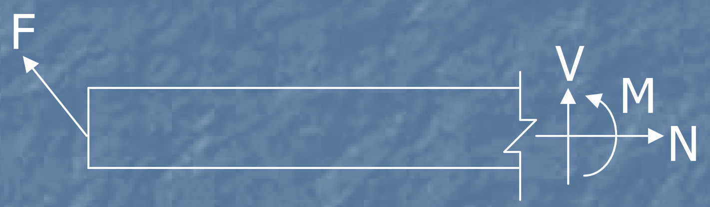
FE Review
Mechanics of Materials

FE Mechanics of Materials Review

Stress

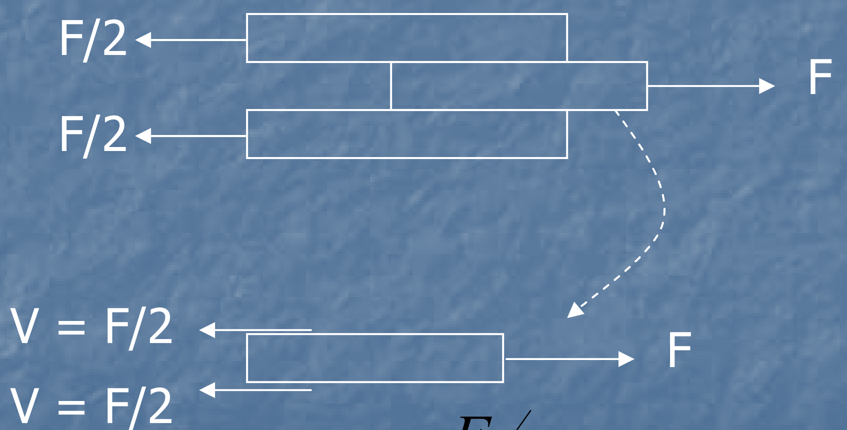


N = internal normal force (or P)
V = internal shear force
M = internal moment

Normal Stress = $\sigma = \frac{N}{A} = \frac{P}{A}$

Average Shear Stress = $\tau = \frac{V}{A}$

Double Shear



$$\tau = \frac{F/2}{A}$$

FE Mechanics of Materials Review

Strain

Normal Strain $$\varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0} = \frac{\delta}{L_0}$$ Units of length/length

ε = normal strain
ΔL = change in length = δ
L_0 = original length
L = length after deformation (after axial load is applied)

Percent Elongation = $$\frac{\Delta L}{L_0} \times 100$$

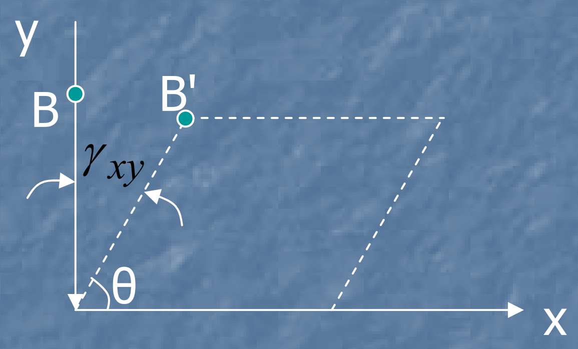
Percent Reduction in Area = $$\frac{A_i - A_f}{A_i} \times 100$$

A_i = initial cross- sectional area
A_f = final cross-sectional area

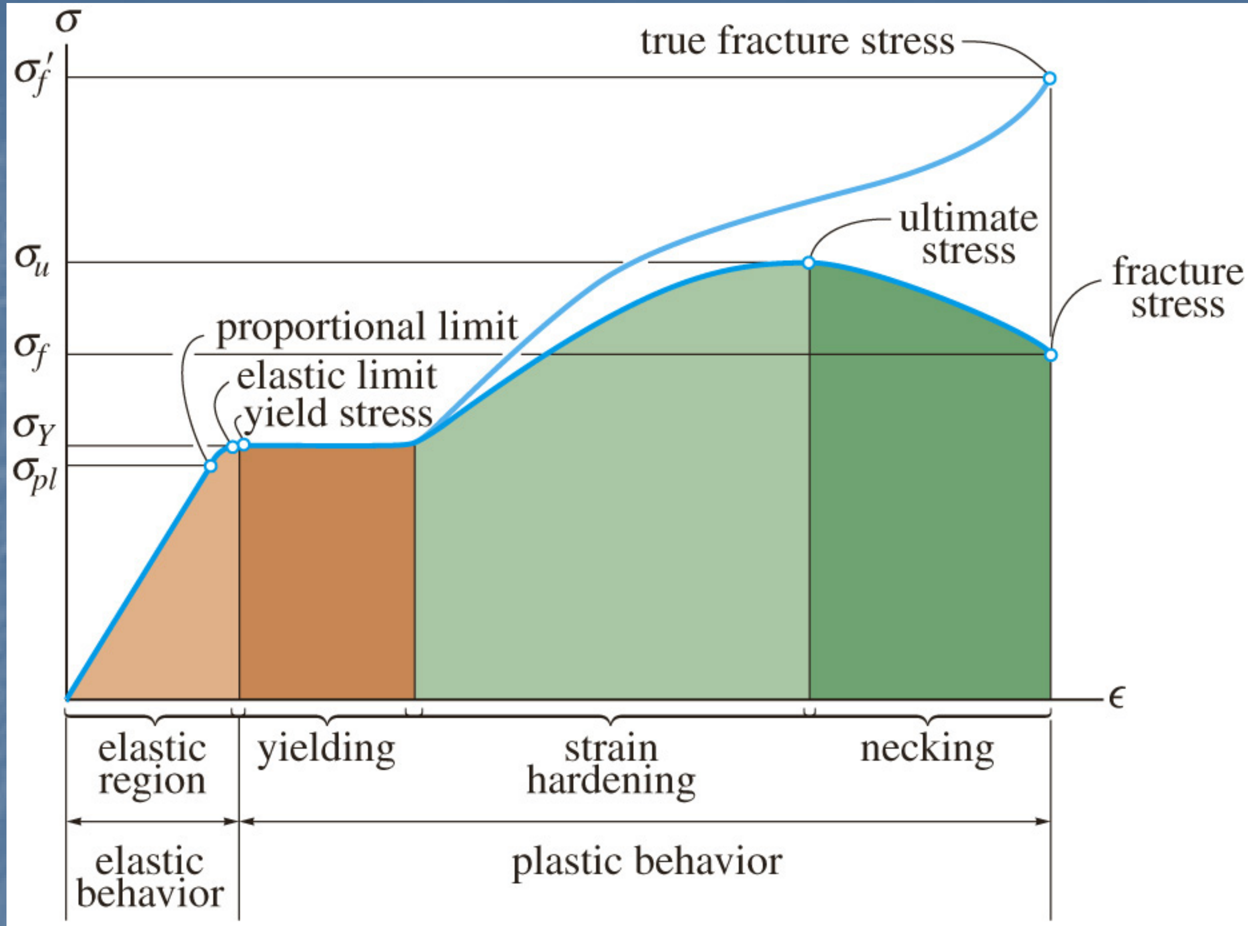
FE Mechanics of Materials Review

Strain

Shear Strain = change in angle , usually expressed in radians



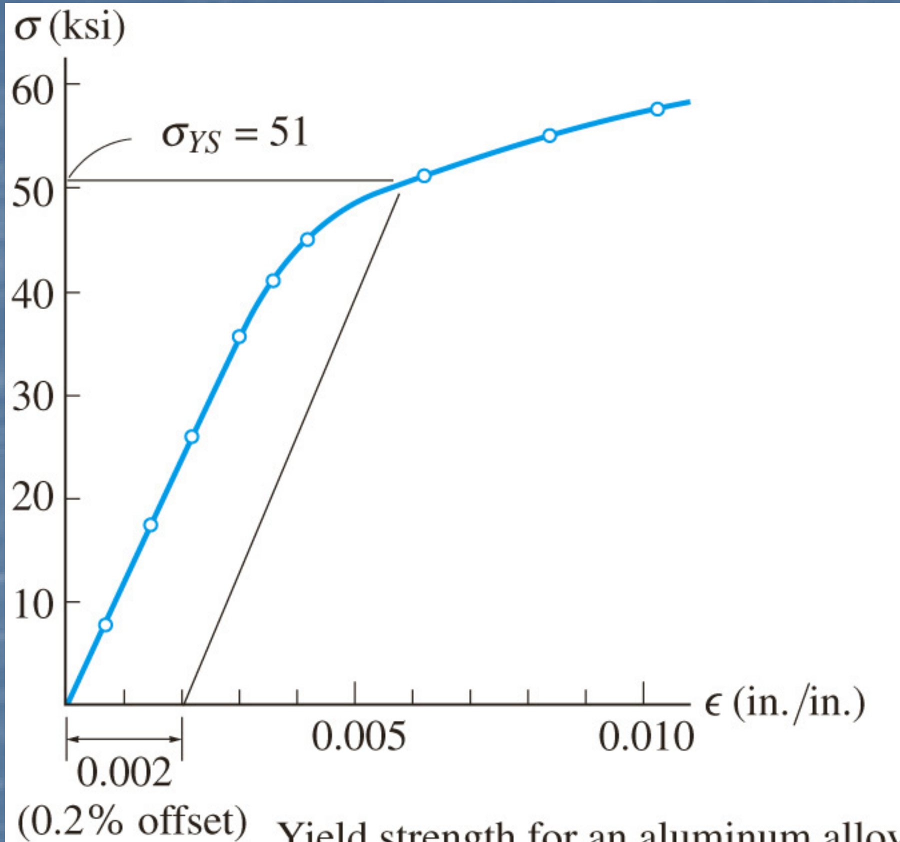
FE Mechanics of Materials Review



Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

Stress-Strain Diagram for Normal Stress-Strain

FE Mechanics of Materials Review



Yield strength for an aluminum alloy

FE Mechanics of Materials Review

Hooke's Law (one-dimension)

$$\sigma = E\varepsilon$$

σ = normal stress, force/length^2
E = modulus of elasticity, force/length^2
ε = normal strain, length/length

$$\tau = G\gamma$$

τ = shear stress, force/length^2
G = shear modulus of rigidity, force/length^2
γ = shear strain, radians

FE Mechanics of Materials Review

$$G = \frac{E}{2(1+\nu)}$$

ν = Poisson's ratio = -(lateral strain)/(longitudinal strain)

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

$$\varepsilon_{lat} = \frac{\delta'}{r}$$ change in radius over original radius

$$\varepsilon_{long} = \frac{\delta}{L}$$ change in length over original length

FE Mechanics of Materials Review

Axial Load

If A (cross-sectional area), E (modulus of elasticity), and P (load) are constant in a member (and L is its length):

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\delta/L} \Rightarrow \qquad \delta = \frac{PL}{AE}$$ Change in length

If A, E, or P change from one region to the next:

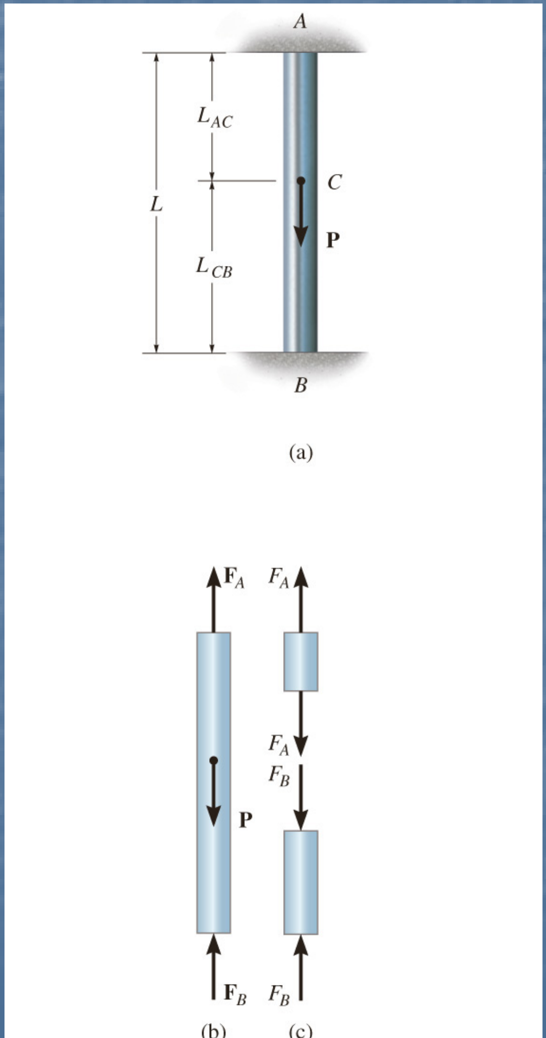
$$\delta = \sum \frac{PL}{AE}$$ Apply to each section where A, E, & P are constant

$\delta_{A/B}$ = displacement of pt A relative to pt B

δ_{A} = displacement of pt A relative to fixed end

FE Mechanics of Materials Review

-Remember principle of superposition used for indeterminate structures
- equilibrium/compatibility



FE Mechanics of Materials Review

Thermal Deformations

$$\delta_t = \alpha(\Delta T)L = \alpha(T - T_0)L$$

δ_t = change in length due to temperature change, units of length

α = coefficient of thermal expansion, units of 1/°

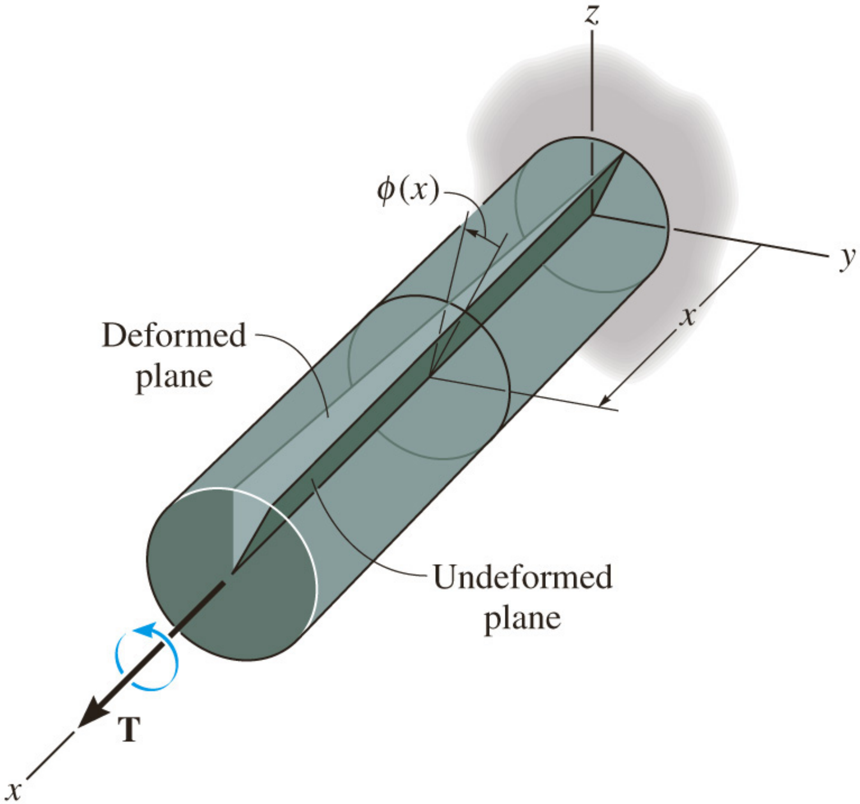
T = final temperature, degrees

T_0 = initial temperature, degrees

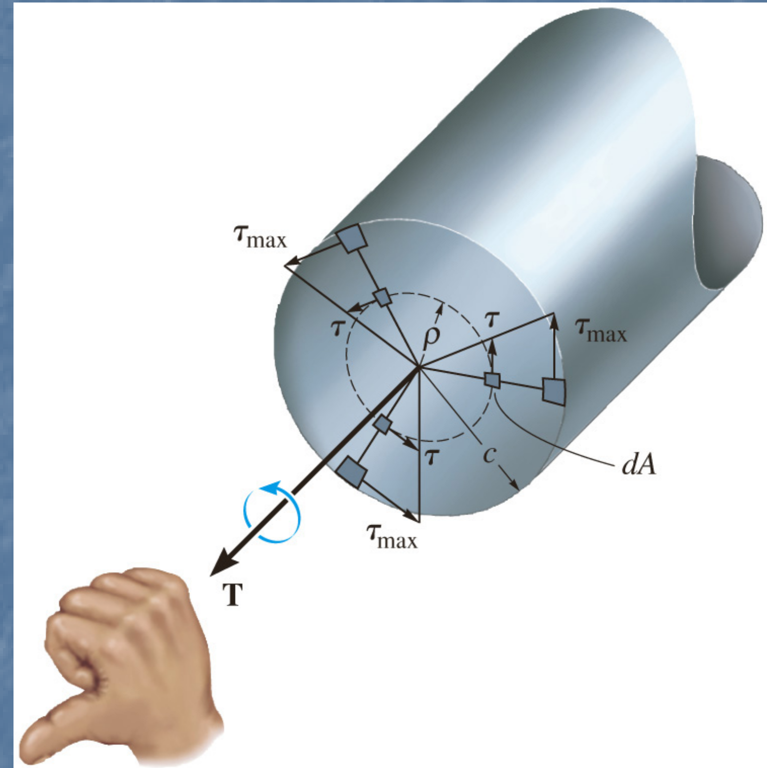
FE Mechanics of Materials Review

Torsion

Torque – a moment that tends to twist a member about its longitudinal axis



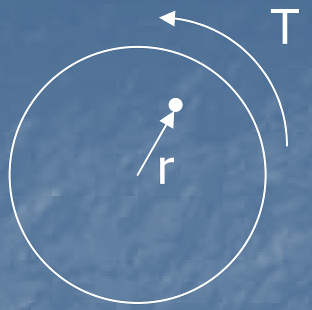
The angle of twist $\phi(x)$ increases as x increases.



Shear stress varies linearly along each radial line of the cross section.

Shear stress, τ, and shear strain, γ, vary linearly from 0 at center to maximum at outside of shaft

FE Mechanics of Materials Review



$$\tau = \frac{Tr}{J}$$

τ = shear stress, force/length^2

T = applied torque, force·length

r = distance from center to point of interest in cross-section (maximum is the total radius dimension)

J = polar moment of inertia (see table at end of STATICS section in FE review manual), length^4

$$\phi = \frac{TL}{JG}$$

ϕ = angle of twist, radians

L = length of shaft

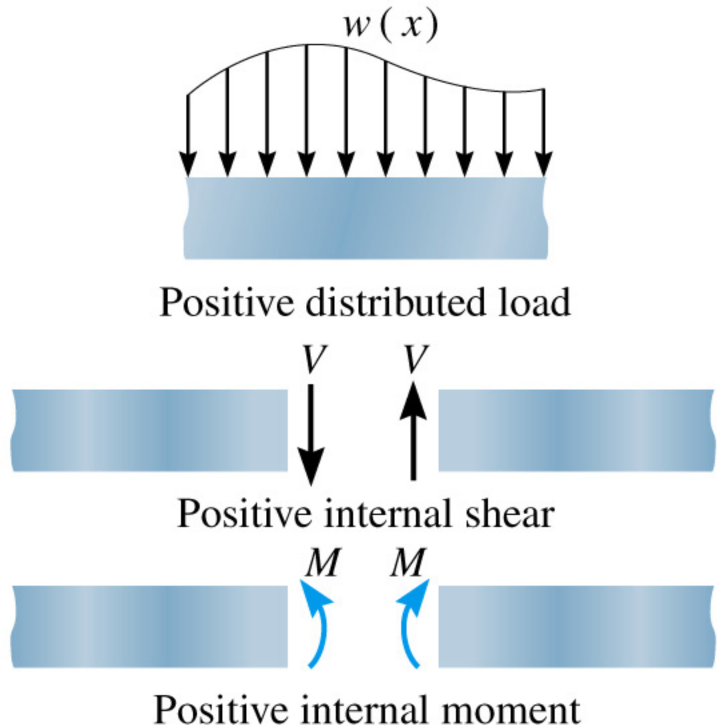
G = shear modulus of rigidity, force/length^2

$$\tau_{\phi z} = G\gamma_{\phi z} = Gr(d\phi / dz)$$

$(d\phi / dz)$ = twist per unit length, or rate of twist

FE Mechanics of Materials Review

<u>Bending</u>



Beam sign convention

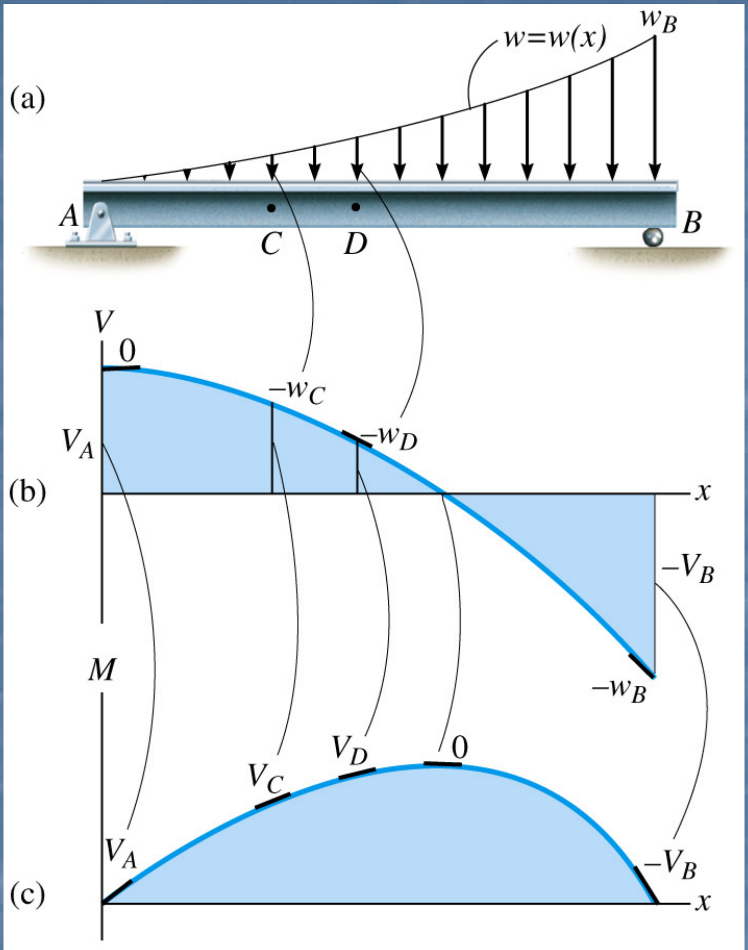
Positive Bending

Makes compression in top fibers and tension in bottom fibers

Negative Bending

Makes tension in top fibers and compression in bottom fibers

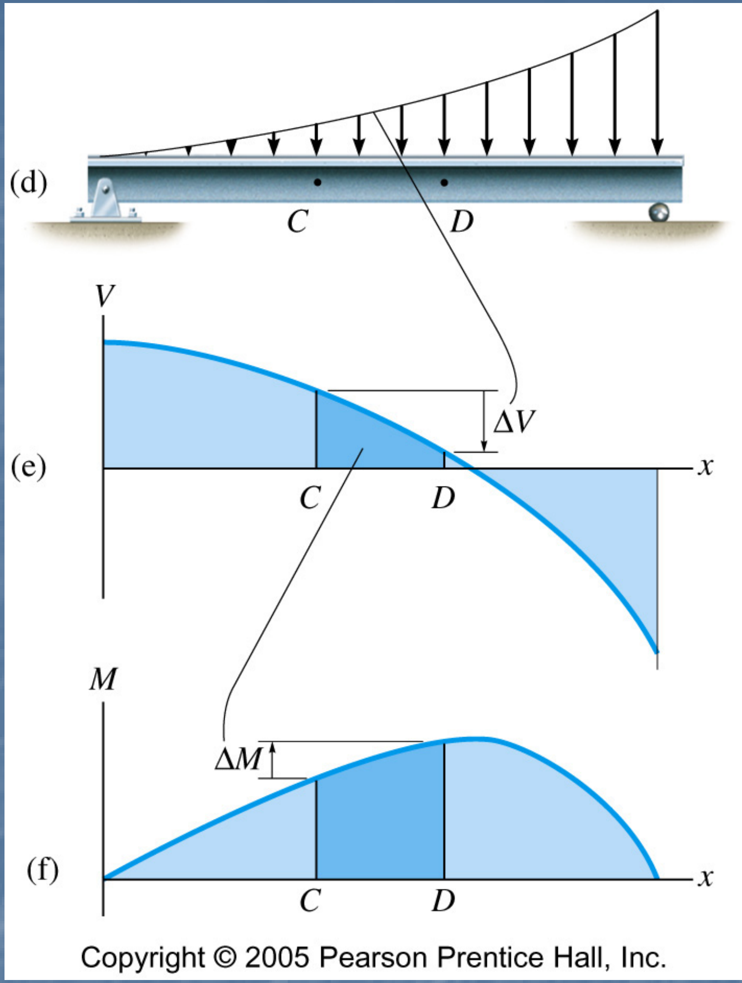
FE Mechanics of Materials Review



Slope of shear diagram = negative of distributed loading value → $-\frac{dV}{dx} = q(x)$

Slope of moment diagram = shear value → $\frac{dM}{dx} = V$

FE Mechanics of Materials Review

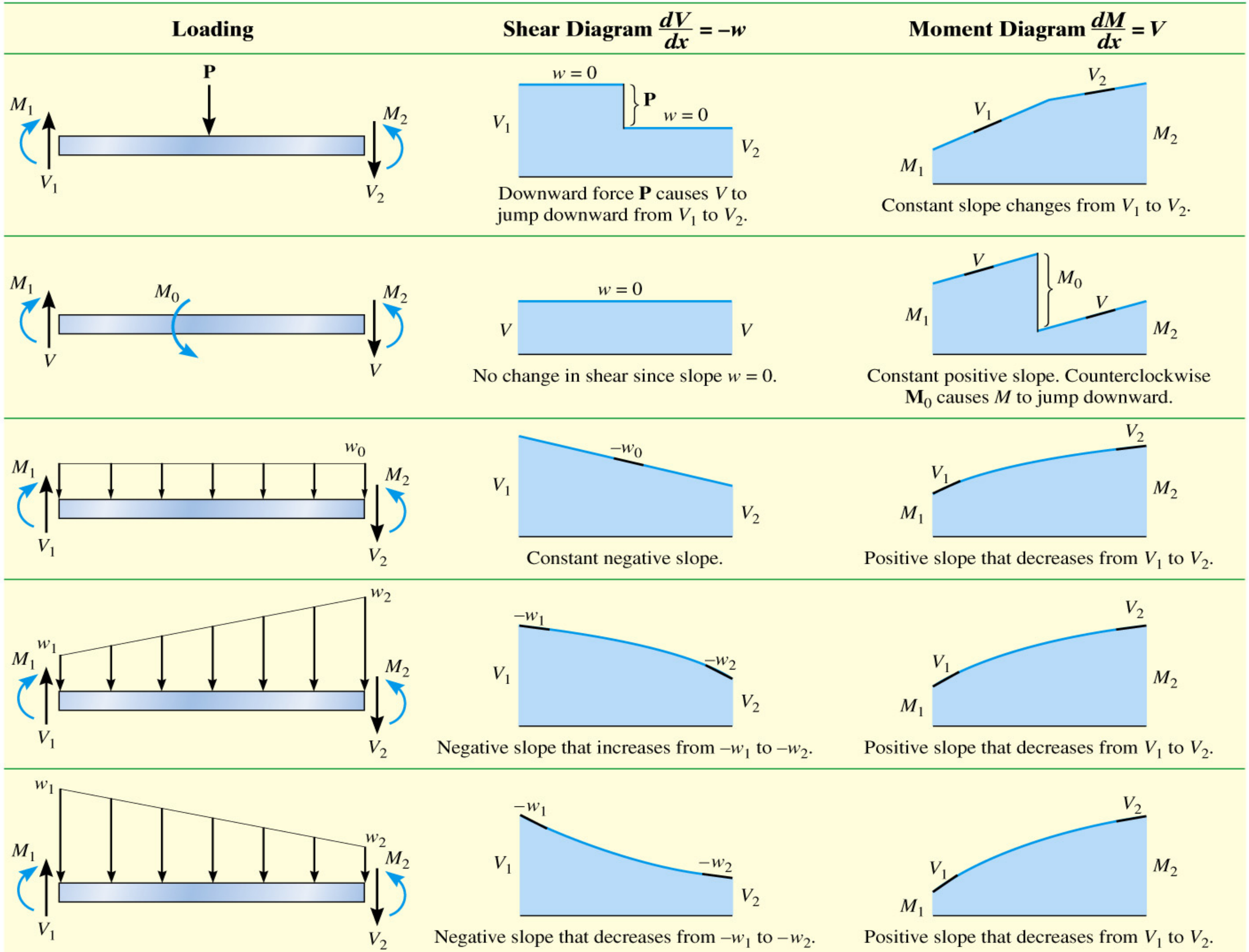


Change in shear between two points = neg. of area under distributed loading diagram between those two points ➔

$$V_2 - V_1 = \int_{x_1}^{x_2} [-q(x)]dx$$

Change in moment between two points = area under shear diagram between those two points ➔

$$M_2 - M_1 = \int_{x_1}^{x_2} [V(x)]dx$$


Loading
Shear Diagram $\frac{dV}{dx} = -w$
Moment Diagram $\frac{dM}{dx} = V$
Downward force **P** causes V to jump downward from V_1 to V_2.
Constant slope changes from V_1 to V_2.
No change in shear since slope $w = 0$.
Constant positive slope. Counterclockwise $\mathbf{M}_0$ causes M to jump downward.
Constant negative slope.
Positive slope that decreases from V_1 to V_2.
Negative slope that increases from $-w_1$ to $-w_2$.
Positive slope that decreases from V_1 to V_2.
Negative slope that decreases from $-w_1$ to $-w_2$.
Positive slope that decreases from V_1 to V_2.

FE Mechanics of Materials Review

Stresses in Beams

$$\sigma = -\frac{My}{I}$$

σ = normal stress due to bending moment, force/length^2

y = distance from neutral axis to the longitudinal fiber in question, length (y positive above NA, neg below)

I = moment of inertia of cross-section, length^4

$$\sigma_{\max} = \pm\frac{Mc}{I}$$

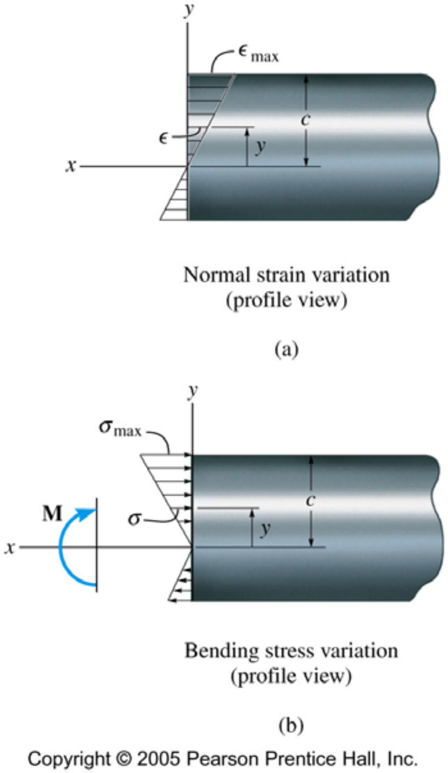
c = maximum value of y; distance from neutral axis to extreme fiber

$$\varepsilon_x = -y/\rho$$

ρ = radius of curvature of deflected axis of the beam

From $\sigma = E\varepsilon = -E\, y/\rho$ and $\frac{1}{\rho} = \frac{M}{EI}$

➔ $\sigma = -\frac{My}{I}$



Normal strain variation
(profile view)

(a)

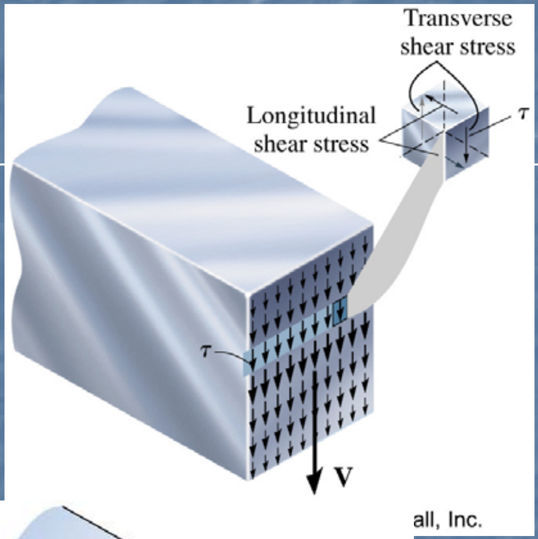
Bending stress variation
(profile view)

(b)

FE Mechanics of Materials Review

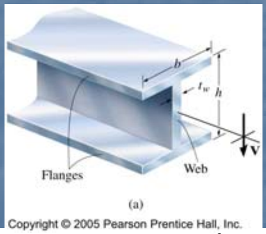
$$S = I/c$$ S = elastic section modulus of beam

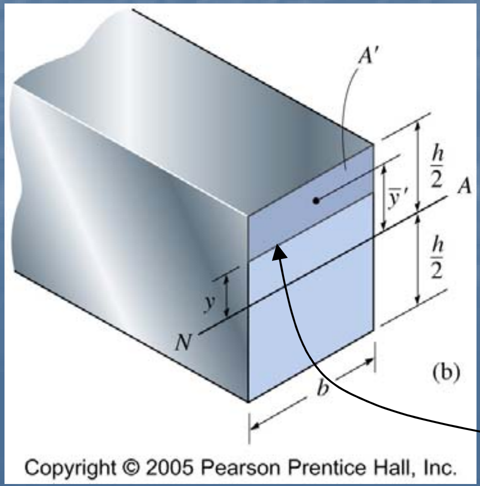
Then $$\sigma_{max} = \pm\frac{Mc}{I} = \pm\frac{M}{S}$$



Transverse Shear Stress: $$\tau = \frac{VQ}{It}$$

Transverse Shear Flow: $$q = \frac{VQ}{I}$$

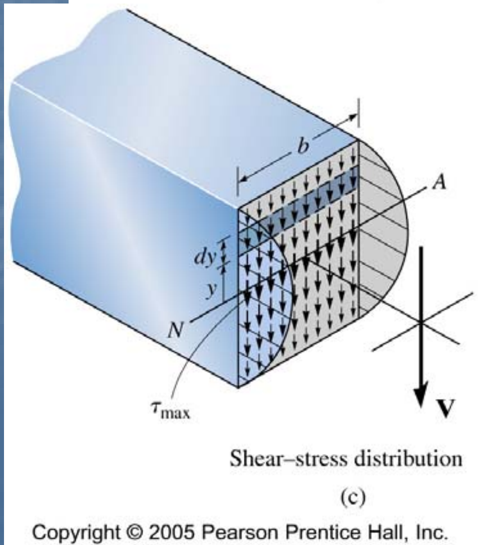


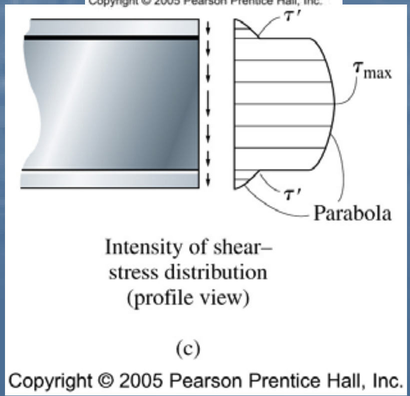


$$Q = \bar{y}' A'$$

t = thickness of cross-section at point of interest

t = b here





FE Mechanics of Materials Review

Thin-Walled Pressure Vessels (r / t >= 10)

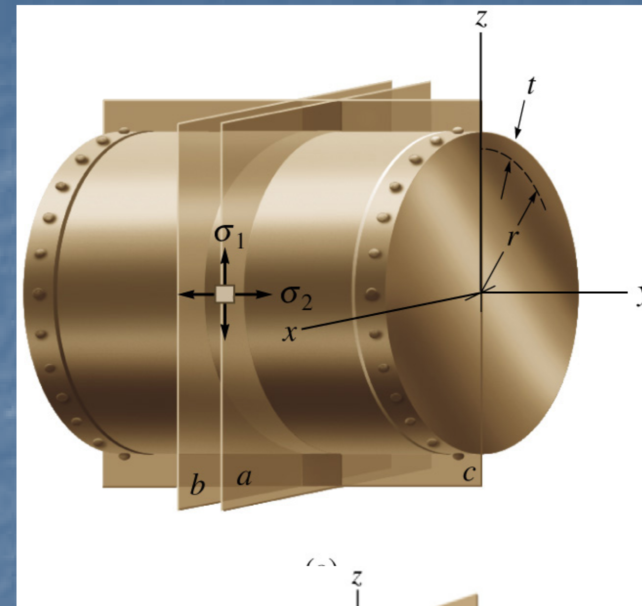
Cylindrical Vessels

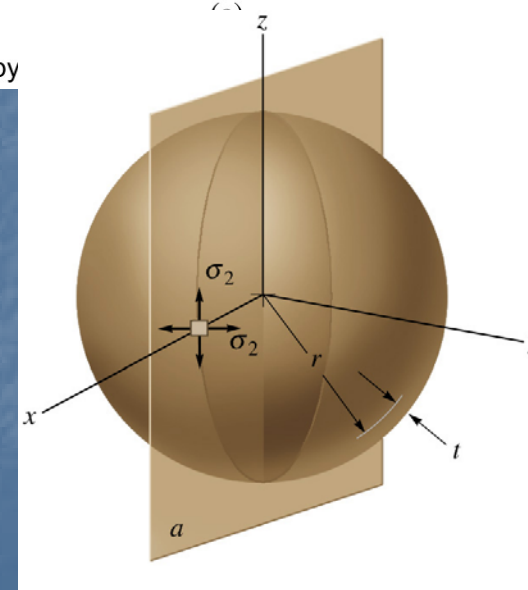
$$\sigma_t = \frac{pr}{t} = \sigma_1$$

σ_1 = hoop stress in circumferential direction
p = gage pressure, force/length^2
r = inner radius
t = wall thickness

$$\sigma_a = \frac{pr}{2t} = \sigma_2$$ = axial stress in longitudinal direction

See FE review manual for thick-walled pressure vessel formulas.





(a)

FE Mechanics of Materials Review

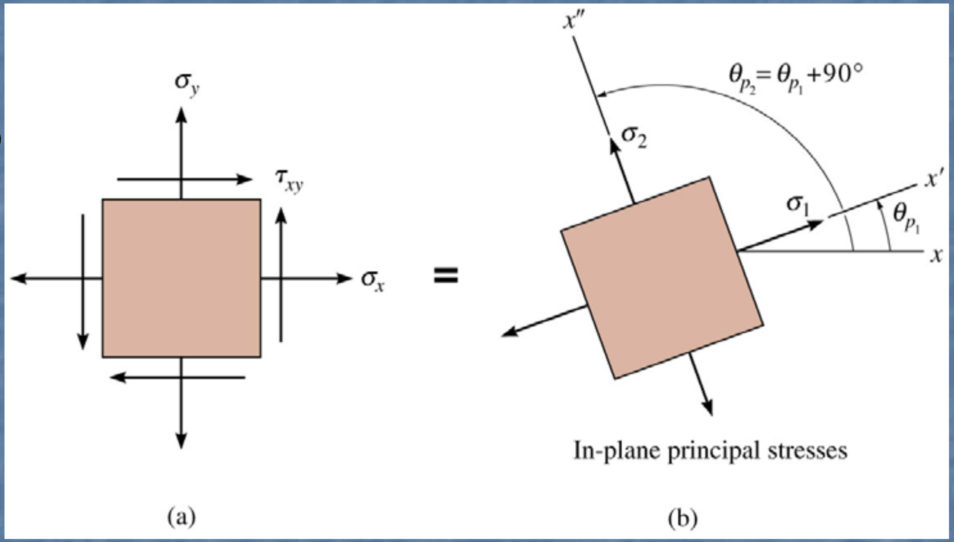
2-D State of Stress

Stress Transformation

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



Principal Stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{\left(\frac{\sigma_x - \sigma_y}{2}\right)}$$

No shear stress acts on principal planes!

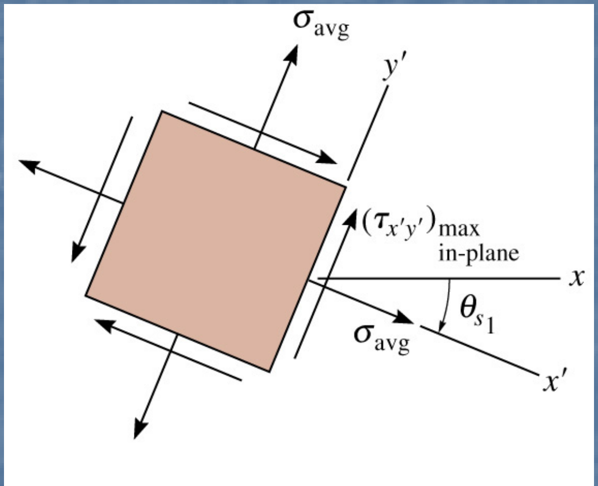
FE Mechanics of Materials Review

Maximum In-plane Shear Stress

$$\tau_{in-plane}^{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$\tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2}\right) / \tau_{xy}$$



(d)

FE Mechanics of Materials Review

Mohr's Circle – Stress, 2D

σ, positive to the right
tau, positive downward!

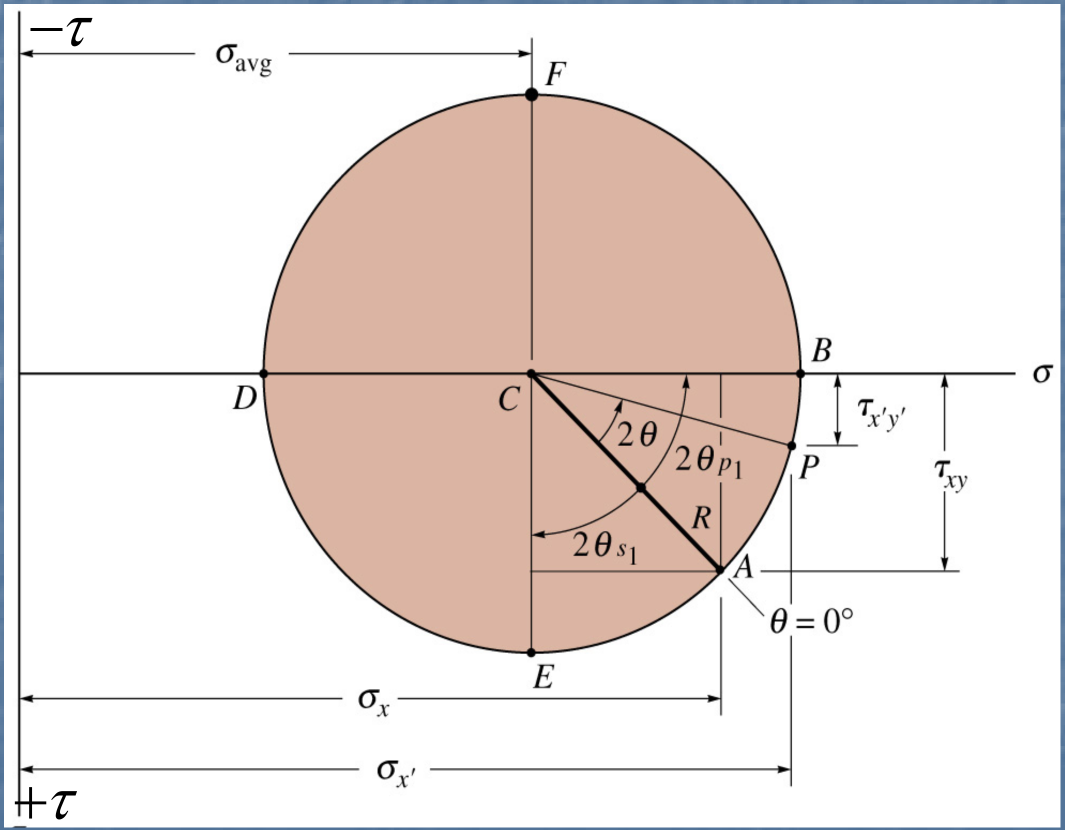
Center: Point C($\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$,0)

$$R = \sqrt{(\sigma_x - \sigma_{avg})^2 + (\tau_{xy})^2}$$

$$\sigma_1 = \sigma_{avg} + R = \sigma_a$$

$$\sigma_2 = \sigma_{avg} - R = \sigma_b$$

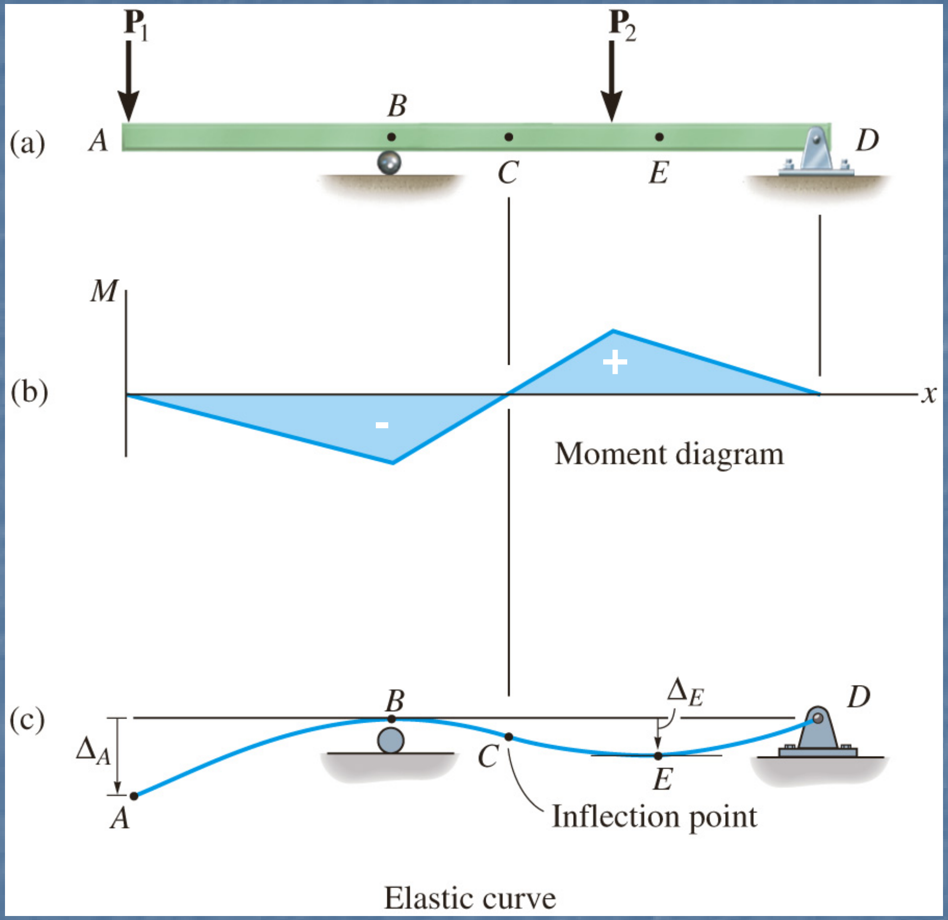
$$\tau_{in-plane}^{max} = R$$

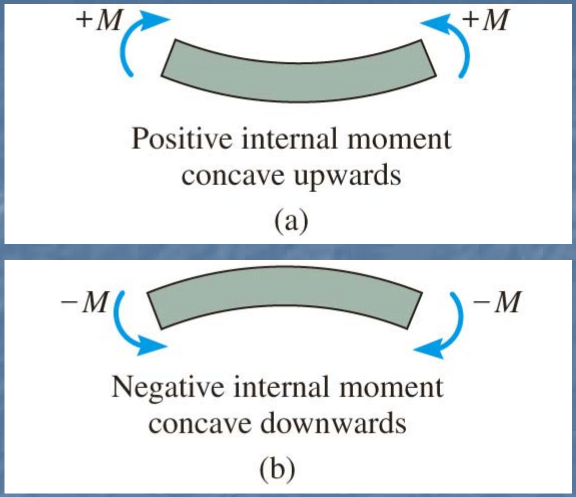


A rotation of θ to the x' axis on the element will correspond to a rotation of 2θ on Mohr's circle!

FE Mechanics of Materials Review

Beam Deflections





Inflection point is where the elastic curve has zero curvature = zero moment

$$\frac{1}{\rho} = -\frac{\varepsilon}{y} \quad \text{Also } \varepsilon = \frac{\sigma}{E} \quad and \quad \sigma = \frac{-My}{I} \Rightarrow \boxed{\frac{1}{\rho} = \frac{M}{EI}}$$

ρ = radius of curvature of deflected axis of the beam

FE Mechanics of Materials Review

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{d^2 y}{dx^2} \Rightarrow \quad M(x) = EI\,\frac{d^2 y}{dx^2}$$

from calculus, for very small curvatures

$$V = \left(\frac{dM(x)}{dx}\right) \Rightarrow \quad V(x) = EI\,\frac{d^3 y}{dx^3}$$

for EI constant

$$-w(x) = \frac{dV(x)}{dx} \Rightarrow -w(x) = EI\,\frac{d^4 v}{dx^4} = -q$$

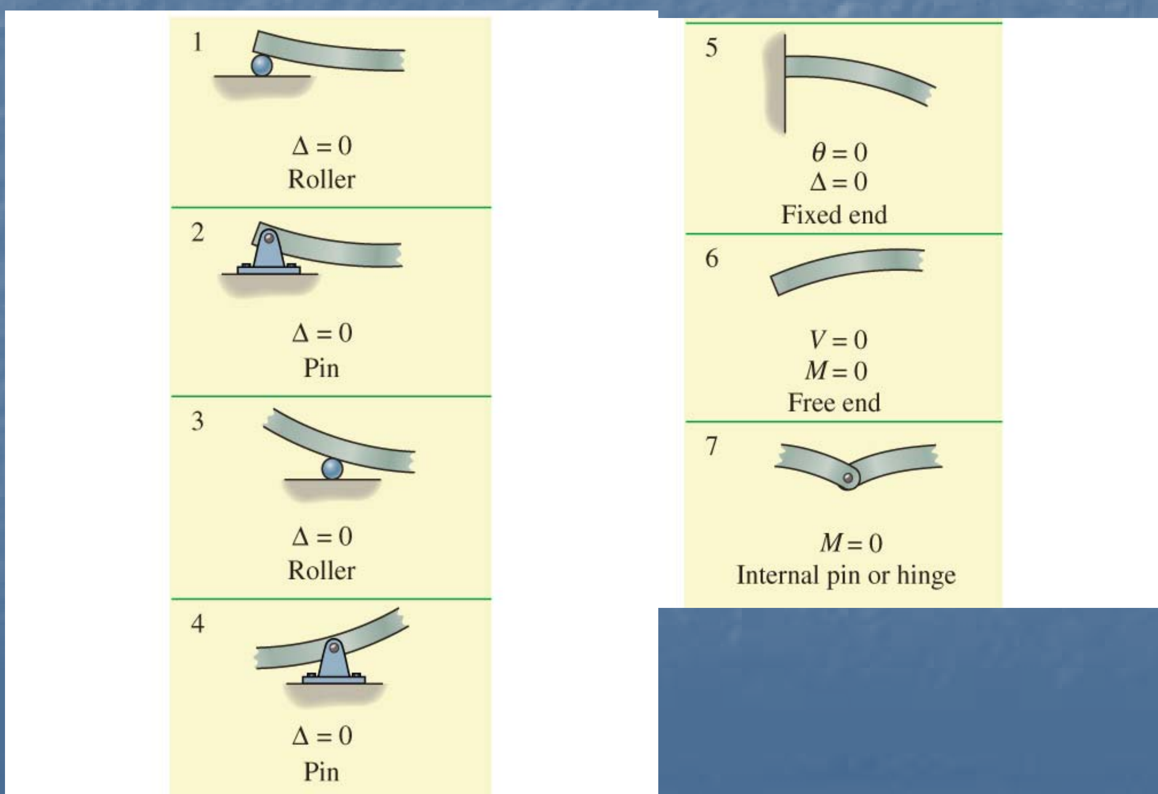
for EI constant

Double integrate moment equation to get deflection; use boundary conditions from supports ➔ rollers and pins restrict displacement; fixed supports restrict displacements *and* rotations

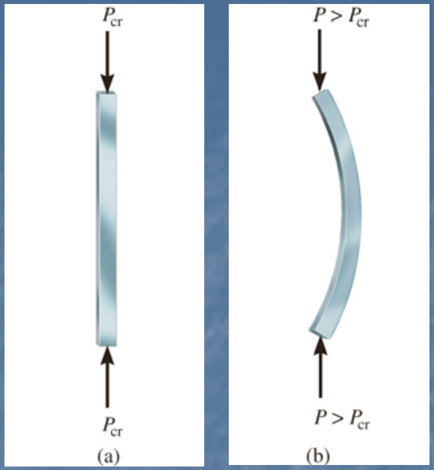
FE Mechanics of Materials Review

$$M(x) = EI\frac{d^2y}{dx^2} \Rightarrow \quad y = \frac{\int [\int M(x)dx]dx}{EI}$$

- For each integration the "*constant of integration*" has to be defined, based on boundary conditions



FE Mechanics of Materials Review

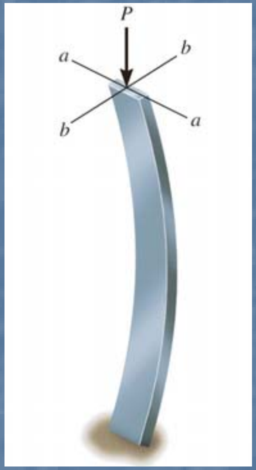


Column Buckling

$$P_{cr} = \frac{\pi^2 EI}{\ell^2}$$

Euler Buckling Formula
(for ideal column with pinned ends)

P_{cr} = critical axial loading (maximum axial load that a column can support just before it buckles)



I = the smallest moment of inertia of the cross-section

ℓ = unbraced column length

$r = \sqrt{\frac{I}{A}}$ = radius of gyration, units of length

$$r = \sqrt{\frac{I}{A}} \Rightarrow I = r^2 A \Rightarrow \quad \sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(\ell / r)^2}$$

= critical buckling stress

ℓ / r = slenderness ratio for the column

FE Mechanics of Materials Review

Euler's formula is only valid when $\sigma_{cr} \leq \sigma_{yield}$.

When $\sigma_{cr} > \sigma_{yield}$, then the section will simply yield.

For columns that have end conditions other than pinned-pinned:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

K = the effective length factor (see next page)

KL = L_e = the effective length

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

KL/r = the effective slenderness ratio

FE Mechanics of Materials Review

Effective Length Factors