=1}^M w_i f^*(u_i) \quad (18) \end{aligned}$$

where,

$$f^*(u) \triangleq \frac{1}{\sqrt{\pi}} Q\left(\frac{\eta - \mu_2 - e^{\mu_1 + \sqrt{2}\sigma_1 u}}{\sigma_2}\right) \quad (19)$$

and we have used the Gauss-Hermite quadrature method ($\int_{-\infty}^\infty e^{-x^2} f(x) dx = \sum_{i=1}^M w_i f(x_i) + R_M$), with M assumed to be a large integer. The abscissas (x_i) and weights (w_i) for different values of M are tabulated in [6].

If we define

$$f_u^*(x) \triangleq \frac{1}{\sqrt{\pi}} Q\left(\frac{\eta - \mu_2 - e^{\mu_{1,u} + \sqrt{2}\sigma_{1,u} x}}{\sigma_2}\right), \quad (20)$$

we have $f^*(x) < f_u^*(x)$, for any x . So, we can write

$$P_{exc} < \sum_{i=1}^M w_i f_u^*(x_i). \quad (21)$$

IV. INTERFERENCE AVOIDANCE STRATEGIES

The expression depends on μ_2 , σ_2 , and the upper bounds of μ_1 and σ_1 . The bounds for μ_1 and σ_1 are determined by the choice of p_1 (see (14), (15) and (16)). On the other hand μ_2 and σ_2 are infinite summations which depend on the choice of $P_{S,i}$, $i \geq N + 1$. In following subsections, we propose two different approaches to determine the power levels which make these summations converge and the interference constraint satisfied.

A. Fixed Power Levels

For fixed power levels, (i.e. $P_{S,i} = p_2$, $i \geq N + 1$), the equations (7) and (8) can be written as

$$\mu_2 = \frac{E\{\xi\} p_2}{\lambda^{\alpha/2}} \left(\zeta(-\alpha/2) - \sum_{i=1}^N i^{\alpha/2} \right), \quad (22)$$

$$\sigma_2^2 = \frac{\text{var}\{\xi\} p_2^2}{\lambda^\alpha} \left(\zeta(-\alpha) - \sum_{i=1}^N i^\alpha \right). \quad (23)$$

where $\zeta(\cdot)$ is the Riemann-Zeta function which is convergent when its argument is larger than 1 [15]. In a practical path loss models, we always have $\alpha \leq -2$ with equality for the free space model. So, μ_2 and σ_2^2 are convergent even with fixed power levels. In this case, the degrees of freedom are the parameters p_1 and p_2 . Using the obtained bound in (21), p_1 and p_2 are determined such that the interference constraint is satisfied.

B. Distance-dependent Power Levels

A typical $P_{S,i}$, that assures the convergence of the series in (7) and (8) is

$$P_{S,i} = k i^\beta, \quad i \geq N + 1 \quad (24)$$

where k and β are constant and $\beta < -\alpha/2 - 1$. In this case,

$$\mu_2 = \frac{E\{\xi\} k}{\lambda^{\alpha/2}} \left(\zeta(-\alpha/2 - \beta) - \sum_{i=1}^N i^{\alpha/2 + \beta} \right), \quad (25)$$

$$\sigma_2^2 = \frac{k^2 \text{var}\{\xi\}}{\lambda^\alpha} \left(\zeta(-\alpha/2 - \beta) - \sum_{i=1}^N i^{\alpha + 2\beta} \right). \quad (26)$$

Note that for $\alpha < -2$, $-\alpha/2 - 1 > 0$ and β can be chosen either positive or negative, which correspond to power levels steadily increasing or decreasing for $i \geq N + 1$. In this case, the degrees of freedom are p_1 , k and β , which should be adjusted such that the interference constraint is satisfied using the upper bound in (21).

V. SIMULATION RESULTS

We consider a primary node at the origin and secondary nodes which are distributed according to a spatial Poisson process with the parameter λ_c around it such that $\lambda = \pi\lambda_c = .025$. We consider a disc with radius $R = 1000$ units as the underlying area in which points are present. To avoid singularities which is present in the path loss model for $r = 0$, a small disc of radius r_0 is considered as the forbidden area around the primary node, inside which no secondary node exists. For $r_0 \ll 1$, there is a small probability that a node exists within the disc with radius r_0 and the results for $E\{R_{PS,n}^\alpha\}$ are still valid. In our simulation we have set $r_0 = 0.1$. We consider a $\sigma_s = 6$ dB Log-normal shadowing. By defining $\sigma_s = \sigma_s \frac{\ln 10}{10}$, the mean and variance of ξ are then calculated as $E\{\xi\} = e^{\sigma_s^2/2} = 2.597$, $E\{\xi^2\} = e^{2\sigma_s^2} = 45.484$, and $\text{var}\{\xi\} = E\{\xi^2\} - E^2\{\xi\} = 38.74$ [14]. The value of M

used in the Gauss-Laguerre and Gauss-Hermite quadratures, used in (2) and (18) respectively, is set to be 32.

In Figure 1, we have plotted the empirical CDF of $I_1 = \sum_{i=1}^N \xi_i P_{S,i} R_{P_{S,i}}^\alpha$ for $P_{S,i} = p_1 = 1$, $N = 1000$ and $\alpha = -3.2$. Alongside, the CDFs of $\mathcal{LN}(\mu_{1,l}, \sigma_{1,l})$ and $\mathcal{LN}(\mu_{1,u}, \sigma_{1,u})$ are plotted. The CDFs are practically the same. This shows that the obtained upper and lower bounds for s_1^2 (see equations (14), (15) and (16)) are very tight. Also the Log-normal assumption (either $\mathcal{LN}(\mu_{1,l}, \sigma_{1,l})$ or $\mathcal{LN}(\mu_{1,u}, \sigma_{1,u})$) for I_1 is a close approximation.

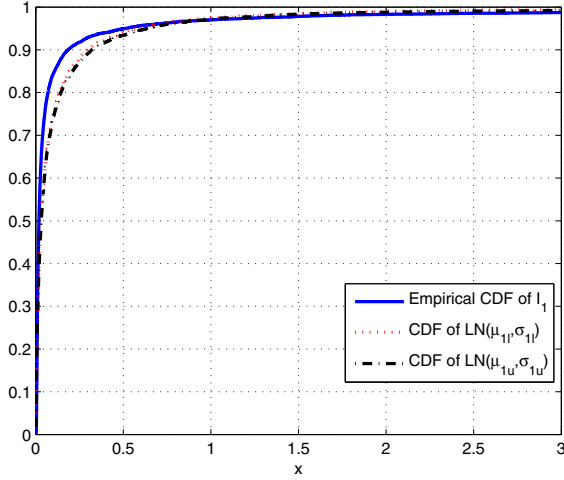


Fig. 1: The Empirical CDF of $I_1 = \sum_{i=1}^N \xi_i P_{S,i} R_{P_{S,i}}^\alpha$ for $P_{S,i} = p_1 = 1$, $N = 1000$ and $\alpha = -3.2$ and the CDFs of $\mathcal{LN}(\mu_{1,l}, \sigma_{1,l})$ and $\mathcal{LN}(\mu_{1,u}, \sigma_{1,u})$.

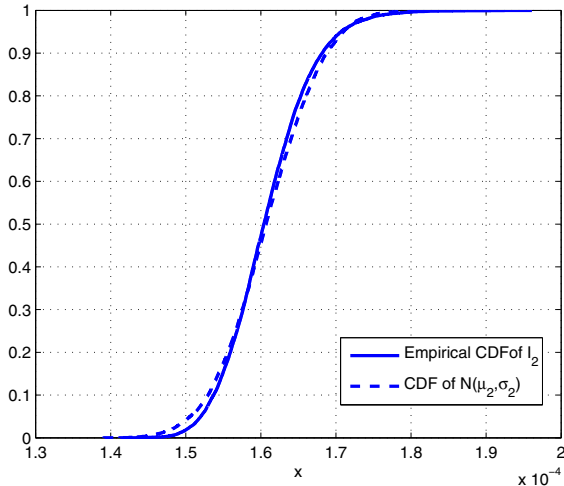


Fig. 2: The Empirical CDF of $I_2 = \sum_{i=N+1}^\infty \xi_i P_{S,i} R_{P_{S,i}}^\alpha$ for $P_{S,i} = p_2 = 1$, $N = 1000$ and $\alpha = -3.2$ and the CDF of $\mathcal{N}(\mu_2, \sigma_2)$.

In Figure 2, we have plotted the empirical CDF of $I_2 =$

$\sum_{i=N+1}^\infty \xi_i P_{S,i} R_{P_{S,i}}^\alpha$ ³. We again assume $P_{S,i} = p_1 = 1$. In the same graph, the CDF of $\mathcal{N}(\mu_2, \sigma_2)$ is plotted which shows that the Gaussian assumption is a close approximation for distribution of I_2 .

In Figure 3, we have investigated the tightness of the obtained bound for $P_{exc} = Pr\{I > \eta\}$ in (21). The empirical CCDF of $I = I_1 + I_2$ (i.e, the relative frequency of the event $\{I > \eta\}$) is plotted along with the obtained upper bound for P_{exc} . We have assumed that $p_1 = p_2 = 1$. The figure shows that the obtained bound for P_{exc} is, specially for $\eta > 0.5$, quite a tight bound.

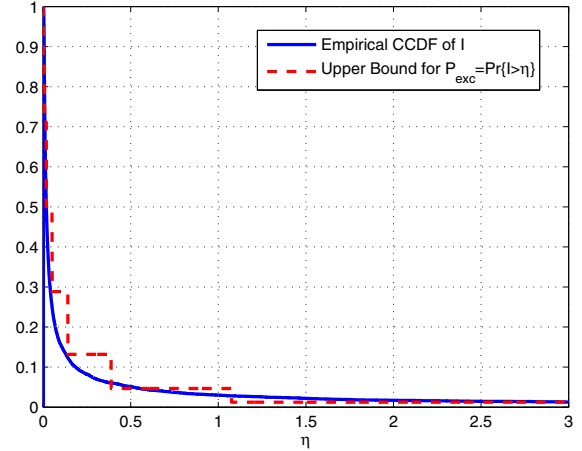


Fig. 3: Empirical CCDF of $I = I_1 + I_2$ and the upper bound for P_{exc} (using (17)). We assume that $p_1 = p_2 = 1$

In Figure 4, probability of excess interference (P_{exc}) is plotted versus path loss exponent for the interference avoidance strategy with fixed power levels. We have set $p_1 = p_2$ and chosen its value such that the CCDF bound (i.e., (17)) is satisfied. The (η, ϵ) pair is set to be $(1, 0.01)$. The results show that the interference constrained is always satisfied and probability of excess interference never exceeds the required 0.01 value.

In Figure 5, probability of excess interference (P_{exc}) is plotted versus path loss exponent for the interference avoidance strategy with distance-dependent power levels. We have assumed $\beta = -\alpha/2 - 1.05$ to satisfy the $\beta < -\alpha/2 - 1$ convergence condition. This leads to increasing power levels for $-\alpha > 2.1$ which includes the range of our simulations. To simplify, we have set $k = p_1 N^{-\beta}$ which leads to the continuity of power levels. p_1 is chosen such that the CCDF bound is satisfied. The (η, ϵ) pair is again set to be $(1, 0.01)$. The results show that the interference constrained is always satisfied.

VI. CONCLUSION

The goal of this paper is statistical modeling of the aggregate interference in a spectrum underlay cognitive wireless network. We consider a model that includes the transmit power

³Note that in our simulation ∞ is replaced by index of the farthest neighbor.

levels of secondary nodes as controlling means, by adjustment of which the interference constraint is satisfied.

In our approach, we show that the aggregate secondary interference to a primary node is composed of two independent components. Secondary interferers close to a primary node generate a non-Gaussian interference while CLT can be applied to model the interference originated from rest of the secondary nodes as a Gaussian RV. We argue that in a Log-normal fading scenario, a suitable model for interference is the sum of a Normal and Log-normal random variables and verify the model by using simulation. By considering this model, we obtain an upper bound for the CCDF of interference and propose strategies to fulfil the interference constraint. We show through simulation that the CCDF bound is tight and using these strategies, the interference constraint is always satisfied.

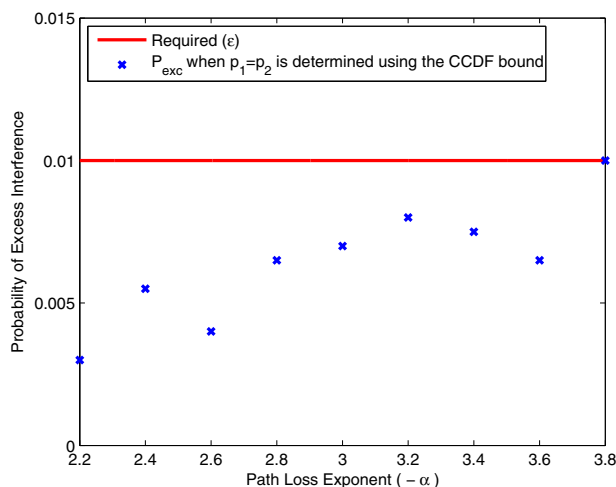


Fig. 4: P_{exc} vs. Path Loss Exponent for the strategy with fixed power levels. $p_1 = p_2$ are chosen such that the CCDF bound is satisfied.

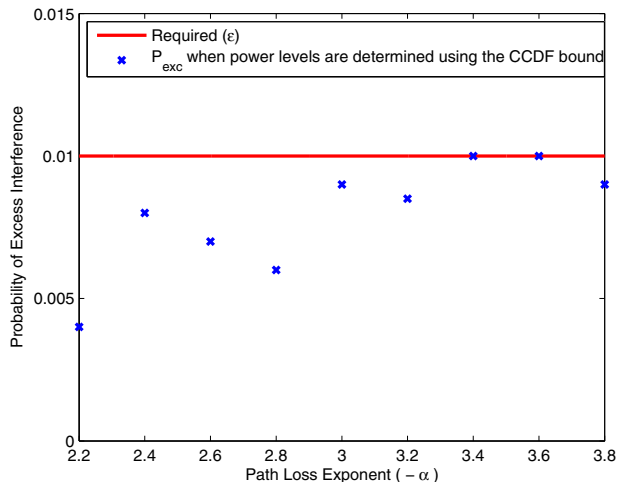


Fig. 5: P_{exc} vs. Path Loss Exponent for the strategy with distance-dependent power levels. We have set $\beta = -\alpha/2 - 1.05$ (increasing power levels), and $k = p_1 N^{-\beta}$. p_1 is chosen such that the CCDF bound is satisfied.

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