AUBURN UNIVERSITY

INSTRUCTION

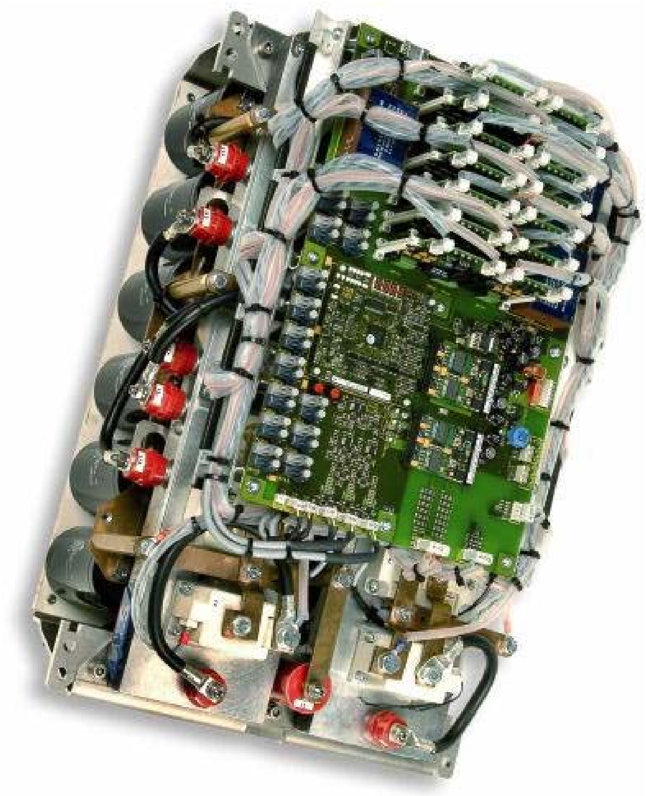
RESEARCH

EXTENSION

FOR THE ADVANCEMENT OF SCIENCE AND ARTS

ESTABLISHED 1856

FUNDAMENTALS OF ENGINEERING (FE) EXAMINATION REVIEW



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ELECTRICAL ENGINEERING

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FE EXAM FORMAT

- 8 hours long
  - 4-hour morning session; 120 multiple-choice questions.
  - 4-hour afternoon session; 60 multiple-choice questions.
- Closed book
  - Reference material is supplied
- Morning session
  - material common to all engineering disciplines
- Afternoon session, examinees opt to take
  - a general exam, or
  - a discipline-specific exam

For more information:
http://www.ncees.org/exams/fundamentals/

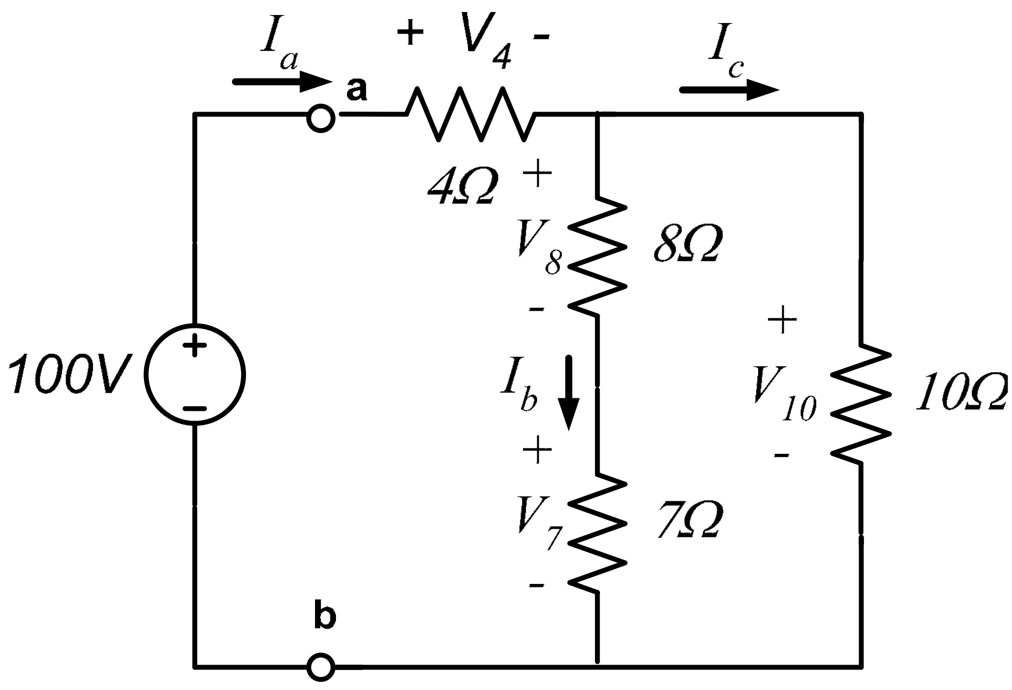
EE Content (9 areas)

1.	Circuits	16%
2.	Power	13%
3.	Electromagnetics	7%
4.	Control Systems	10%
5.	Communications	9%
6.	Signal Processing	8%
7.	Electronics	15%
8.	Digital Systems	12%
9.	Computer Systems	10%

EE Review Problems

1. dc Circuits
2. ac Circuits
3. 1^{st} Order Transients
4. 3-phase Circuits; pf correction
5. Control
6. Signal Processing
7. Electronics
8. Digital Systems

1. dc Circuits:



Find all voltages, currents, and powers.

Solution

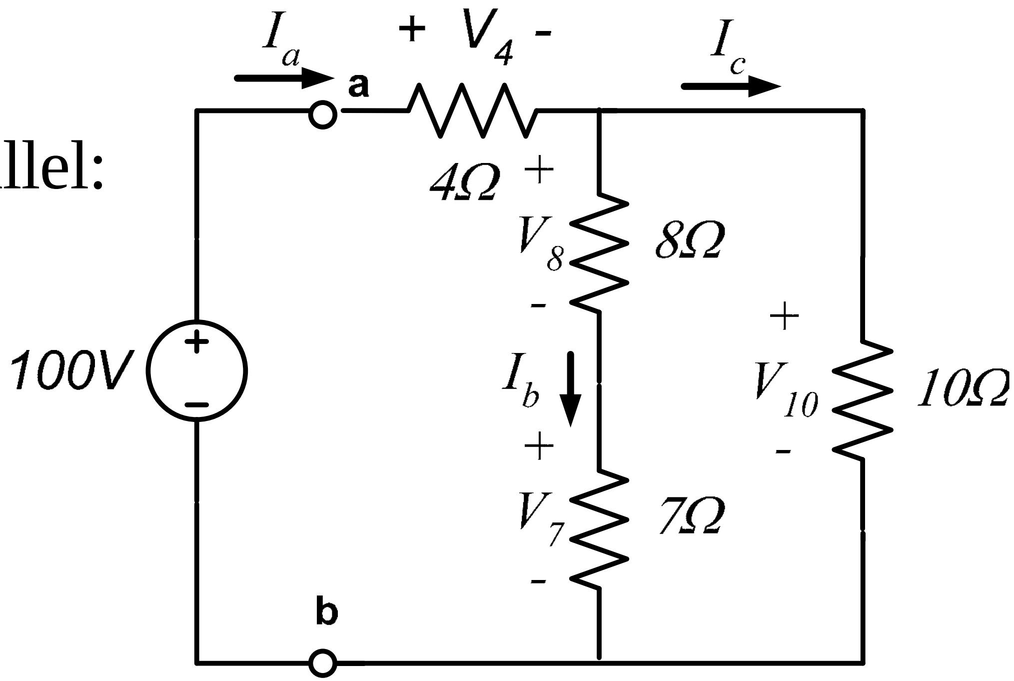
The 8Ω and 7Ω resistors are in series:

$$\boldsymbol{R}1 = 8 + 7 = 15\Omega$$

R1 and 10Ω are in parallel:

$$\boldsymbol{R}2 = \frac{1}{\dfrac{1}{10} + \dfrac{1}{\boldsymbol{R}1}}$$

$$= \frac{10(\boldsymbol{R}1)}{10 + \boldsymbol{R}1} = 6\Omega$$



Solution

4Ω and R2 are in series:

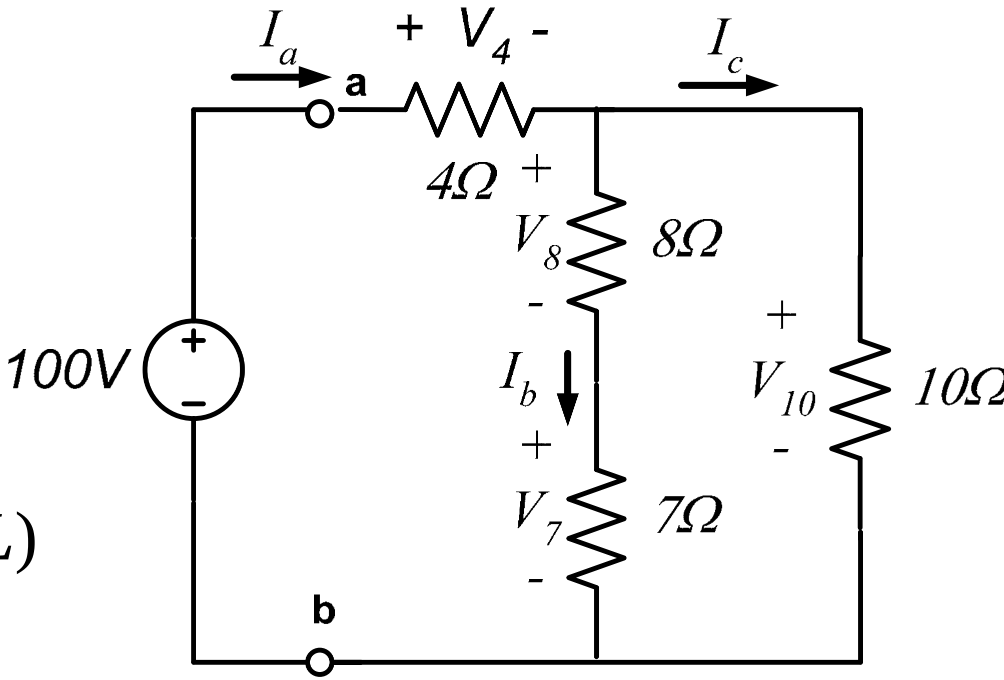
$$R_{ab} = 4 + R2 = 10\Omega$$

ΩL :

$$I_a = \frac{V_{ab}}{R_{ab}} = \frac{100}{10} = 10A$$

$$V_4 = 4 \cdot I_a = 40V \quad (\Omega L)$$

$$V_{10} = 100 - 40 = 60V \quad (KVL)$$

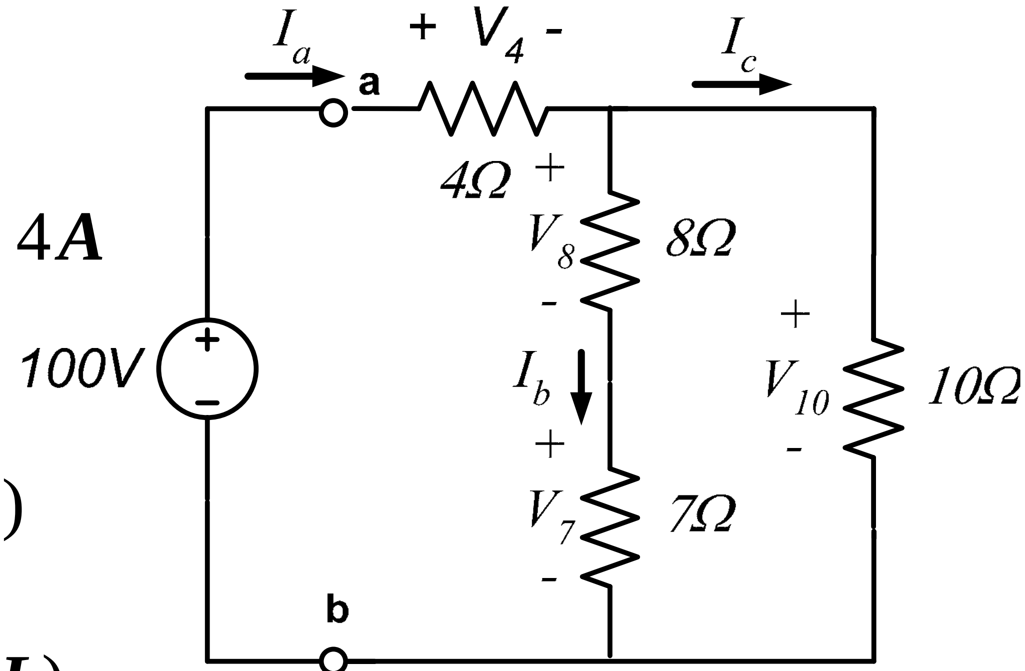


Solution

$$I_c = \frac{V_{10}}{10} = \frac{60}{10} = 6A \quad (\Omega L)$$

$KCL:$

$$I_b = I_a - I_c = 10 - 6 = 4A$$



$$V_8 = 8 \cdot I_b = 32V \quad (\Omega L)$$

$$V_7 = 7 \cdot I_b = 28V \quad (\Omega L)$$

Absorbed Powers...

$$R_4 \cdot I_a^2 = 4(10)^2 = 400W$$

$$R_{10} \cdot I_c^2 = 10(6)^2 = 360W$$

$$R_7 \cdot I_b^2 = 7(4)^2 = 112W$$

$$R_8 \cdot I_b^2 = 8(4)^2 = 128W$$

$$\text{Total Absorbed Power} = 1000W$$

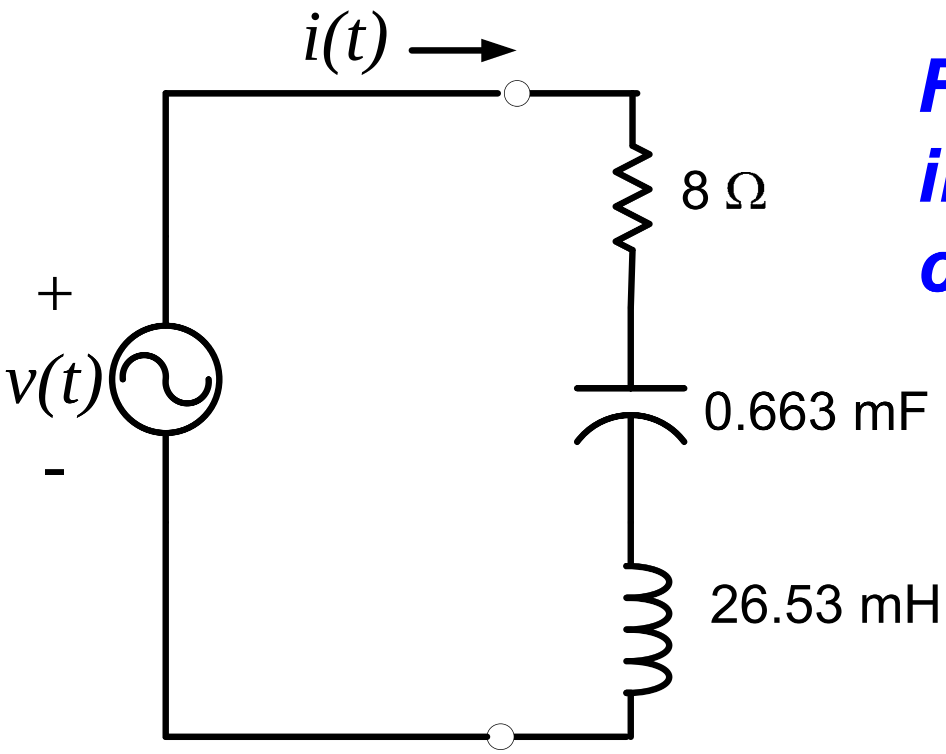
In General:

$P_{ABS} = P_{DEV}$

(Tellegen's Theorem)

$$\text{Power Delivered by Source} = V_s \cdot I_a = 100(10) = 1000W$$

2. ac Circuits



Find "everything" in the given circuit.

$$v(t) = 141.4\cos(377t) \quad V$$

Solution

$$v(t) = 141.4\cos(377t) \quad V$$

$$\text{(radian) frequency} = \omega = 377 \; rad/s$$

$$\text{(cyclic) frequency} = f = \frac{\omega}{2\pi} = 60 \; Hz$$

$$Period = \frac{1}{f} = \frac{1}{60} = 16.67 \; ms$$

Solution

To solve the problem, we convert the circuit into an "ac circuit":

$$\boldsymbol{R}, \boldsymbol{L}, \boldsymbol{C} \text{ elements} \rightarrow \bar{\boldsymbol{Z}} \text{(impedance)}$$

$$\boldsymbol{v}, \boldsymbol{i} \text{ sources} \rightarrow \bar{\boldsymbol{V}}, \bar{\boldsymbol{I}} \text{(phasors)}$$

$$\boldsymbol{R}: \quad \bar{\boldsymbol{Z}}_{\boldsymbol{R}} = \boldsymbol{R} + \boldsymbol{j}0 = 8 + \boldsymbol{j}0$$

$$\boldsymbol{L}: \quad \bar{\boldsymbol{Z}}_{\boldsymbol{L}} = 0 + \boldsymbol{j\omega L} = 0 + \boldsymbol{j}(0.377)(26.53) = 0 + \boldsymbol{j}10$$

$$\boldsymbol{C}: \quad \bar{\boldsymbol{Z}}_{\boldsymbol{C}} = 0 + \frac{1}{\boldsymbol{j\omega C}} = 0 - \boldsymbol{j}\frac{1}{0.377(0.663)} = 0 - \boldsymbol{j}4$$

Solution

$$v(t) = V_{MAX}\cos(\omega t + \alpha)$$

To convert to a phasor...

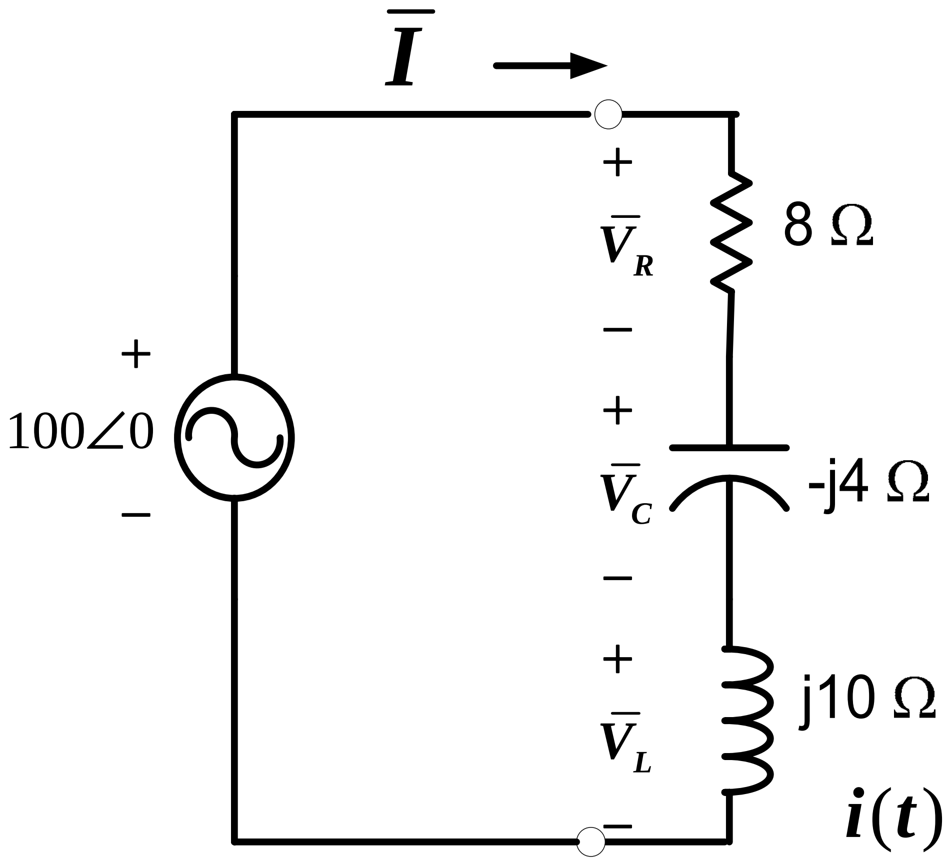
$$\bar{V} = \frac{V_{MAX}}{\sqrt{2}}\angle\alpha$$

For example..

$$v(t) = 141.4\cos(377t)$$

$$\bar{V} = \frac{V_{MAX}}{\sqrt{2}}\angle\alpha = 100\angle 0^{o}$$

The "ac circuit"



$$\Omega L(ac):$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}}$$

$$= \frac{100}{8 + j10 - j4}$$

$$= 10\angle -36.9^{o}$$

$$i(t) = 14.14 \cdot \cos(377t - 36.9^{o})$$

Solving for voltages

$$\bar{V}_R = \bar{Z}_R \cdot \bar{I} = (8)(10\angle -36.9^o) = 80\angle -36.9^o \ V$$

$$v_R(t) = 113.1 \cdot \cos(377t - 36.9^o)$$

$$\bar{V}_C = \bar{Z}_C \cdot \bar{I} = (-j4)(10\angle -36.9^o) = 40\angle -126.9^o \ V$$

$$v_C(t) = 56.57 \cdot \cos(377t - 126.9^o)$$

$$\bar{V}_L = \bar{Z}_L \cdot \bar{I} = (j10)(10\angle -36.9^o) = 100\angle 53.1^o \ V$$

$$v_L(t) = 141.4 \cdot \cos(377t + 53.1^o)$$

Absorbed powers

$$\bar{S} = \bar{V} \cdot \bar{I}^* = P + jQ$$

$$\bar{S}_R = \bar{V}_R \cdot \bar{I}^* = 80\angle -36.9^o (10\angle -36.9^o)^* = 800 + j0$$

$$\bar{S}_C = \bar{V}_C \cdot \bar{I}^* = 40\angle -126.9^o (10\angle -36.9^o)^* = 0 - j400$$

$$\bar{S}_L = \bar{V}_L \cdot \bar{I}^* = 100\angle 53.1^o (10\angle -36.9^o)^* = 0 + j1000$$

$$\bar{S}_{TOT} = \bar{S}_R + \bar{S}_C + \bar{S}_L = 800 + j600$$

$$P_{TOT} = 800 \text{ watts}; \qquad Q_{TOT} = 600 \text{ vars};$$

$$S_{TOT} = |\bar{S}_{TOT}| = 1000\, VA$$

Delivered power

$$\bar{S}_S = \bar{V}_S \cdot \bar{I}^* = 100\angle 0^o (10\angle -36.9^o)^* = 800 + j600$$

$$\bar{S}_S = \bar{S}_{TOT} = 800 + j600$$

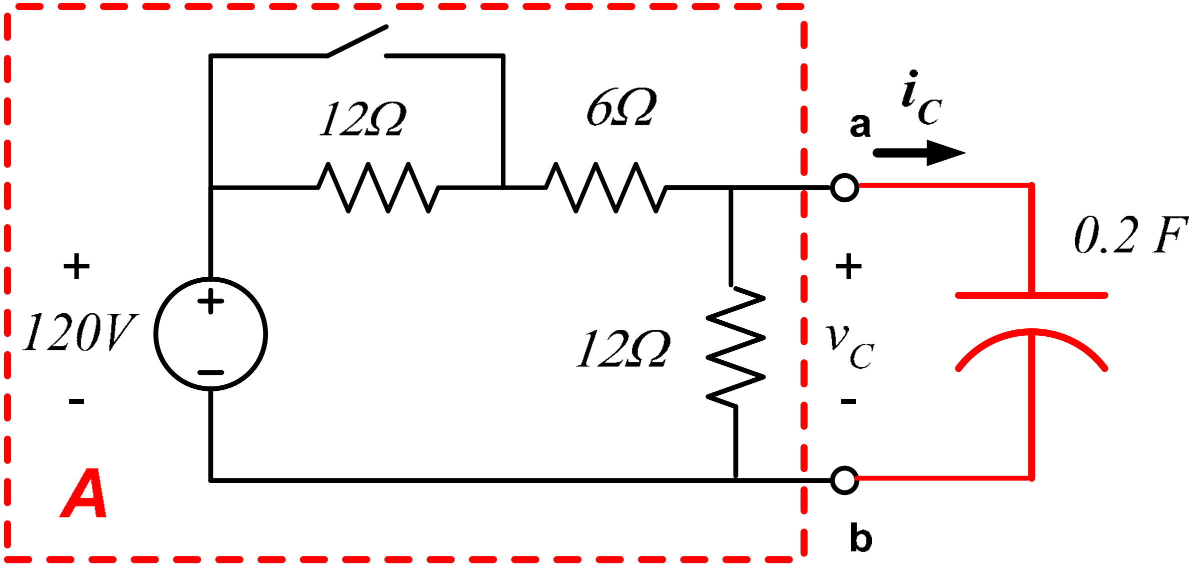
$$P_S = P_{TOT} = 800 \text{ watts}$$

$$Q_S = Q_{TOT} = 600 \text{ vars}$$

In General: $P_{ABS} = P_{DEV}$ $Q_{ABS} = Q_{DEV}$

(Tellegen's Theorem)

3. 1st Order Transients



a. The switch is closed at t = 0. Find and plot $v_C(t)$.

Solution....

In general:

$$t \le 0: \qquad v_C(t) = K_1 + K_2$$

$$t > 0: \qquad v_C(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

$$\tau = R_{ab} \cdot C$$

Our job is to determine K_1, K_2, and $\tau\,(R_{ab})$

Solution....

For a capacitor:

$$i_C = C \cdot \frac{dv_C}{dt}$$

C's are OPENS to dc

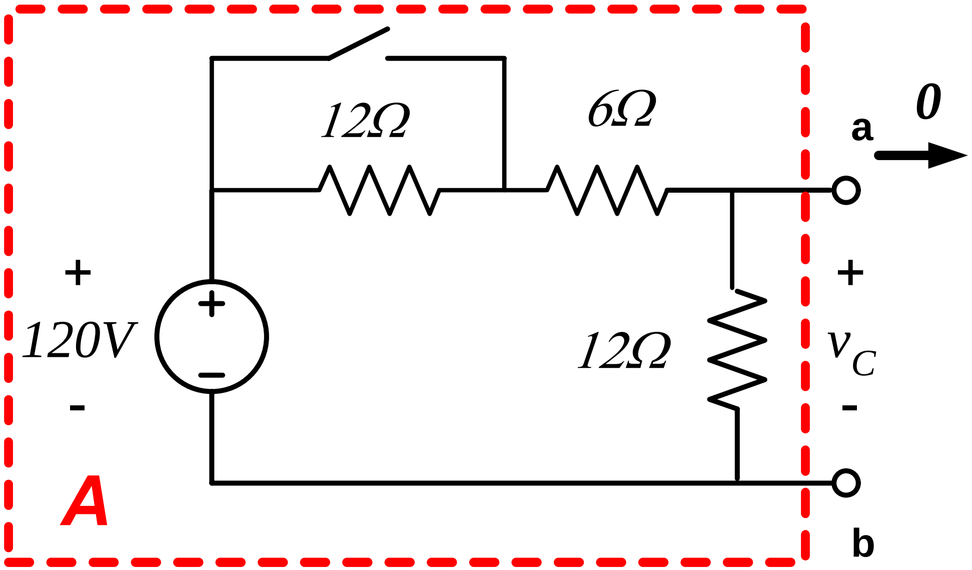
$v_C(t)$ cannot change in zero time

$$v_C(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

$$v_C(0^-) = v_C(0) = v_C(0^+) = K_1 + K_2$$

$$v_C(\infty) = K_1$$

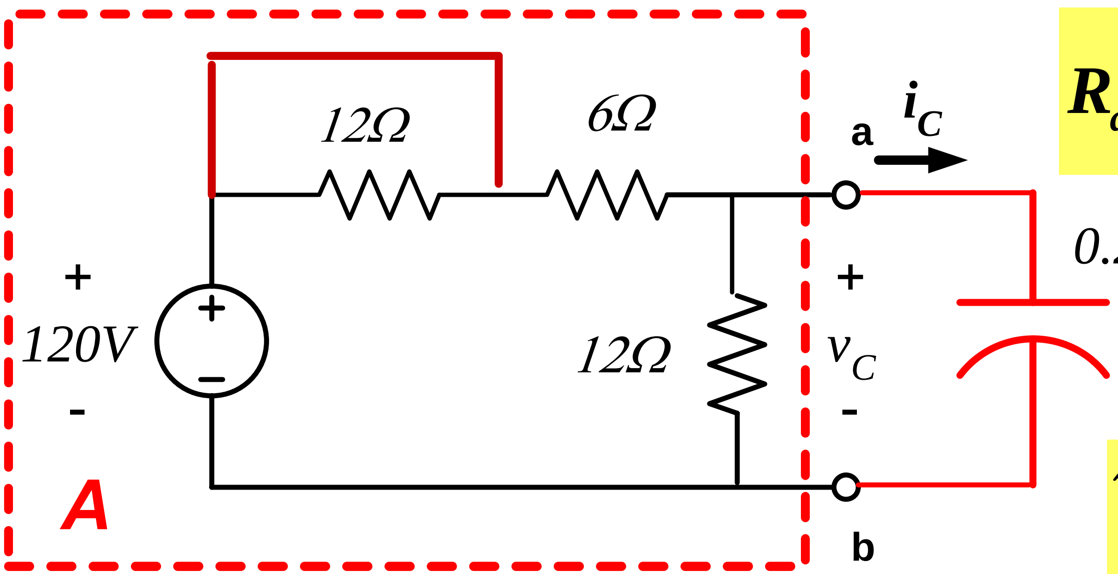
Solution: T < 0; switch and "C" OPEN



$$K_1 + K_2 = 48$$

$$v_C = \frac{120}{12+6+12}(12) = 48\,V$$

Solution: T > 0; switch CLOSED



$$R_{ab} = \frac{6 \cdot 12}{6+12} = 4\,\Omega$$

$$\tau = R_{ab}C$$
$$= 4(0.2) = 0.8\ s$$

$$v_C(\infty) = \frac{120}{0+6+12}(12) = 80\,V$$

$$K_1 = 80$$

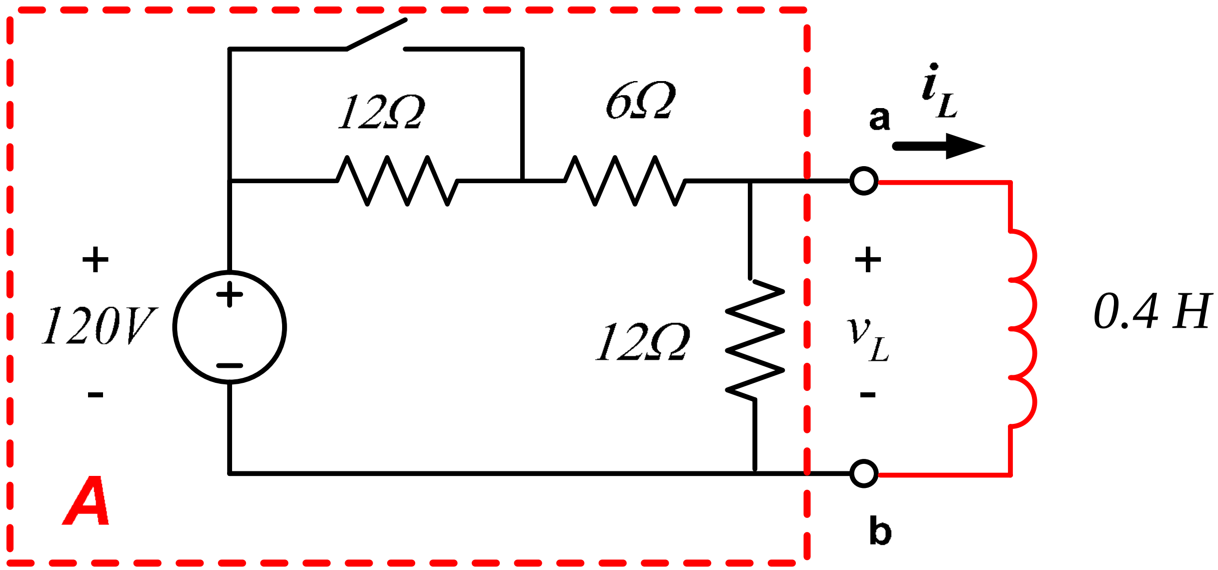
Solution....

$$t \le 0: \qquad v_C(t) = K_1 + K_2 = 48$$

$$t > 0: \qquad v_C(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

$$v_C(t) = 80 - 32 \cdot e^{-1.25t}$$

3. 1st Order Transients



b. The switch is closed at t = 0. Find and plot $i_L(t)$.

Solution....

In general:

$$t \le 0: \qquad i_L(t) = K_1 + K_2$$

$$t > 0: \qquad i_L(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

$$\tau = \frac{L}{R_{ab}}$$

Our job is to determine K_1, K_2, and $\tau\,(R_{ab})$

Solution....

For an inductor:

$$v_L = L \cdot \frac{di_L}{dt}$$

L's are SHORTS to dc

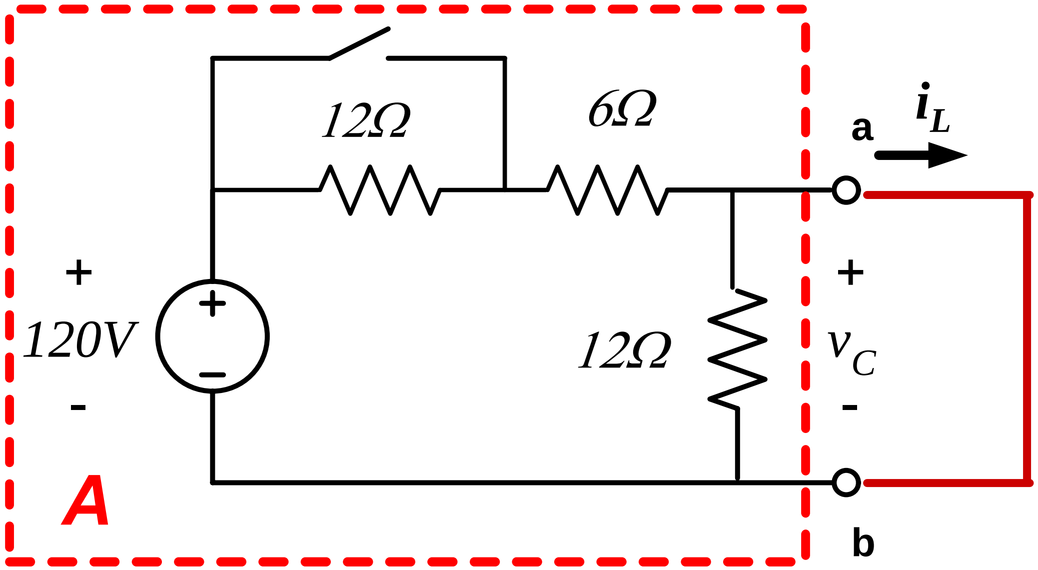
$i_L(t)$ cannot change in zero time

$$i_L(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

$$i_L(0^-) = i_L(0) = i_L(0^+) = K_1 + K_2$$

$$i_L(\infty) = K_1$$

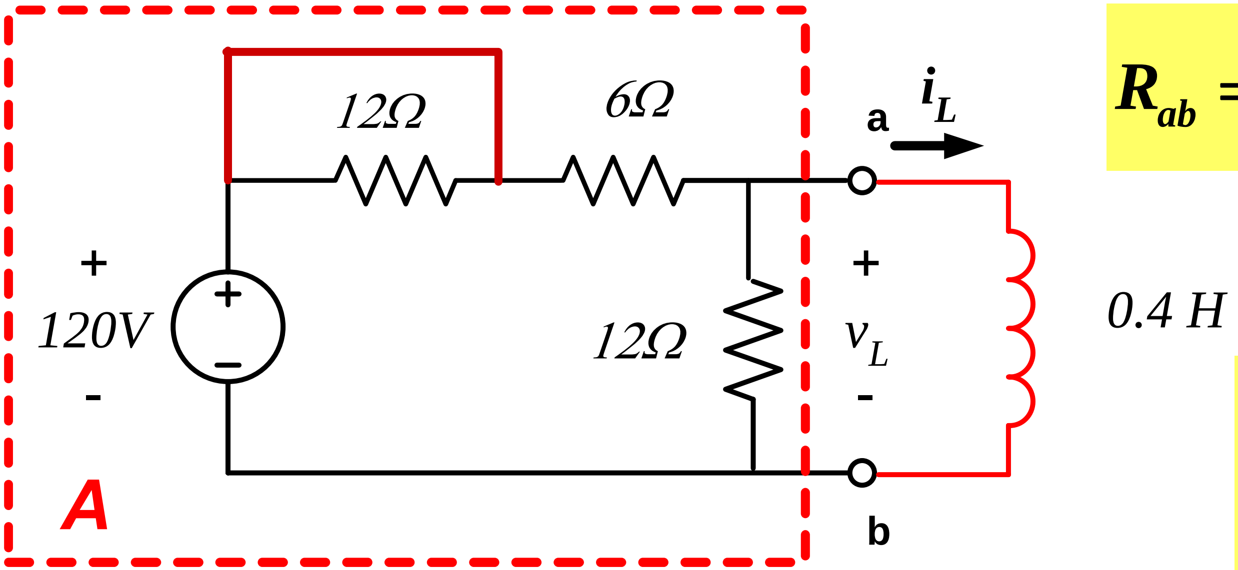
Solution: T < 0; switch OPEN; L SHORT



$$i_L = \frac{120}{12+6} = 6.667\ A$$

$$K_1 + K_2 = 6.667$$

Solution: T > 0; switch CLOSED



$$R_{ab} = \frac{6 \cdot 12}{6+12} = 4\,\Omega$$

$$\tau = \frac{L}{R_{ab}} = \frac{0.4}{4} = 0.1\,s$$

$$i_L(\infty) = \frac{120}{0+6+0} = 20\ A$$

$$K_1 = 20$$

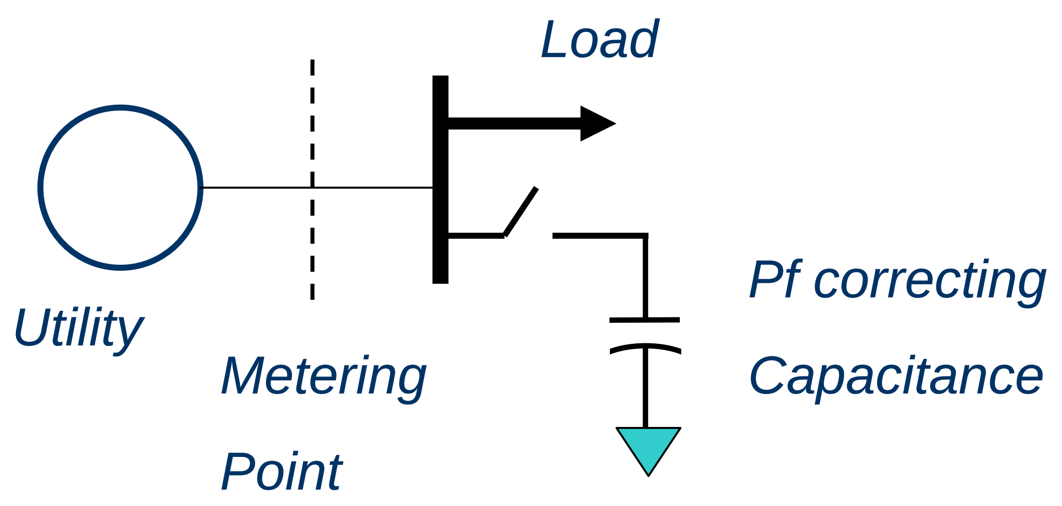
Solution....

$$t \le 0: \qquad i_L(t) = K_1 + K_2 = 6.667$$

$$t > 0: \qquad i_L(t) = K_1 + K_2 \cdot e^{-t/\tau}$$

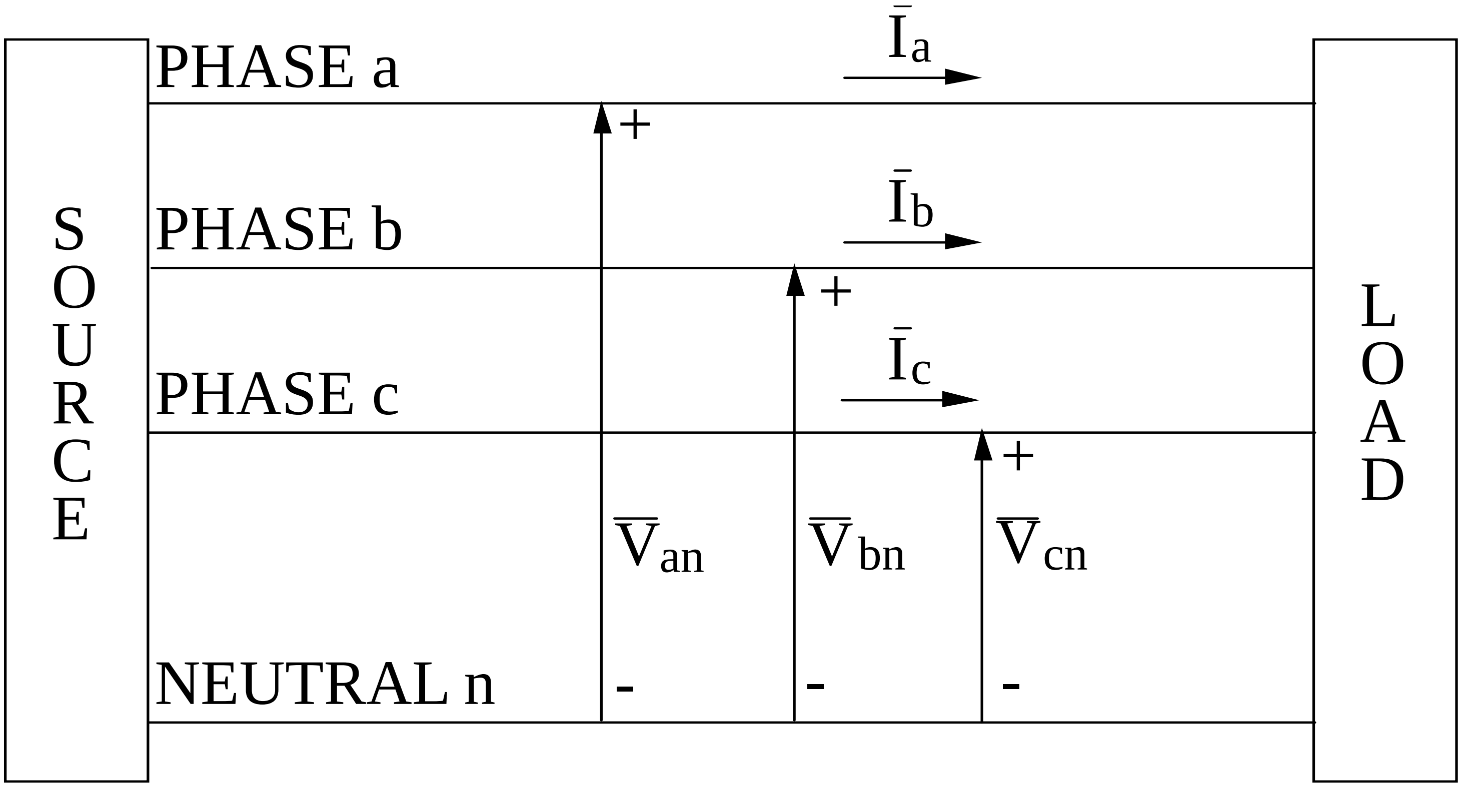
$$i_L(t) = 20 - 13.33 \cdot e^{-10t}$$

4. Three-Phase Circuits: pf correction



A 3ph-load of 3500 kVA @ pf = 0.7754 lag is supplied from a 12.47 kV system. The Utility will provide a discount, if pf ≤ 0.93. Design an appropriate pf-correcting bank.

a. Find all voltages



"PHASE" CONDUCTORS ARE ALSO CALLED "LINES"

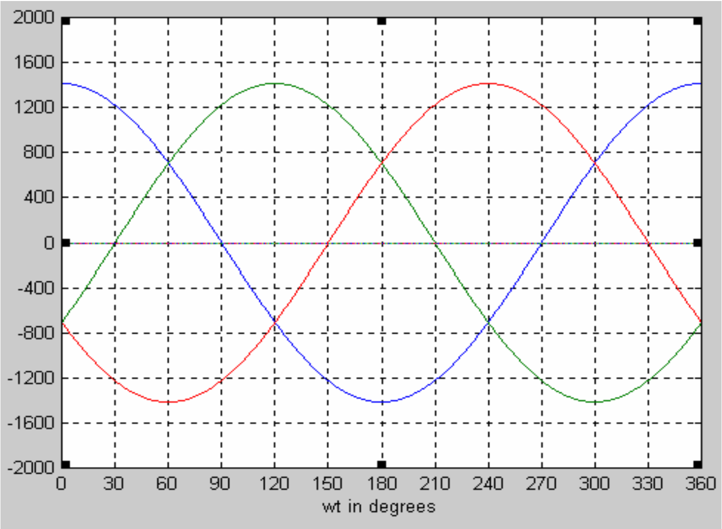
"Balanced" voltage means equal in magnitude, 120º separated in phase

$$v_{an}(t) = V_{\max}\cos(\omega t) = \sqrt{2}\cdot V\cdot\cos(\omega t)$$

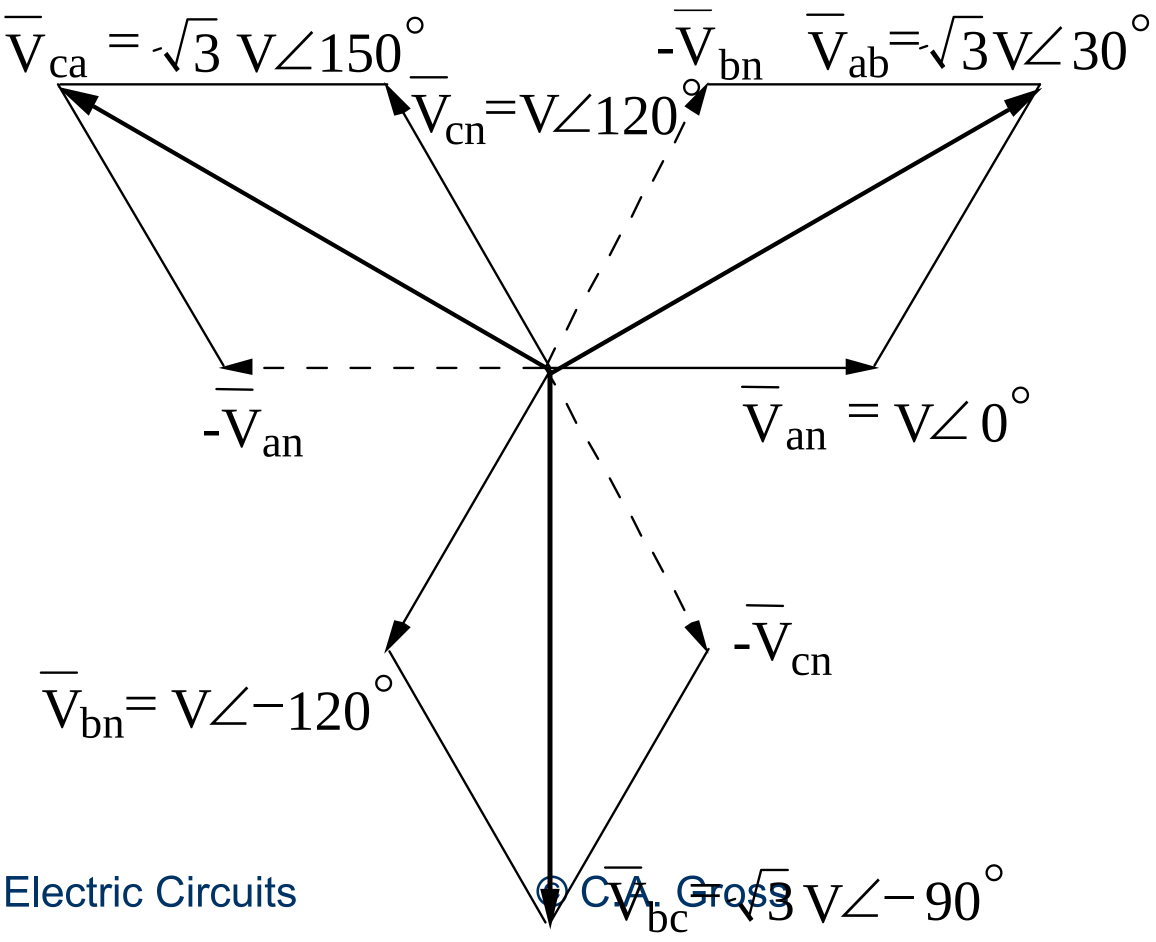
$$v_{bn}(t) = V_{\max}\cos(\omega t - 120^0) = \sqrt{2}\cdot V\cdot\cos(\omega t - 120^0)$$

$$v_{cn}(t) = V_{\max}\cos(\omega t + 120^0) = \sqrt{2}\cdot V\cdot\cos(\omega t + 120^0)$$

e.g.
V=1000 V;
V_{max} = 1414 V



Phasor Diagram



For our problem: $V_L = V_{ab} = 12.47\ kV$

$$V = \frac{V_L}{\sqrt{3}} = 7.2\ kV$$

$$\bar{V}_{an} = 7.2\angle 0^0\ kV \qquad \bar{V}_{ab} = 12.47\angle 30^0\ kV$$

$$\bar{V}_{bn} = 7.2\angle -120^0\ kV \qquad \bar{V}_{bc} = 12.47\angle -90^0\ kV$$

$$\bar{V}_{cn} = 7.2\angle +120^0\ kV \qquad \bar{V}_{ca} = 12.47\angle +150^0\ kV$$

assumes phase sequence abc and V_{an} as reference

b. Draw the Load power triangle

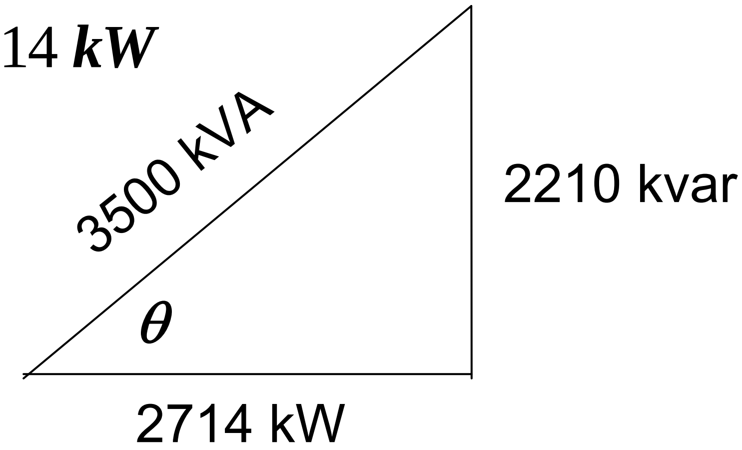
Consider the given load….

3ph-load of 3500 kVA @ pf = 0.7754 lag

$$\theta = \cos^{-1}(0.7754) = 39.2^0$$

$$\boldsymbol{P} = \boldsymbol{S} \cdot \cos(\theta) \ = \ 3500(0.7754) \ = \ 2714\ \boldsymbol{kW}$$

$$\boldsymbol{Q} = \boldsymbol{S} \cdot \sin(\theta) \ = \ 2210\ \boldsymbol{k}\,\mathrm{var}$$



c. Compute the load currents

$$I_a = \frac{S_a}{V_{an}} = \frac{S_a/3}{V_{an}} = \frac{3500/3}{7.2} = 162\ A$$

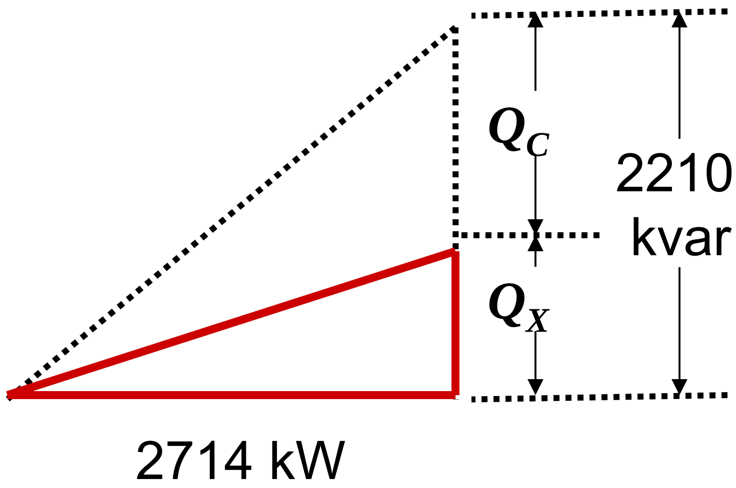
$$\bar{I}_a = 162\angle -39.2^0\ A$$

$$\bar{I}_b = 162\angle(-39.2 - 120) = 162\angle -159.2^0\ A$$

$$\bar{I}_c = 162\angle(-39.2 + 120) = 162\angle 80.2^0\ A$$

assumes phase sequence abc and V_{an} as reference

d. Draw the corrected power triangle and determine the requisite Q_C



Load power triangle

The desired pf:

$$\theta = \cos^{-1}(0.93) = 21.6^0$$

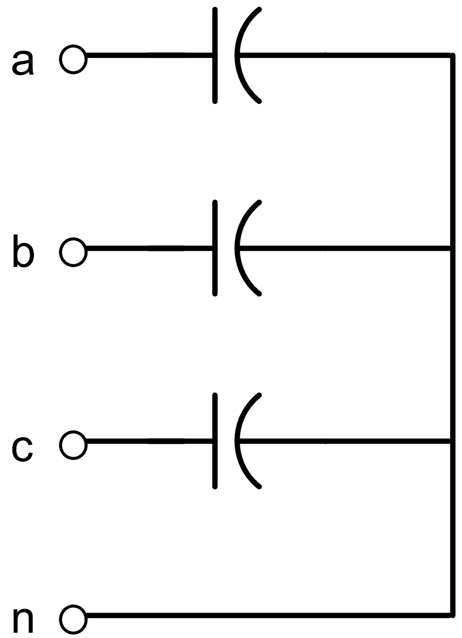
$$Tan(21.6^0) = \frac{Q_X}{2714}$$

$$Q_X = 1073 \text{ kvar}$$

$$Q_C = 2110 - Q_X$$

$$= 1137 \; k\text{var}$$

e. *If the capacitors are connected in wye, find each capacitance and draw the circuit.*



$$Q_{an} = \frac{Q_C}{3} = \frac{1137}{3} = 379 \text{ kvar}$$

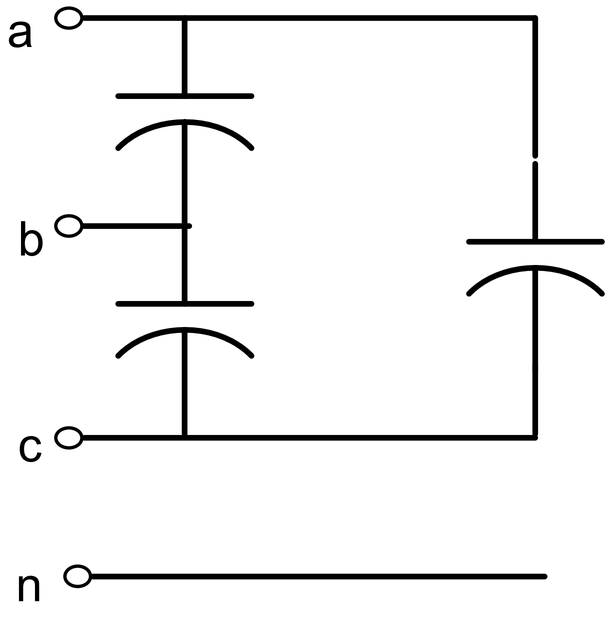
$$I_{an} = \frac{Q_{an}}{V_{an}} = \frac{379}{7.2} = 52.64\ A$$

$$Z_{an} = Z_Y = \frac{V_{an}}{I_{an}} = 136.8\,\Omega$$

$$\bar{Z}_Y = -j136.8\,\Omega$$

$$C_Y = \frac{1}{\omega Z_Y} = \frac{1}{0.377 \cdot 0.1368} = 19.39\ \mu F$$

f. If the capacitors are connected in delta, find each capacitance and draw the circuit.



$$Q_{ab} = \frac{Q_C}{3} = \frac{1137}{3} = 379 \text{ kvar}$$

$$I_{ab} = \frac{Q_{ab}}{V_{ab}} = \frac{379}{12.47} = 30.39 \, A$$

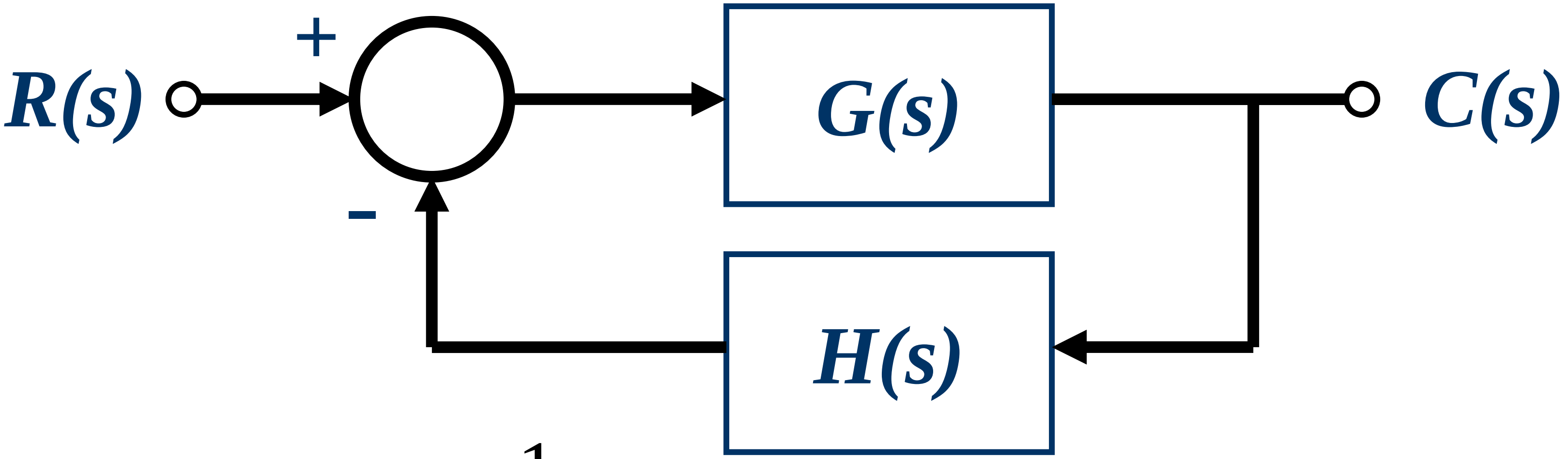
$$Z_{ab} = Z_\Delta = \frac{V_{ab}}{I_{ab}} = 410.3\,\Omega$$

$$\bar{Z}_\Delta = -j410.3\,\Omega$$

$$C_\Delta = \frac{1}{\omega Z_\Delta} = \frac{1}{0.377 \cdot 0.4103} = 6.465\ \mu F$$

5. Control

Given the following feedback control system:



$$G(s) = \frac{1}{(s-1)(s+4)} \qquad H(s) = K$$

a. Write the closed loop transfer function in rational form

$$\frac{C}{R} = \frac{G}{1+GH} = \frac{\dfrac{1}{(s-1)(s+4)}}{1+\dfrac{K}{(s-1)(s+4)}}$$

$$\frac{C}{R} = \frac{1}{(s-1)(s+4)+K} = \frac{1}{s^2+3s+(K-4)}$$

b. Write the characteristic equation

$$s^2 + 3s + (K - 4) = 0$$

c. What is the system order? **2**

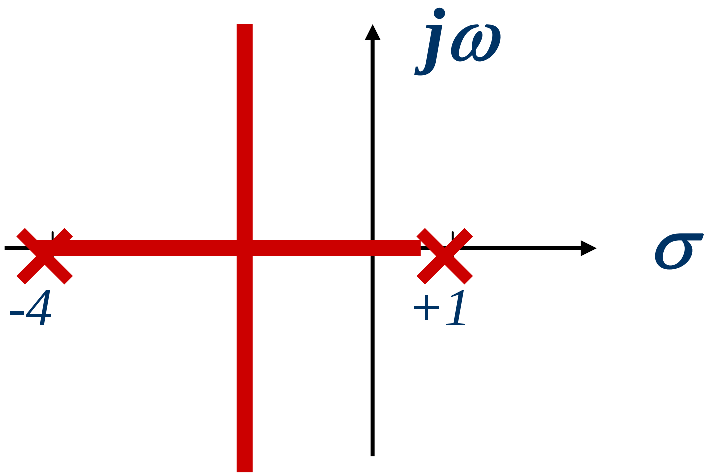
d. For K = 0, where are the poles located?

$$s^2 + 3s - 4 = (s - 1) \cdot (s + 4) = 0$$

s = +1; s = - 4

e. For K = 0, is the system stable? ***NO***

f. Sketch the root locus



g. Find the range on K for system stability.

If K = 4:

$$s^2 + 3s + 0 = (s)\cdot(s+3) = 0$$

Poles at s = 0; s = - 3

Therefore for K>4, poles are in LH s-plane and system is stable.

$K \geq 4$

h. *Find K for critical damping*

$$\boldsymbol{CE}: \qquad \boldsymbol{s}^2 + 3\boldsymbol{s} + (\boldsymbol{K} - 4) = 0$$

$$\text{Solving the CE:} \quad \boldsymbol{s} = \frac{-3 \pm \sqrt{9 - 4(\boldsymbol{K} - 4)}}{2}$$

Critical damping occurs when the poles are real and equal

$$\sqrt{9 - 4(\boldsymbol{K} - 4)} = 0$$

$$\boldsymbol{K} - 4 = 9/4;$$

$$\boldsymbol{K} = 4 + 2.25 = 6.25$$

6. Signal Processing

a. periodic time-domain functions have
continuous discrete frequency spectra.
(circle the correct adjective)

b. aperiodic time-domain functions have
continuous discrete frequency spectra.
(circle the correct adjective)

c. Matching

Laplace Transform	d	*Fourier Transform*	c
Fourier Series	a	*Convolution integral*	b
Inverse FT	e		

a. $x(t) = \sum_{n=-N}^{N} \bar{D}_n \exp(jn\omega_0 t)$

b. $y(t) = \int_{-\infty}^{t} x(\tau) \cdot h(t-\tau) \cdot d\tau$

c. $\bar{X}(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \cdot dt$

d. $X(s) = \int_{0}^{\infty} x(t) \cdot e^{-st} \cdot dt$

e. $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{X}(j\omega) \cdot e^{+j\omega t} \cdot d\omega$

c. Matching

Z-Transform	d	*DFT*	c
Inverse ZT	a	*Discrete Convolution*	b
Inverse DFT	e		

a. $\bar{X}(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jn\Omega}$

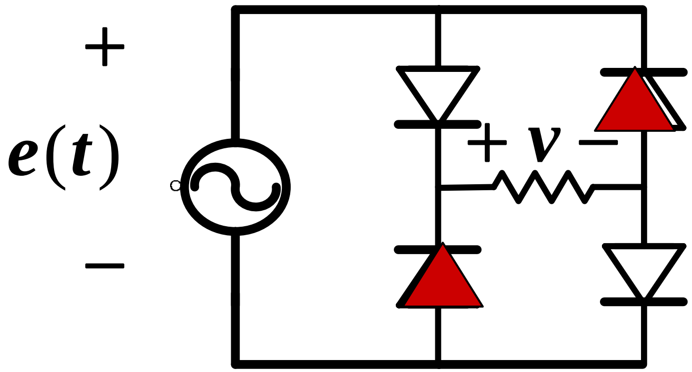
b. $y[k] = \sum_{n=-\infty}^{k} x[n] \cdot h[n-k]$

c. $\bar{X}_k = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}$

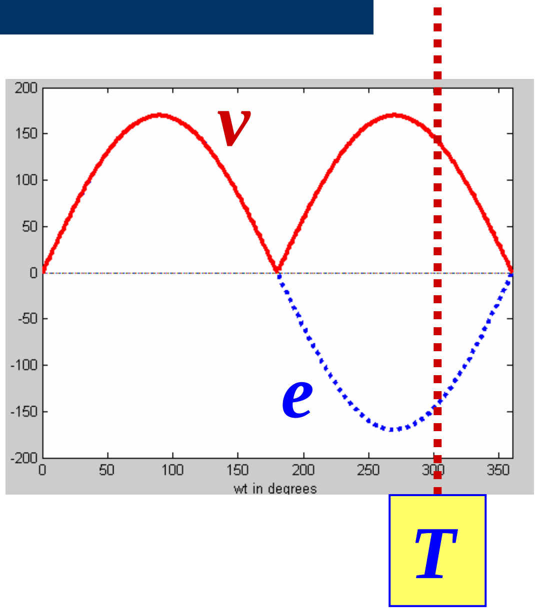
d. $X(z) = \sum_{n=0}^{\infty} x[n] \cdot z^{-n}$

e. $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} \bar{X}_k \cdot e^{+j2\pi kn/N}$

7. Electronics

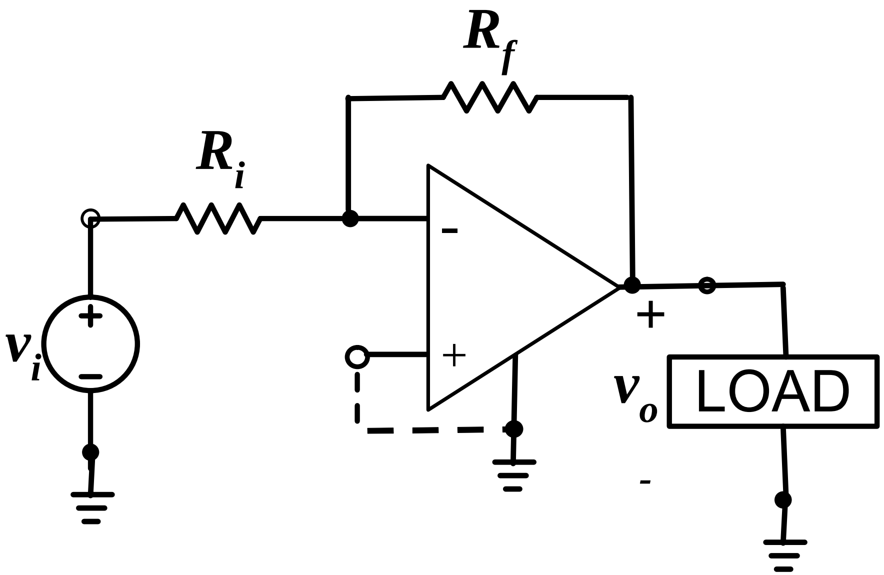


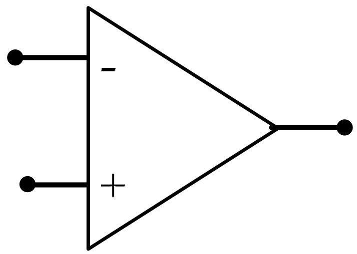
$$e(t) = 169.7 \cdot \sin(\omega t)$$



a. Darken the conducting diodes at time T

b. Given the "OP Amp" circuit

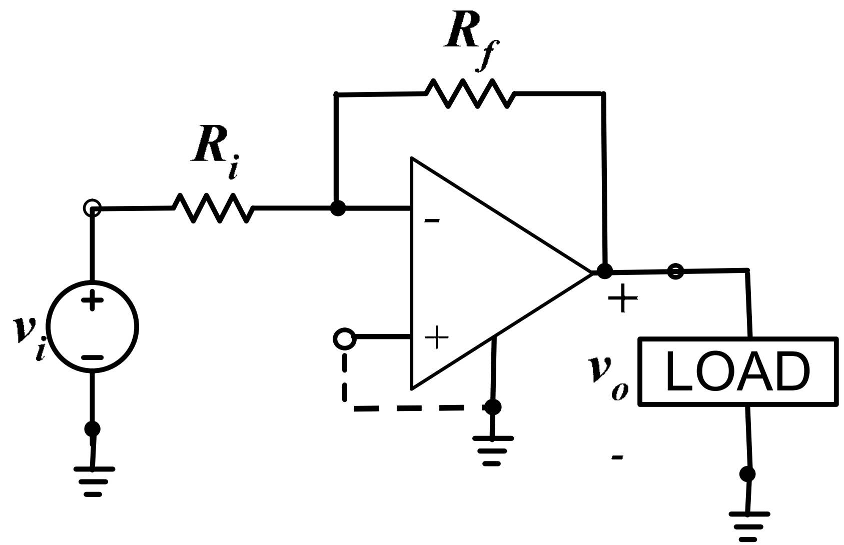




Ideal OpAmp....

- infinite input resistance
- zero input voltage
- infinite gain
- zero output resistance

Find the output voltage.



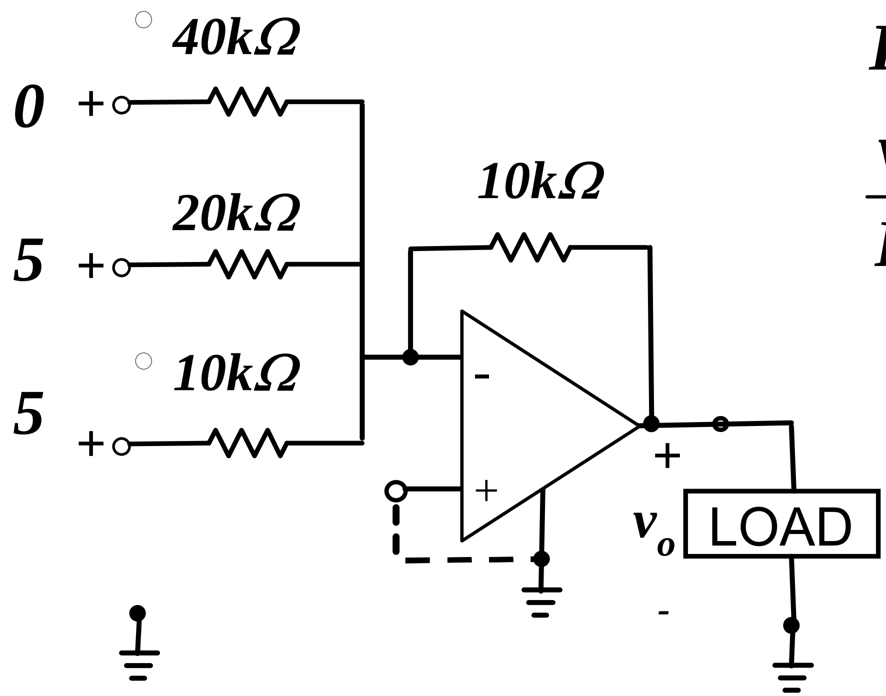
$$v_i = 5\,V$$
$$R_i = 10\,k\Omega$$
$$R_f = 50\,k\Omega$$

$$KCL: \quad \frac{v_i}{R_i} + \frac{v_0}{R_f} = 0$$

$$v_0 = -\left(\frac{R_f}{R_i}\right) \cdot v_i$$

$$v_0 = -\left(\frac{50}{10}\right) \cdot 5 = -25\,V$$

c. Find the output voltage.

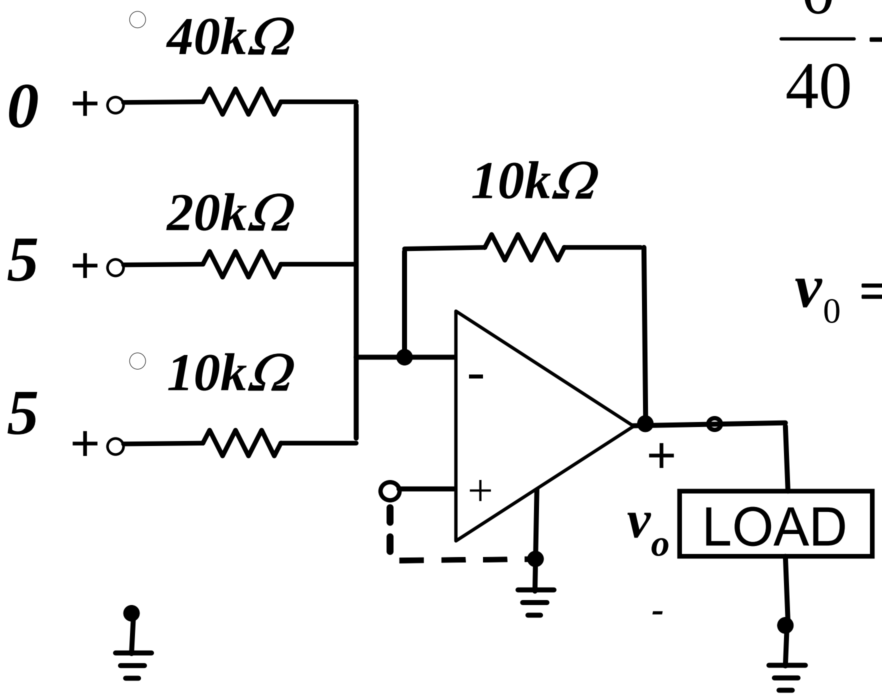


$$KCL:$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} + \frac{v_0}{R_f} = 0$$

"SUMMER"

Solution:



$$\frac{0}{40}+\frac{5}{20}+\frac{5}{10}+\frac{v_0}{10}=0$$

$$v_0=-\left(\frac{10}{10}\right)\cdot 5-\left(\frac{10}{20}\right)\cdot 5-\left(\frac{10}{40}\right)\cdot 0$$

$$v_0=-7.5\,V$$

8. Digital Systems

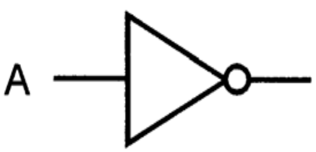
Logic Gates

AB	AND $(A \cdot B)$	NAND	OR	XOR	NOR	NOR
00	0					
01	0					
10	0					
11	1					

NO

A	INV
0	1
1	0

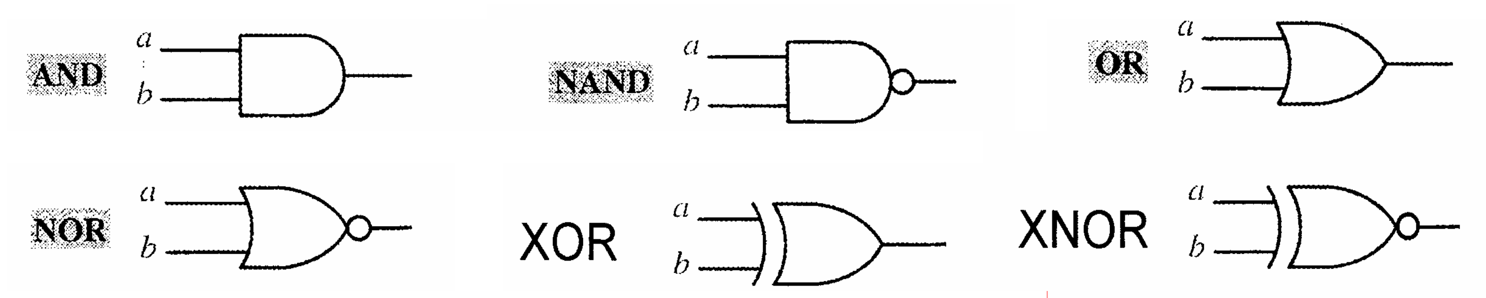
INV



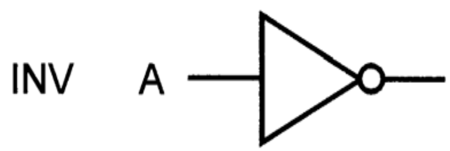
8. Digital Systems

Logic Gates

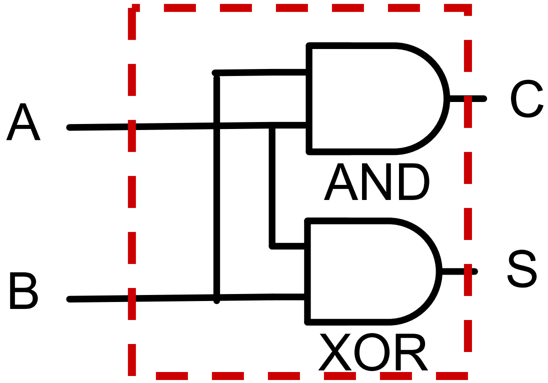
A	B	AND	NAND	OR	NOR	XOR	XNOR
0	0	0	1	0	1	0	1
0	1	0	1	1	0	1	0
1	0	0	1	1	0	1	0
1	1	1	0	1	0	0	1



A	INV
0	1
1	0



a. Complete the Truth Table



Half Adder (HA)

A	B	C	S
0	0	**0**	**0**
0	1	**0**	**1**
1	0	**0**	**1**
1	1	**1**	**0**

A + B = CS

0 + 0 = 00

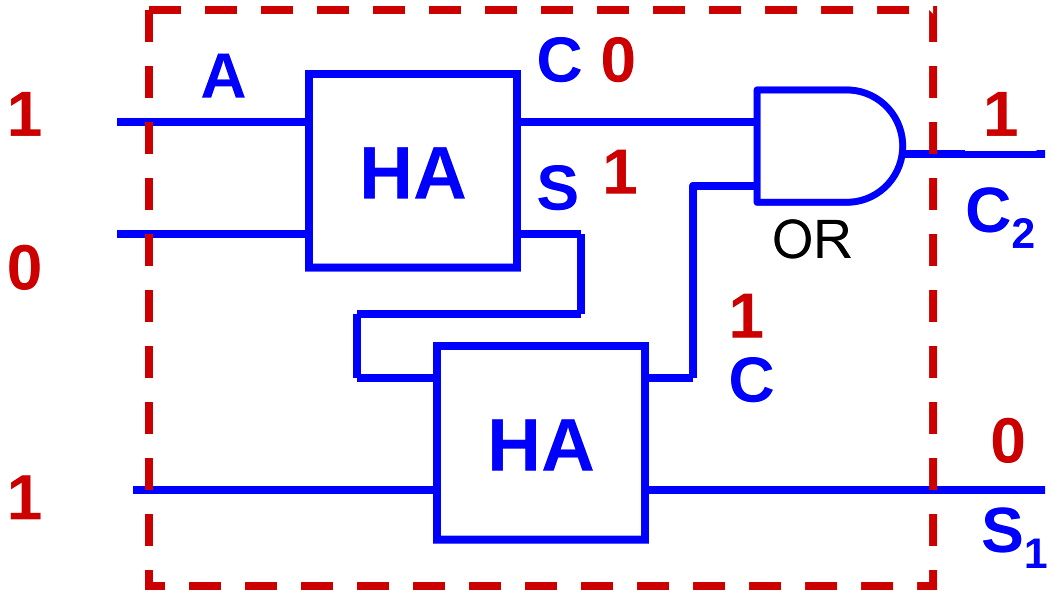
0 + 1 = 01

1 + 0 = 01

1 + 1 = 10

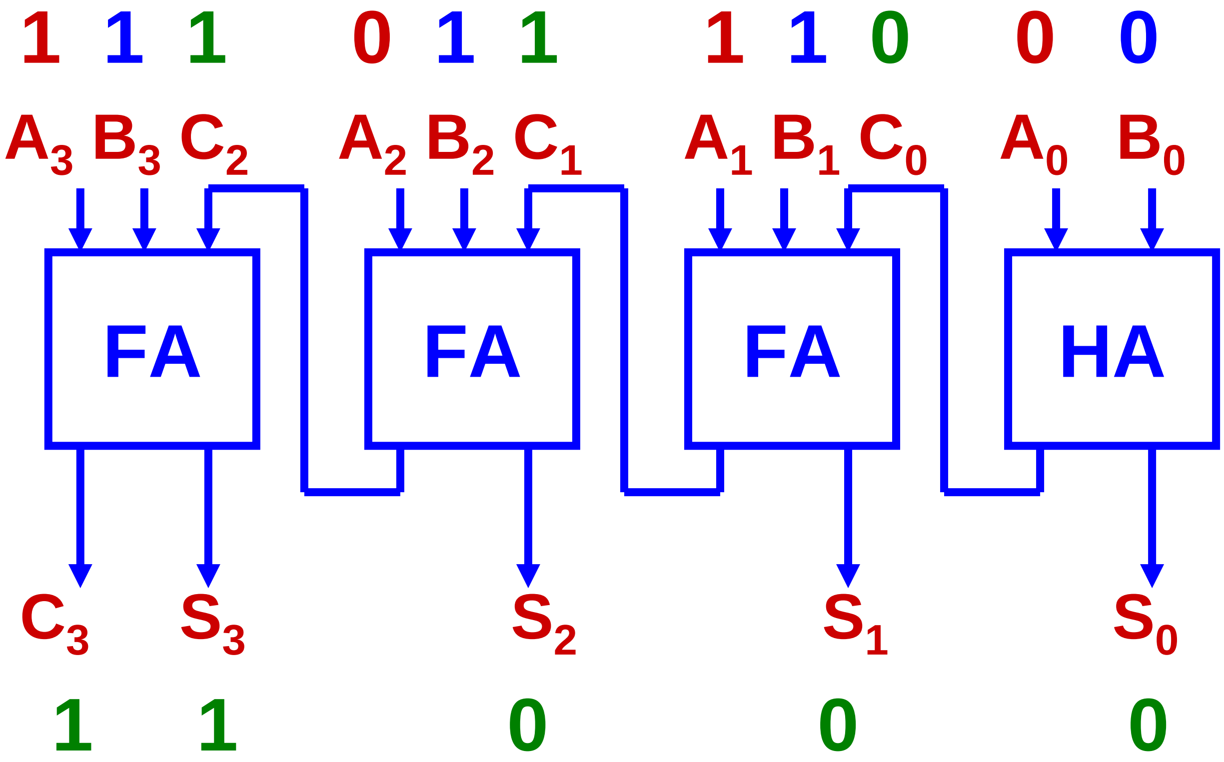
b. Complete the indicated row in the TT

C_1	A_1	B_1	C_2	S_1
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



Full Adder (FA)

c. Indicate the inputs and outputs to perform the given sum in a 4-bit adder



$$\begin{array}{r} 1010 \\ +1110 \\ \hline 11000 \end{array}$$

d. Design a D/A Converter to accomodate 3-bit digital inputs (5 volt logic)

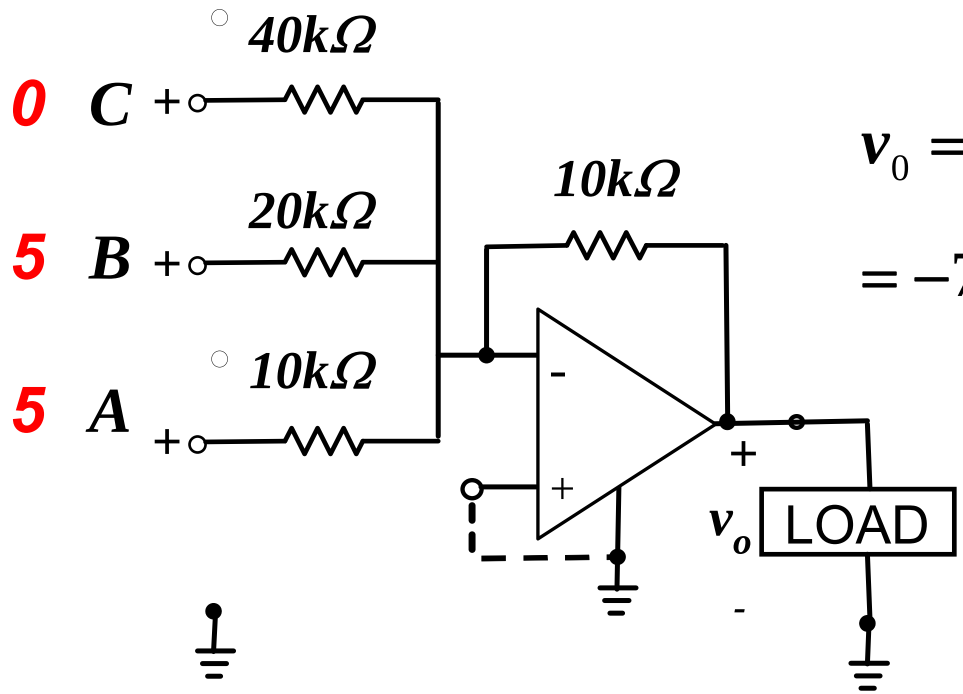
Resolution: 3-bits
(2^3 = 8 levels;
10 V scale)

Digital	*Analog (V)*
000	*0.00*
001	*1.25*
010	*2.50*
011	*3.75*
100	*5.00*
101	*6.25*
110	*7.50*
111	*8.75*

Binary Word: ABC
(A msb; C lsb)

Example...
Convert "110"
to analog

d. Finished Design



$$v_0 = -\frac{10}{10}\cdot 5 - \frac{10}{20}\cdot 5 - \frac{10}{40}\cdot 0$$

$$= -7.50$$

Good Luck on the Exam!

If I can help with any ECE material, come see me (7:30 - 11:00; 1:15 - 2:30)

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Good Evening...