This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2010 proceedings.

Throughput Optimization in Cognitive Random Wireless Ad hoc Networks

Alireza Babaei
Dept. of Electrical and Computer Engineering
Auburn University
Auburn, Alabama 36849
Email: ababaei@auburn.edu

Bijan Jabbari
Dept. of Electrical and Computer Engineering
George Mason University
Fairfax, Virginia 22030
Email: bjabbari@gmu.edu

***Abstract*—We consider coexistence of overlapping but distinct primary and secondary random ad hoc networks. A secondary node has node-level and network-level obligations to avoid interference on primary receiving nodes. At the node level, it is allowed to transmit only if it does not detect data reception from its primary neighbors. At the network level, it needs to make sure that the aggregate secondary interference on an arbitrary primary receiving node satisfies an interference constraint. In this paper, we show that both of these obligations are non-trivially related to transmission attempt probabilities in primary and secondary networks and our aim is the optimization of throughput in these networks. We show that the interference constraint limits the feasible space for transmission probabilities of both systems. Looking at the secondary network, as a network plugged into the primary, we propose a progressive transmission probability optimization algorithm and show through simulation that it converges to the optimum throughput in a few time slots. Results show that at the optimum transmission probabilities, while the primary network deviates slightly from its optimal throughput (obtained when secondary network is not present), a considerable throughput can be gained by the secondary network. Results also show that the sum throughput also increases as a result of this optimization.**

I. Introduction

Cognitive radio is a novel approach to improve the utilization of the scarce radio spectrum [1]. Unlicensed secondary nodes can coexist with a primary licensed network so long as the aggregate interference inflicted by them on a primary node does not harshly degrade its detection performance. Various methods have been considered to achieve this coexistence goal.

MAC layer and cross-layer optimization have been proposed for cognitive wireless networks [2, 3]. The proposed approaches mainly focus on the optimization of sensing times and transmission durations in secondary networks and do not consider the effect of transmission attempt probability on the interference and throughput when the networks employ random access techniques. Reference [4] proposes an adaptive transmission protocol by limiting transmission probability (LTP) of secondary network in a 802.11 environment. The model, however, does not consider a random network scenario and the effect of interference on detection and throughput is not investigated.

We consider a scenario where primary and secondary are overlapping ad hoc networks and nodes in each are randomly and independently distributed with different densities. Recently, there has been much interests in obtaining theoretical results on throughput scaling in such cognitive wireless networks, e.g., [5]. However, when the networks use random access methods, the effect of transmission probabilities on achievable primary and secondary throughput has not yet been investigated.

Throughput in each system is limited by the self interference as well as the interference from the other network. In this paper, we consider a simple Gaussian model for interference. We show that the throughput in each system is a nontrivial function of transmission probabilities of both primary and secondary networks. We also show that the interference constraint limits the feasible space of the probabilities. We consider the maximization of throughput in these networks by obtaining the optimum transmission probabilities in each network and show that the problem is a Game with constraints in multi-strategies [6]. We propose a progressive transmission probability optimization algorithm to find the optimum transmission probabilities in a distributed manner.

II. System Model and Notations

Network Model: The primary and secondary nodes form uniformly random networks and are independently distributed in a region with densities λ_p and λ_s, respectively. We denote the distance between a primary node and its nth nearest secondary and primary neighbor as $R_{PS,n}$ and $R_{P,n}$ respectively. Similarly, $\{R_{SP,n}\}$ and $\{R_{S,n}\}$ denote the distances between a secondary node and its primary and secondary neighbors. Due to the independence assumption of networks, we have $R_{PS,n} \stackrel{d}{=} R_{S,n}$ and $R_{SP,n} \stackrel{d}{=} R_{P,n}$, where $\stackrel{d}{=}$ denotes equality in distribution.

Channel Model: We consider deterministic distance-dependent path loss as $r^{-\alpha}$, where α is called the path loss exponent and Rayleigh fading (x with $E\{x^2\} = 1$) which is assumed to be i.i.d. across the interferers (i.e., the power decays as $x^2 r^{-\alpha}$). We use the notations: $x_{PS,n}$, $x_{P,n}$, $x_{SP,n}$ and $x_{S,n}$ to denote the fading component along the corresponding internodal links.

Spectrum Sensing: We assume each primary receiving node sends a beacon on a control channel indicating that it is actively engaged in receiving primary data [7]. Secondary nodes sense this channel and may transmit only if the SNR of the aggregate of beacons is above a threshold. The other way would be that a secondary node detects individual beacons and abstains from transmission if at least one primary beacon is detected. This brings about the contention problem in the control channel. Due to the exponential decaying of power of beacons, the aggregate power of beacons will be significant only if a secondary node receives beacon signals from its nearby primary nodes, which are mostly affected by the secondary node's transmission. The aggregate beacons detection, as opposed to individual beacon detection, obviates the contention problem that arises in the control channel and simplifies the protocol.

Access Model: We consider the access method to be slotted ALOHA in both systems. The attempt probability in the primary network is assumed to be p_1. In the event that a secondary node does not detect the beacon signal (assume that this happens with probability ρ), it attempts to transmit with probability p_2'. The effective transmit probability of the secondary nodes is therefore $p_2 = p_2'.\rho$.

Interference and Throughput: Assuming P_P and P_S are the transmit power levels of the primary and secondary nodes respectively, we define following variables as the interference power inflicted by each system on its own nodes or on the nodes of the other system:

$$I_P = \sum_n P_P.\phi_{P,n}.x_{P,n}^2.R_{P,n}^{-\alpha} \quad (1)$$

$$I_{P\to S} = \sum_n P_P.\phi_{P,n}.x_{SP,n}^2.R_{SP,n}^{-\alpha} \quad (2)$$

$$I_S = \sum_n P_S.\phi_{S,n}.x_{S,n}^2.R_{S,n}^{-\alpha} \quad (3)$$

$$I_{S\to P} = \sum_n P_S.\phi_{S,n}.x_{PS,n}^2.R_{PS,n}^{-\alpha} \quad (4)$$

where $\phi_{P,n}$ and $\phi_{S,n}$ are Bernoulli random variables with parameters p_1 and p_2 respectively. It is clear from the above definitions that $I_P \overset{d}{=} I_{P\to S}$ and $I_S \overset{d}{=} I_{S\to P}$. Assuming a noise power of σ^2, the Signal power to Interference and Noise power Ratio (SINR) at a destination which is R meters away from its primary or secondary transmitter can be written as:

$$SINR^P = \frac{P_P.x_P^2.R^{-\alpha}}{I_P + I_{S\to P} + \sigma^2}, \quad (5)$$

$$SINR^S = \frac{P_S.x_S^2.R^{-\alpha}}{I_S + I_{P\to S} + \sigma^2}. \quad (6)$$

With a half-duplex assumption (a node can either transmit or receive data), the probabilistic throughput (i.e., the probability that the transmitter is in the transmission mode, the intended receiver is in the reception mode and the link SINR is above a threshold) in the primary and secondary networks will be

$$\tau_P = p_1(1-p_1)Pr\{SINR^P > \theta\}, \text{and} \quad (7)$$

$$\tau_S = p_2(1-p_2)Pr\{SINR^S > \theta\}, \quad (8)$$

respectively, where θ is the $SINR$ threshold for successful detection which is assumed to be the same in both systems. The transmitter is assumed to have no channel information and can not therefore estimate the SNR values at the receivers.

III. Statistical Models and Results

τ_P, given in (7), is the probability that a primary node receives successfully and can be interpreted as the probability that a primary node is actively receiving data. Primary receiving nodes therefore form a thinned Poisson process with density $\lambda' = \lambda_p \tau_P$ [11]. We denote the distance between a secondary node and its ith nearest primary receiving node as R_i, the power of the beacon signal as P_b, and the power of aggregate beacon signals as Y. Assuming that the power of aggregate beacons is sum of the power levels of the beacons (e.g., by using orthogonal signals for the beacons), we have

$$Y = P_b \sum_i x_i^2 R_i^{-\alpha}. \quad (9)$$

The aggregate interference in equation (1)-(4) and the aggregate beacons power in (9) are aggregation of random variables and have the same form. The exact distribution of interference in a network with a Poisson field of interferers is not known and approximation methods have been proposed in the literature [8-10]. In this paper we assume that the power of interference or beacon signals add up independently and therefore Central Limit Theorem can be used to model them as Gaussian random variables.

For a random Poisson network with density λ, transmit probability of p, transmit power level of P and Rayleigh fading channel with $E\{x^2\} = 1$, the mean and variance of interference is found in [9] as

$$m = \frac{\lambda p d_0^2}{1 - 2/\alpha} \frac{P}{d_0^\alpha}, \quad (10)$$

$$v^2 = \frac{2\lambda\pi p d_0^2}{1 - 1/\alpha} \left(\frac{P}{d_0^\alpha}\right)^2 \quad (11)$$

where d_0 is the near field cut-off radius. In other words, no other nodes within radius d_0 around destination can transmit in the same time slot.

The distribution of $I_{S\to P}$ (or I_S) and $I_{P\to S}$ (or I_P) can be obtained by substituting the corresponding parameters of the secondary and primary networks in (10) and (11).

Calculation of ρ: If at least one primary receiving node exists within the radius d_0 of a secondary node (which happens with probability $1 - e^{-\lambda'\pi d_0^2}$), we assume that the secondary node will detect the beacon and therefore $\rho = 0$. Otherwise, it abstains from transmission if the SNR of aggregate beacons is above a threshold (γ_t). Using the relation for the power of aggregate beacon signals in (9), ρ can be calculated as

$$\Pr\left\{\frac{Y}{N} < \gamma_t\right\}, \quad (12)$$

where N is the noise power and γ_t is the SNR threshold. The parameters of the distribution of Y (i.e., (m_y, v_y^2)) can be found by substituting the corresponding parameters in (10) and (11), i.e., P_b and λ' for P and λ and setting $p = 1$. ρ can therefore be calculated as

$$\rho = e^{-\lambda' \pi d_0^2} \left(1 - Q\left(\frac{\gamma_t N - m_y}{v_y}\right)\right) \tag{13}$$

Throughput Calculation: We denote the mean and variance of $I_{S\to P}$ (or I_S) as (m_S, v_S^2) and those of $I_{P\to S}$ (or I_P) as (m_P, v_P^2). Due to the independence assumption of networks, $I_P + I_{S\to P}$ in (5) (or $I_S + I_{P\to S}$ in (6)) will be Gaussian random variable with parameters $(m_P + m_S, v_P^2 + v_S^2)$. From (10) and (11), mean and variance depend on both p_1 and p_2.

Let us denote $SINR = \frac{Px^2R^{-\alpha}}{I+\sigma^2}$, where x is a Rayleigh random variable with $E\{x^2\} = 1$ and I is a Gaussian random variable with parameters (m, v^2). We have

$$\Pr\{SINR > \theta\} = \Pr\{\frac{x^2}{I+\sigma^2} > \frac{\theta R^\alpha}{P}\}. \tag{14}$$

Denoting $Z = X^2$,

$$\begin{aligned}\Pr\{SINR > \theta\} &= E\left\{F_{c,Z}\left(\frac{\theta(I+\sigma^2)R^\alpha}{P}\right)\right\} \\ &= E\left\{\exp\left(\frac{-\theta(I+\sigma^2)R^\alpha}{P}\right)\right\}, \end{aligned}\tag{15}$$

where $F_c(.)$ denotes the Complementary Cumulative Distribution Function (CCDF). Above result is found by noting that Z is an exponential random variable and $F_{c,Z}(z) = e^{-z}$. After some mathematical manipulation, (15) will reduce to

$$\begin{aligned}\Pr\{SINR > \theta\} &= \exp\left(-\frac{\theta(m+v^2)}{P}\right)\exp\left(\frac{\theta^2 v^2}{2P^2}\right) \\ &\times Q\left(\frac{\theta v}{P} - \frac{m}{v}\right)\end{aligned} \tag{16}$$

Using this result, $\Pr\{SINR^P > \theta\}$ and $\Pr\{SINR^S > \theta\}$ can be found by substituting the corresponding parameters. Consequently, τ_p and τ_s can be found from (7) and (8).

IV. Optimization Problem

From previous section, τ_p and τ_s depend on both p_1 and p_2 and we can explicitly write them as $\tau_p(p_1, p_2)$ and $\tau_s(p_1, p_2)$. The choice of p_1 by primary node and p_2' (and thereby p_2 through $p_2 = p_2'\rho$) by the secondary network affects the throughput of both systems.

Interference Constraint: The probability that secondary interference on primary receiving nodes, i.e. primary nodes which are successfully receiving data, is larger than a threshold should be small. The interference constraint at a primary node can be written as:

$$\Pr\{I_{S\to P} > \eta, \text{ primary node is actively receiving}\} < \epsilon, \tag{17}$$

where (η, ϵ) are system-defined values. The two events in (17) are not independent. (17) can also be written as

$$p_1(1-p_1)Pr\{I_{S\to P} > \eta, SINR^P > \theta\} < \epsilon. \tag{18}$$

Denoting the constraint space obtained through (18), i.e. the (p_1, p_2) for which (18) is satisfied, as $\mathcal{C}$, we have $\mathcal{C} \subseteq [0,1]^2$. The primary and secondary networks choose p_1 and p_2 respectively such that $(p_1, p_2) \in \mathcal{C}$ and each try to optimize its own throughput. The primary and secondary nodes can compete for transmission as long as the chosen (p_1, p_2) belongs to $\mathcal{C}$. This is a game with constraints on multi-strategies [6].

V. Optimization of Throughput

```
Set p_2 = 0
p_1* = argmax_{0<=p_1<=1} tau_p(p_1, 0), find tau_p* = tau_p(p_1*, 0)
repeat
    TIME SLOT i: secondary optimization
    from constraint: tau_p* g(p_2) < eps, find the range of p_2:
    0 <= p_2 < p_{2,max}
    p_2* = argmax_{0<=p_2<p_{2,max}} tau_s(p_1*, p_2),
    BROADCAST p_2*
    TIME SLOT i+1: primary optimization
    from constraint: tau_p g(p_2*) < eps, find the range of tau_p:
    tau_p < tau_{p,max}
    p_1* = argmax_{tau_p<tau_{p,max}} tau_p(p_1, p_2*), find tau_p* = tau_p(p_1*, p_2*)
    BROADCAST p_1* and tau_p*
until sufficient convergence of p_1* and p_2*
lambda' = tau_p* lambda_p, calculate rho from (13), p_2' = p_2*/rho
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Algorithm 1: Progressive Transmission Probability Optimization Algorithm

In section II, the problem was formulated as a game with constraints on multi-strategies. Obtaining the constraint space through (18) is an involved task which may not lead into a practical result.

In this section, we discuss a subproblem which is amenable to a practical solution. We consider a primary network alone which is working in its optimal transmission probability and think of the secondary network as a network plugged into the primary. The secondary network, gradually (i.e., in a slot-by-slot fashion) increases its attempt probability while making sure that the interference constraint is not violated. Since the optimization is done slot by slot, the secondary interference in each time slot is assumed to be independent from the successful primary reception in the previous time slot and (18) can be written as:

$$\Pr\{I_{S\to P} > \eta\}Pr\{\text{primary node is actively receiving}\} < \epsilon, \tag{19}$$

or,

$$\Pr\{I_{S\to P} > \eta\}\tau_p < \epsilon. \tag{20}$$

Defining $g(p_2) \triangleq Pr\{I_{S\to P} > \eta\}$, we have

$$g(p_2) = Q\left(\frac{\eta - m_S}{v_S}\right) \tag{21}$$

which is dependent on p_2 through m_S and v_S.

The optimization algorithm consists of few steps. At each step, optimization is performed either in the primary or in the secondary network. At the end of each step, some information needs to be communicated to the other network. We propose that at least for the time duration that the algorithm is executed (The algorithm converges quite fast as we will see), primary and secondary networks establish a common control channel to communicate this information. This is beneficial to both networks as both will reach their optimal throughput and deviation from optimum transmission probabilities will degrade throughput in both networks[1]. When the secondary network is plugged, it uses its knowledge of p_1 and τ_p from the primary network and finds the optimum value p_2 which does not violate the interference constraint and also optimizes its throughput. This causes a secondary interference which was not present beforehand for the primary nodes. In the next time slot, primary network uses its knowledge of p_2 to optimize its throughput by finding the optimal value of p_1. The new choice of p_1 and the resulting τ_p will affect the secondary throughput and the secondary network tries to re-optimize its throughput in the next time slot. Secondary and primary networks sequentially adjust their transmission probabilities one after the other until convergence takes place (See Algorithm 1).

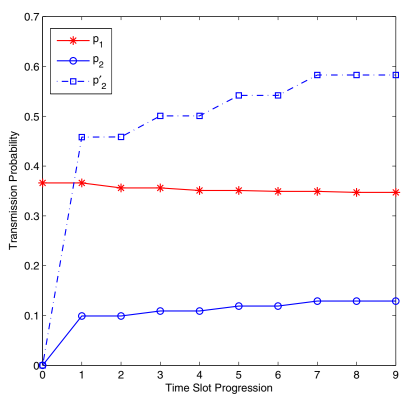

Fig. 1: Transmission probabilities vs time slot progression ($\alpha = 4$)

VI. Simulation Results

We consider primary and secondary ad hoc networks randomly and independently distributed with parameters $\lambda_p = 1$ and $\lambda_s = 2$ respectively. The transmission power levels in secondary and primary networks is assumed to be $P_S = 100\mu$W and $P_P = 200\mu$W, respectively[2]. The noise power is considered to be $\sigma^2 = 5$fW and SINR threshold for signal detection is assumed to be $\theta = 7$dB. The system defined values for (η, ϵ) in the interference constraint is set to $(5 \times 10^{-4}, 0.001)$. d_0 is assumed to be 1m and R is set to be 0.5m.

In Figure 1, for the path loss exponent $\alpha = 4$, we use the progressive transmission probability optimization algorithm to find the optimum transmission probabilities. The result shows that the primary transmission probability gradually decreases (starting from $p_1 = 0.366$) and the secondary transmission probability gradually increases (starting from $p_2 = 0$) to reach their stable values of $p_1 = 0.1290$ and $p_2 = .3470$ after 8 time slots. The values of $p_2' = p_2/\rho$ are shown in the same graph. In Figure 2, we have shown the corresponding values of throughput in each time slot. The primary throughput gradually decreases (starting from $\tau_P = 0.1022$) and the secondary throughput gradually increases (starting from $\tau_S = 0$) and reach the stable values of $\tau_P = 0.0894$ and $\tau_S = 0.0229$. It indicates that at the optimum transmission probabilities, the primary network is deviated slightly from its optimum throughput, while the secondary network gains a considerable amount of throughput. The sum of primary and secondary optimum throughput is also shown to be larger than the optimum primary throughput when the secondary network is not present.

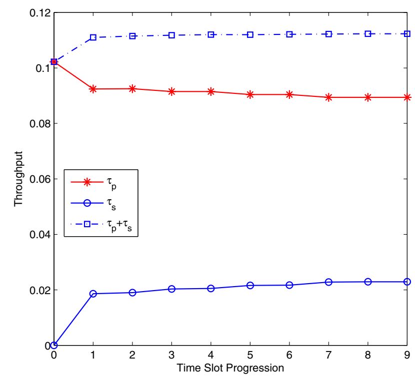

Fig. 2: Throughput vs time slot progression ($\alpha = 4$)

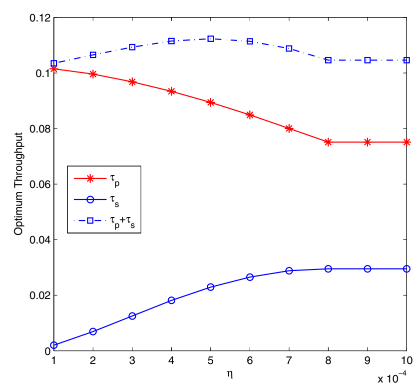

Fig. 3: Optimum throughput vs η (interference threshold)

[1] This is in fact the equilibrium of the constrained game described in section IV.

[2] Secondary network is generally assumed to have lower power and to be more densely distributed compared to the primary.

In Figure 3, optimum throughput is plotted versus η in the interference constraint when it varies from 10^{-4} to 10^{-3}. As η increases, i.e., as the the constraint becomes looser, more primary throughput is lost and more secondary throughput is gained. The sum throughput, on the other hand, has less variation. The same observation can be seen in Figure 4, where optimum throughput is plotted versus ϵ in the interference constraint.

In Figure 5, the optimum transmission probabilities is shown versus the path loss exponent (α). In the same graph, the values of $p_2' = p_2/\rho$ are shown. The corresponding values of throughput (using the converged values of p_1 and p_2) are shown in Figure 6. The results indicate that the optimum transmission probabilities and the values of throughput increase with path loss exponent.

VII. Conclusion

The throughput in the coexisting primary and secondary ad hoc networks depend on the choices of transmission probabilities of both networks. Meanwhile, secondary network needs to be alert in its choice of transmission probability so that it does not violate the interference constraint on the primary nodes. We consider a simple Gaussian model for interference and incorporate a sensing mechanism by secondary nodes to avoid interference with the primary receiving nodes. We assume that the secondary network is plugged into a primary network which is already operating at its optimum parameters and propose a progressive transmission probability optimization algorithm which progressively optimizes the primary and secondary transmission probabilities while ensuring that the interference constraint is satisfied. Simulation results indicate that at the optimum transmission probabilities, the primary network is deviated slightly from its optimum throughput, while the secondary network gains a considerable amount of throughput. In addition, the sum throughput is found to increase by using these optimum transmission probabilities.

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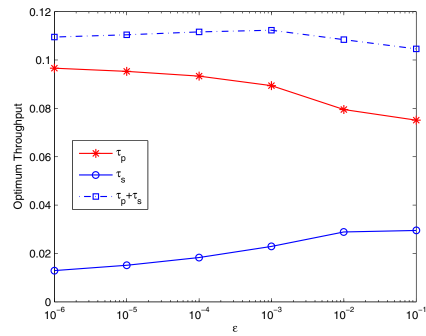

Fig. 4: Optimum throughput vs ϵ

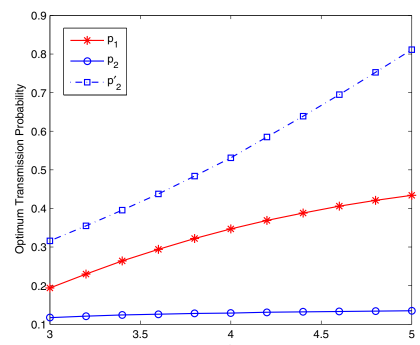

Fig. 5: Optimum transmission probabilities vs path loss exponent

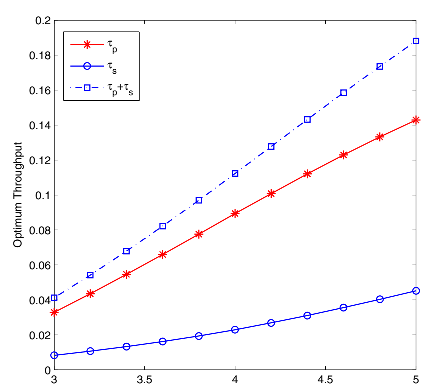

Fig. 6: Optimum throughput vs path loss exponent