FE Review
Mechanics of Materials
Stress

N = internal normal force (or P)
V = internal shear force
M = internal moment

Normal Stress = \( \sigma = \frac{N}{A} = \frac{P}{A} \)

Average Shear Stress = \( \tau = \frac{V}{A} \)

Double Shear

\( \tau = \frac{F}{2A} \)
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**Strain**

Normal Strain  
\[ \varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0} = \frac{\delta}{L_0} \]  
Units of length/length

\( \varepsilon \) = normal strain  
\( \Delta L \) = change in length = \( \delta \)  
\( L_0 \) = original length  
\( L \) = length after deformation (after axial load is applied)

Percent Elongation  
\[ \text{Percent Elongation} = \left( \frac{\Delta L}{L_0} \right) \times 100 \]

Percent Reduction in Area  
\[ \text{Percent Reduction in Area} = \left( \frac{A_i - A_f}{A_i} \right) \times 100 \]

\( A_i \) = initial cross-sectional area  
\( A_f \) = final cross-sectional area
Strain

Shear Strain = change in angle, usually expressed in radians
Stress-Strain Diagram for Normal Stress-Strain

Conventional and true stress-strain diagrams for ductile material (steel) (not to scale)

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Yield strength for an aluminum alloy

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Hooke's Law (one-dimension)

\[ \sigma = E \varepsilon \]

\( \sigma \) = normal stress, force/length\(^2\)
\( E \) = modulus of elasticity, force/length\(^2\)
\( \varepsilon \) = normal strain, length/length

\[ \tau = G \gamma \]

\( \tau \) = shear stress, force/length\(^2\)
\( G \) = shear modulus of rigidity, force/length\(^2\)
\( \gamma \) = shear strain, radians
$G = \frac{E}{2(1 + \nu)}$

$\nu = \text{Poisson's ratio} = -(\text{lateral strain})/(\text{longitudinal strain})$

$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$

$\varepsilon_{lat} = \frac{\delta'}{r}$  \hspace{1cm} \text{change in radius over original radius}

$\varepsilon_{long} = \frac{\delta}{L}$  \hspace{1cm} \text{change in length over original length}
**Axial Load**

If A (cross-sectional area), E (modulus of elasticity), and P (load) are constant in a member (and L is its length):

\[
E = \frac{\sigma}{\varepsilon} = \frac{P}{A} \Rightarrow \delta = \frac{PL}{AE} \quad \text{Change in length}
\]

If A, E, or P change from one region to the next:

\[
\delta = \sum \frac{PL}{AE} \quad \text{Apply to each section where A, E, & P are constant}
\]

\[
\delta_{A/B} = \text{displacement of pt A relative to pt B}
\]

\[
\delta_A = \text{displacement of pt A relative to fixed end}
\]
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- Remember principle of superposition used for indeterminate structures
  - equilibrium/compatibility
**Thermal Deformations**

\[ \delta_t = \alpha(\Delta T)L = \alpha(T - T_0)L \]

- \( \delta_t \) = change in length due to temperature change, units of length
- \( \alpha \) = coefficient of thermal expansion, units of 1/°
- \( T \) = final temperature, degrees
- \( T_0 \) = initial temperature, degrees
Torsion

Torque – a moment that tends to twist a member about its longitudinal axis

Shear stress, $\tau$, and shear strain, $\gamma$, vary linearly from 0 at center to maximum at outside of shaft.
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\[ \tau = \frac{Tr}{J} \]

\( \tau \) = shear stress, force/length^2

\( T \) = applied torque, force·length

\( r \) = distance from center to point of interest in cross-section
(maximum is the total radius dimension)

\( J \) = polar moment of inertia (see table at end of STATICS section in FE review manual), length^4

\[ \phi = \frac{TL}{JG} \]

\( \phi \) = angle of twist, radians

\( L \) = length of shaft

\( G \) = shear modulus of rigidity, force/length^2

\[ \tau \phi_z = G \gamma \phi_z = Gr(d\phi/dz) \]

\[ (d\phi/dz) = \text{twist per unit length, or rate of twist} \]
Bending

Positive Bending
Makes compression in top fibers and tension in bottom fibers

Negative Bending
Makes tension in top fibers and compression in bottom fibers
Slope of shear diagram = negative of distributed loading value \( \Rightarrow -\frac{dV}{dx} = q(x) \)

Slope of moment diagram = shear value \( \Rightarrow \frac{dM}{dx} = V \)
Change in shear between two points = neg. of area under distributed loading diagram between those two points

\[ V_2 - V_1 = \int_{x_1}^{x_2} [-q(x)]dx \]

Change in moment between two points = area under shear diagram between those two points

\[ M_2 - M_1 = \int_{x_1}^{x_2} [V(x)]dx \]
<table>
<thead>
<tr>
<th>Loading</th>
<th>Shear Diagram $\frac{dV}{dx} = -w$</th>
<th>Moment Diagram $\frac{dM}{dx} = V$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td>$w = 0$ Downward force $P$ causes $V$ to jump downward from $V_1$ to $V_2$.</td>
<td><img src="image2.png" alt="Diagram" /> Constant slope changes from $V_1$ to $V_2$.</td>
</tr>
<tr>
<td><img src="image3.png" alt="Diagram" /></td>
<td>$w = 0$ No change in shear since slope $w = 0$.</td>
<td><img src="image4.png" alt="Diagram" /> Constant positive slope. Counterclockwise $M_0$ causes $M$ to jump downward.</td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram" /></td>
<td>$w_0$ Constant negative slope.</td>
<td><img src="image6.png" alt="Diagram" /> Positive slope that decreases from $V_1$ to $V_2$.</td>
</tr>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td>$-w_1$ Negative slope that increases from $-w_1$ to $-w_2$.</td>
<td><img src="image8.png" alt="Diagram" /> Positive slope that decreases from $V_1$ to $V_2$.</td>
</tr>
<tr>
<td><img src="image9.png" alt="Diagram" /></td>
<td>$w_1$ Negative slope that decreases from $-w_1$ to $-w_2$.</td>
<td><img src="image10.png" alt="Diagram" /> Positive slope that decreases from $V_1$ to $V_2$.</td>
</tr>
</tbody>
</table>

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**Stresses in Beams**

\[ \sigma = -\frac{My}{I} \]

- \( \sigma \): normal stress due to bending moment, force/length\(^2\)
- \( y \): distance from neutral axis to the longitudinal fiber in question, length (y positive above NA, neg below)
- \( I \): moment of inertia of cross-section, length\(^4\)

\[ \sigma_{\text{max}} = \pm \frac{Mc}{I} \]

- \( c \): maximum value of y; distance from neutral axis to extreme fiber

\[ \epsilon_x = -\frac{y}{\rho} \]

- \( \rho \): radius of curvature of deflected axis of the beam

From \[ \sigma = E\epsilon = -E\frac{y}{\rho} \] and \[ \frac{1}{\rho} = \frac{M}{EI} \]

\[ \Rightarrow \sigma = -\frac{My}{I} \]
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\[ S = \frac{I}{c} \]

Then

\[ \sigma_{\text{max}} = \pm \frac{Mc}{I} = \pm \frac{M}{S} \]

Transverse Shear Stress:

\[ \tau = \frac{VQ}{It} \]

Transverse Shear Flow:

\[ q = \frac{VQ}{I} \]

\[ Q = \overline{y'} A' \]

\( t = \text{thickness of cross-section at point of interest} \)

\( t = b \) here
**Thin-Walled Pressure Vessels (r/ t >= 10)**

Cylindrical Vessels

\[ \sigma_t = \frac{pr}{t} = \sigma_1 \]

- \( \sigma_1 \) = hoop stress in circumferential direction
- \( p \) = gage pressure, force/length\(^2\)
- \( r \) = inner radius
- \( t \) = wall thickness

\[ \sigma_a = \frac{pr}{2t} = \sigma_2 \]

= axial stress in longitudinal direction

See FE review manual for thick-walled pressure vessel formulas.
2-D State of Stress

Stress Transformation

\[ \sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \]

\[ \sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \]

\[ \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta \]

Principal Stresses

\[ \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \left( \tau_{xy} \right)^2} \]

\[ \tan 2\theta_p = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} \]

No shear stress acts on principal planes!
Maximum In-plane Shear Stress

\[ \tau_{\text{in-plane}}^{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2} \]

\[ \sigma_{\text{avg}} = \frac{\sigma_x + \sigma_y}{2} \]

\[ \tan 2\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2}\right)/\tau_{xy} \]
Mohr’s Circle - Stress, 2D

Center: Point C( $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}, 0$)

$$R = \sqrt{\left(\sigma_x - \sigma_{avg}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$\sigma_1 = \sigma_{avg} + R = \sigma_a$$

$$\sigma_2 = \sigma_{avg} - R = \sigma_b$$

$$\tau_{max}^{in-plane} = R$$

$\sigma$, positive to the right
tau, positive downward!

A rotation of $\theta$ to the x’ axis on the element will correspond to a rotation of $2\theta$ on Mohr’s circle!
**Beam Deflections**

Inflection point is where the elastic curve has zero curvature = zero moment

\[ \frac{1}{\rho} = - \frac{\varepsilon}{y} \]

Also \( \varepsilon = \frac{\sigma}{E} \) and \( \sigma = \frac{-My}{I} \) \( \Rightarrow \)

\[ \frac{1}{\rho} = \frac{M}{EI} \]

\( \rho \) = radius of curvature of deflected axis of the beam
Double integrate moment equation to get deflection; use boundary conditions from supports ➔ rollers and pins restrict displacement; fixed supports restrict displacements and rotations
For each integration the "constant of integration" has to be defined, based on boundary conditions.
Column Buckling

\[ P_{cr} = \frac{\pi^2 EI}{\ell^2} \]

Euler Buckling Formula
(for ideal column with pinned ends)

- \( P_{cr} \) = critical axial loading (maximum axial load that a column can support just before it buckles)
- \( I \) = the smallest moment of inertia of the cross-section
- \( \ell \) = unbraced column length

\[ r = \sqrt{\frac{I}{A}} \]

= radius of gyration, units of length

\[ r = \sqrt{\frac{I}{A}} \Rightarrow I = r^2 A \Rightarrow \]

\[ \sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(\ell / r)^2} \]

= critical buckling stress

\( \ell / r \) = slenderness ratio for the column
Euler’s formula is only valid when $\sigma_{cr} \leq \sigma_{yield}$.

When $\sigma_{cr} > \sigma_{yield}$, then the section will simply yield.

For columns that have end conditions other than pinned-pinned:

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

$K = \text{the effective length factor (see next page)}$

$KL = L_e = \text{the effective length}$

$$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$$

$KL/r = \text{the effective slenderness ratio}$
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Effective Length Factors

(a) Pinned ends: $K = 1$
(b) Fixed and free ends: $K = 2$
(c) Fixed ends: $K = 0.5$
(d) Pinned and fixed ends: $K = 0.7$