Effect of Initial Condition and Influence of Aspect Ratio Change on Rayleigh-Benard Convection

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Rayleigh Benard convection is an example of thermal instability where temperature difference between the top and bottom caused by heating the fluid from below results in formation of rolls. Rayleigh Benard convection is studied and the effect of change of aspect ratio and change of initial condition is investigated through numerical simulation by the use of a commercial computational fluid dynamics package Fluent. A 2d model of Rayleigh Benard convection was created. The 2d domain was constructed with fixed temperatures on the top and bottom with different aspect ratios. The critical Rayleigh number was found in each case. The effect of initial condition change was imposed in the form of an initial velocity in the whole domain with a fixed number of rolls. In the end a 3 dimensional model was created to find out the change in spatial character of the roll structures in the domain.

Nomenclature

ρ	=	density
U_i	=	velocity in ith direction
τ_{ij}	=	stress tensor
E	=	total energy
P	=	pressure
T	=	temperature
λ	=	coefficient of bulk viscosity
ϕ	=	dissipation function
β	=	coefficient of thermal expansion
d	=	depth of the container
ν	=	kinematic viscosity
α	=	thermal diffusivity
R	=	Rayleigh number
R_{ac}	=	critical Rayleigh number
Nu	=	Nusselt number

I. Introduction

Rayleigh Benard convection has been studied extensively over the years. It occurs when a fluid held in a container is heated from the bottom. A temperature gradient is thus set up between the top and the bottom of the fluid. When this gradient reaches certain value convection rolls begin to form which indicates the onset of thermal instability of the system. This instability occurs due to the interplay among the viscous force, thermal dissipation, gravity and buoyancy force. Due to the heating, fluid in the bottom layer becomes light in weight and tries to rise upward. This upward motion is prevented by gravity, viscous forces and thermal dissipation. But after a certain temperature difference is reached the buoyancy force overcome the opposition of the stabilizing forces and the light fluid layer comes to the top. The heavy fluid layer from the top start coming down to the bottom to occupy the vacant place, afterwards this fluid layer also gets heated and start rising upwards. Thus a cycle of motion sets in which expresses itself in the form of convection rolls. Convection rolls also form in surface tension driven Benard- Marangoni convection. At the onset we can describe convection by a single Fourier mode. When Rayleigh number crosses R_{ac}, more modes are generated through nonlinear interaction. When there are only few modes (say 10 modes or so), one gets convective patterns [1]. The patterns can be square, hexagonal, wavy etc. These patterns could be oscillating as well. When Ra is increased even further, the amplitudes of these modes become large resulting in a chaotic

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behavior. A popular model called Lorenz equation [2] attempts to capture this feature. In Lorenz equation, the temporal behavior is chaotic, but the spatial structure is regular. In Lorenz model the horizontal rolls flip direction of rotation at random intervals, but the rolls do not move in space. When Ra is increased further, many more modes are generated. In this case spatial and temporal variations can be observed. When Rayleigh number increases more the flow becomes turbulent, where many modes interact with each other. The time series of temperature or velocity signal is random. This regime is called turbulent. The domain size is determined by the aspect ratio which is equal to L /d, where L is the length and d is the height of the domain. The effect of aspect ratio on the chaotic dynamics in a system is of great scientific interest. Ahlers and Behringer [3] were one of the first to experimentally study the effects of aspect ratio on the dynamics of Rayleigh Benard convection

This particular instability problem has many important applications in a number of engineering problems, such as in oil extraction from porous media, energy storage in molten salts, crystal growth in space and chemical engineering of paints, colloids and detergents. Apart from that this kind of convection is a crucial factor in the dynamics of ocean and atmosphere. Benard (1901) first reported experimental findings of convection rolls. Subsequently Rayleigh [4] and Jeffrey [5] came up with the mathematical solution of the instability problem. The linear stability analysis of Lord Rayleigh or Sir Harald Jeffrey gives the magnitude of the critical wave director, but it cannot be decided whether the patterns above onset will consist of rolls, hexagons, or squares. Indeed all three patterns occur in RBC. Malkus and Veronis [6] predicted that the stable planform for the case of free boundaries and Boussinesq conditions should be straight rolls rather than e.g. squares or hexagons.

Afterwards there have been a number of attempts to solve the instability problem both experimentally and theoretically. A major aspect of these works was to find out the selection of patterns for various boundary and initial conditions. Most notable are the works of Chandrasekhar [7], Koschmeider [8], etc. The simplest geometric case is the rectangular enclosure. Though attempts have been made in past to investigate other shapes most notably circular and square containers. It has been found that the spatial character of the rolls, i.e. their axis, size depends mainly on the boundary condition and the non uniformity of the temperature distribution on the top and bottom plate. In number of the cases steady circular or hexagonal rolls (for rectangular container) and circular concentric rolls were found for a circular container.

In the work presented here a commercial software package Fluent [9] was employed to simulate the Benard convection. Various parameters like Nusselt number were calculated and compared with the theoretical value. In the end a 3d model was constructed to find the characteristics of 3d instability. The main aim of this work was to find the effect of aspect ratio change and that of implementation of initial perturbation on the instability regime.

II. Basic Parameters and Governing Equations

The basic parameter controlling the onset of instability is the Rayleigh number which is defined as $Ra = \beta \Delta T g d^3 / \nu\, \alpha$. This ratio is a dimensionless number, which is a result of the competition between the destabilizing mechanism and the stabilizing process. Convection develops when the buoyancy is more effective than the dissipative process and thus when Ra is large. The limiting condition after which convection sets in is called critical Rayleigh number. The governing equation for the problem is the continuity, momentum which is famously known as Navier Stokes equation and energy equation.

Continuity equation: $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho U_i}{\partial x_i} = 0;$$

Momentum equation: $$\rho \frac{DU_i}{Dt} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} - \rho g_i$$

Energy equation: $$\rho \frac{DE}{Dt} = -p \frac{\partial U_i}{\partial x_i} + \frac{\partial}{\partial x_j}(\lambda \frac{\partial T}{\partial x_j}) + \phi$$

To these equations of motion, Rayleigh applied the Boussinesq approximation. The approximation states that there are flows in which the density variation is negligible due to small temperature variation. But the flow may be driven by buoyancy. So the variation of density is neglected everywhere except in the buoyancy. On the basis of this approximation for small temperature difference between the top and bottom layer of fluid, we can write, $\rho=\rho_0$ {1-β (T-T_0), where T_0 is the temperature of the lower plate. This approximation works well for Benard convection problem where the temperature difference is small between the top and the bottom plate.

III. Numerical Simulation

To simulate Benard convection numerically a commercial computational fluid dynamics package Fluent was used. Two cases with aspect ratio 2 and 7 were investigated. The mesh was generated in the preprocessor Gambit. For the aspect ratio 2 case, a 2dimensional rectangular geometry was modeled with length 20 mm and height 10 mm. A laminar model was applied since the flow in the instability regime is essentially laminar (unless the Rayleigh number is above 10000). An infinite horizontal span was assumed for ease of solution and this assumption was implemented through periodic boundary condition on the side walls. The temperature of the bottom plate was fixed at 60 ^{o} C and wall boundary condition was applied to them. The temperature of the top plate was determined from the temperature difference which was calculated from the formula for Rayleigh number. This process was repeated with different Rayleigh numbers. The fluid for the simulation was always selected as water with its properties at 60 ^{o} c. For the aspect ratio 7 case the width of the rectangle was taken as 2 mm and length 14 mm. The following table shows the temperature difference applied between the top and the bottom plate for both aspect ratio cases. For both the laminar and turbulence case analysis SIMPLE algorithm was used in Fluent.

Ra	ΔT
700	0.0104
1700	0.03
2500	0.03722
3000	0.0447
6000	0.08932
10000	0.1488
30000	0.4467

Table 1: Temperature difference between the top and bottom plate for different Rayleigh numbers

A. Turbulence modeling:
Turbulent flows are characterized by fluctuating velocity fields. These fluctuations mix transported quantities such as momentum, energy, and species concentration, and cause the transported quantities to fluctuate as well. Since these fluctuations can be of small scale and high frequency, they are too computationally expensive to simulate directly in practical engineering calculations. Instead, the instantaneous (exact) governing equations can be time-averaged, ensemble-averaged, or otherwise manipulated to remove the small scales, resulting in a modified set of equations that are computationally less expensive to solve. However, the modified equations contain additional unknown variables, and turbulence models are needed to determine these variables in terms of known quantities. In cfd a number of turbulence models are available. The selection of a particular turbulence model depends on the problem definition and the flow characteristics. The available options in Fluent are K-epsilon, K-omega, Spallart – Allmaras model etc.

For Ra=10000 and 30000 case the turbulent case were selected for this problem. The K-epsilon turbulence model was implemented with default coefficients and standard wall function approach. The K-epsilon model is a two equations, semi empirical model in which two separate transport equations are solved to obtain the required turbulent velocity and length scale. K-epsilon model is very robust and it gives reasonable accuracy for a wide class of flow problems. .For the 3d simulation a 0.005x0.05x0.05 rectangular parallelepiped was created in Gambit. The Ra was fixed at 1760. In this case also the laminar model was selected for viscosity.

IV. Results and Discussion

A. Effect of Aspect Ratio Change:

Figure 1 shows the static temperature contours at Ra= 700. It can be clearly seen that the isotherms are perfectly horizontal without any distortion which signifies that no instability has set up. This is in accordance with the theory as it can be shown from linear stability analysis that for small disturbances the critical Rayleigh number for an enclosure with aspect ratio 2 is approximately 1708. Figure 2 shows that at Ra=1700 the instability has set in which is evident in the periodic nature of the static temperature plot. The patterns formed are square rolls as indicated by figure 3. Number of rolls in this case is two which corresponds to the minimum stable case. The wavelength of the rolls can be found from figure 4 which shows that the wavelength of the variation of velocity magnitude is around 10 mm which is same as the depth of the fluid. In the turbulent case with Ra=10000 the rolls appeared to be less clear (figure 5) and in figure 6 which is for Ra=30000, the patterns are broken. This is due to the result of increased perturbation in the turbulent case.

For the aspect ratio 7 case almost the same results were obtained. The number of rolls formed were 7 and the wavelength of the rolls were almost 2 mm i.e. same as the depth of the fluid layer. But in this case one important deviation was periodicity in temperature profile at Ra 700 (figure 8) which signifies formation of rolls. So it can be concluded that the critical Ra number comes down with the increase of aspect ratio. A reason for this is the increase in surface area which allows more heating area. As more heating area is available, the same temperature difference between the top and the bottom plate in the increased aspect ratio case causes more heat flux to enter the system. So the fluid gets heated more quickly and the heating enables a fluid lump to overcome the stabilizing effect of dissipation and viscous forces. As an effect the convection rolls appear more quickly in the increased aspect ratio case.

One prime aspect of Benard convection is the heat transfer between the top and bottom surface of the fluid. Before reaching the critical Rayleigh number, when the fluid is undisturbed the heat transfer takes place by diffusion only. But after the system reaches instability heat transfer in the fluid is aided by convection also. So Nusselt number which is actually the ratio of heat transfer in the fluid in the disturbed state to that of the undisturbed state is always greater than one after critical Rayleigh number. The onset of instability can also be identified by the break in the instability curve. The first empirical formulas for the heat transfer by convection were given by Silverstone (1958). According to him for $Nu=0.24(Ra)^{0.25}$. [1]. Figure 9 shows a plot of Nusselt number vs. Rayleigh number in which the aforesaid empirical relation was compared with the data obtained from the numerical study for aspect ratio 2 case. The data shows a good match in the laminar regime. But there is a tendency of deviation in the turbulent case i.e. as the Rayleigh number is increased more and more.

B. Effect of initialization of the velocity in the problem domain:

To find out the effect of initial conditions on the Benard convection, an initial perturbation was applied in the form of an initial velocity to the whole domain. The function selected to achieve the initialization was of the form v=A*(sin (pi*y/0.01))*(cos (N1*pi*x/0.02) + B* cos (N2*pi*x/0.02)), where N1 and N2 are the number of rolls in the x and y direction respectively. This particular form of velocity is actually the result of a linear stability analysis for an infinitely spanned domain in the horizontal direction. The initialization of the velocity was done by hooking a user defined function to fluent. Through this implementation the no of rolls in the x direction was fixed at 6 and in the y direction it was 2.As the result of initialization 6 rolls formed in the domain which is visible in the contour plot of x velocities (figure 10). The contour plot of the y velocity showed 2 rolls as before (figure 11). The Rayleigh number was arbitrarily selected to be 6000. The initialization case can be difficult to attain in practical experiment as it involves a periodic velocity profile.

Figure 12 shows the result from a 3dimensional test case with Ra = 6000. In case of a 3 d domain the wall boundary conditions were applied along the six faces. So the velocity contour plot is actually from a cross section of the domain. It can be observed that there is a periodicity in the number of rolls (5 and 4 consecutively) along the z axis.

One important finding from the results is all the patterns were square in shape. The structure of pattern formation is an active research topic in Benard convection. Square rolls are the result of the simplest boundary condition, i.e. infinite span in the horizontal direction. As discussed previously the structure and shape of the roll depends on the boundary condition, shape of the container etc. Also the presence of thermal noise affects its structure.

V. Figures

A. Aspect Ratio 2

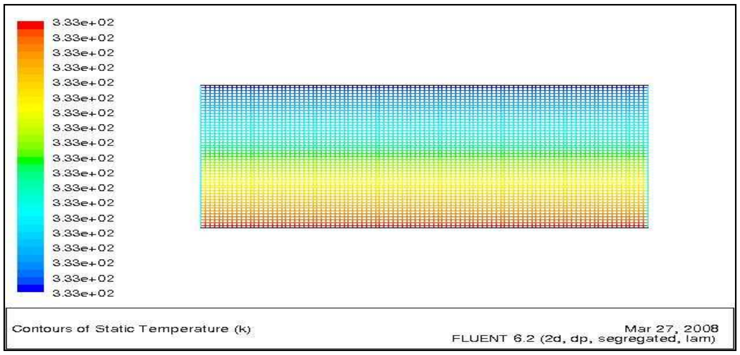

Figure 1. Contour plot of static temperature at Ra = 700.

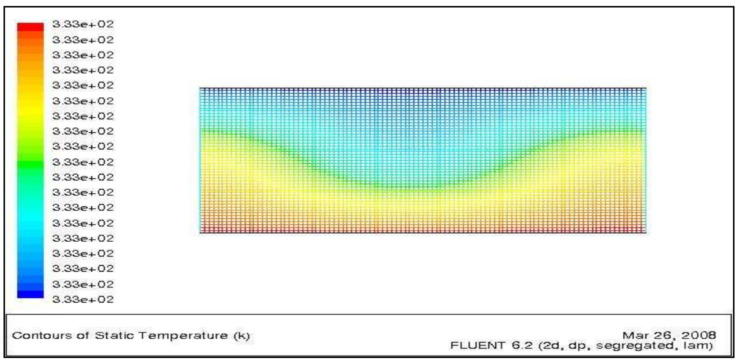

Figure 2. Contour plot of static temperature at Ra = 1700.

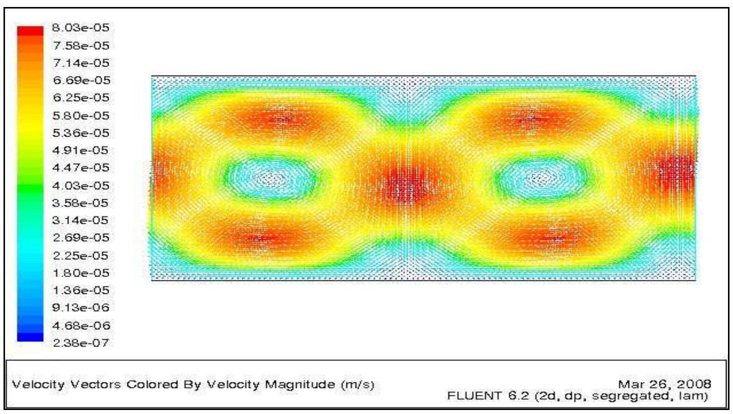

Figure 3. Contour plot of velocity in x direction at Ra = 1700.

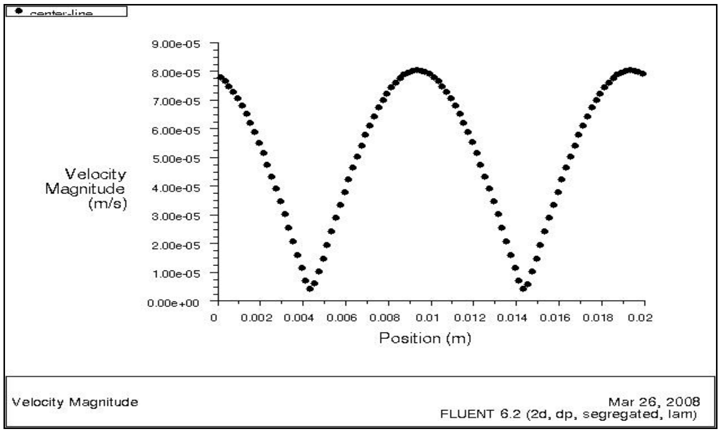

Figure 4. Variation of velocity magnitude along the central line of the rectangular domain along x axis at Ra=1700

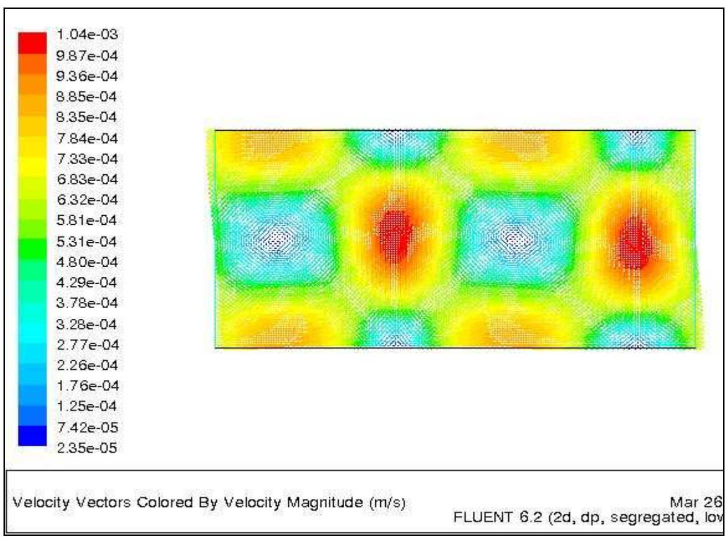

Figure 5. Contour plot of velocity in x direction at Ra = 10000.

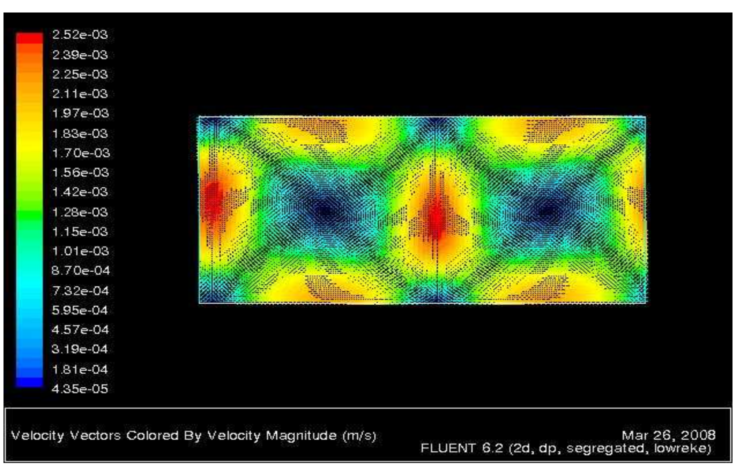

Figure 6. Contour plot of velocity in x direction at Ra = 30000

B. Aspect Ratio 7

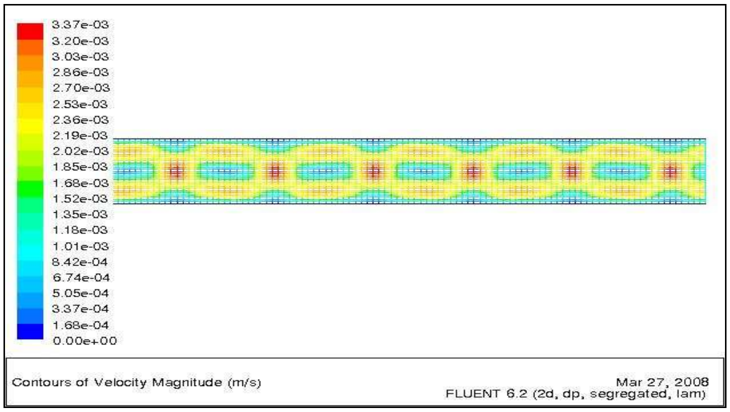

Figure 7. Contour plot of velocity in x direction at Ra = 1700

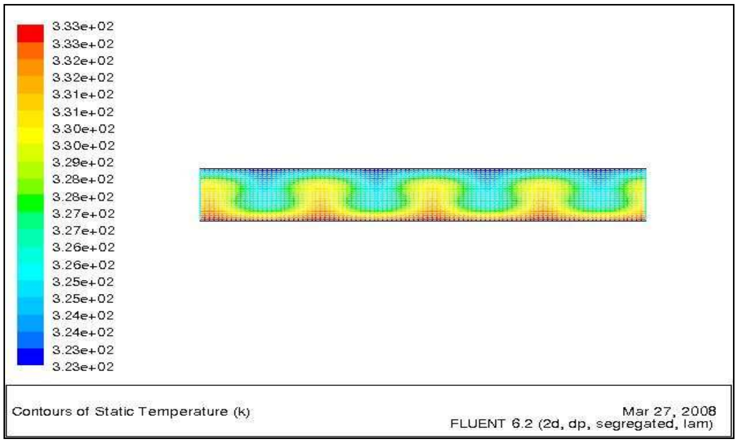

Figure 8. Contour plot of static temperature Ra = 700

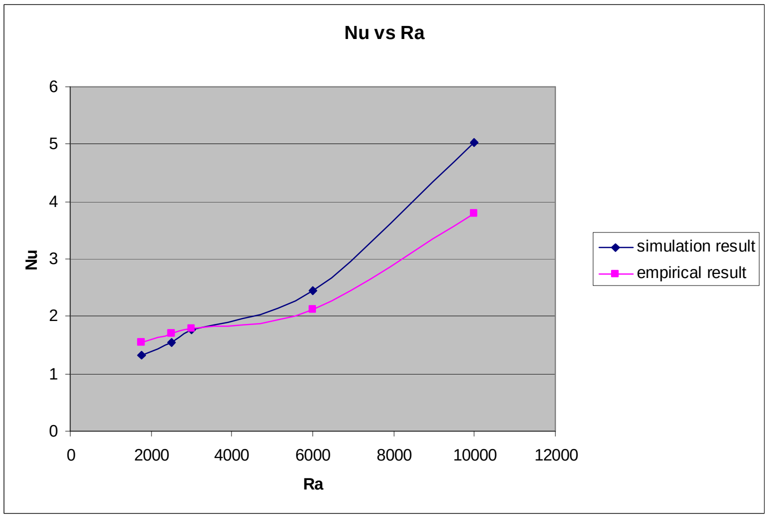

Figure 9. Plot of Nusselt number vs. Rayleigh number

C. Initialization of velocity

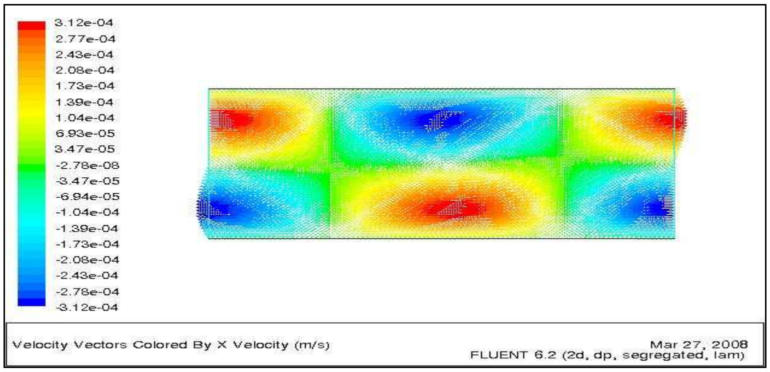

Figure 10. Contour plot of velocity in x direction at Ra = 6000

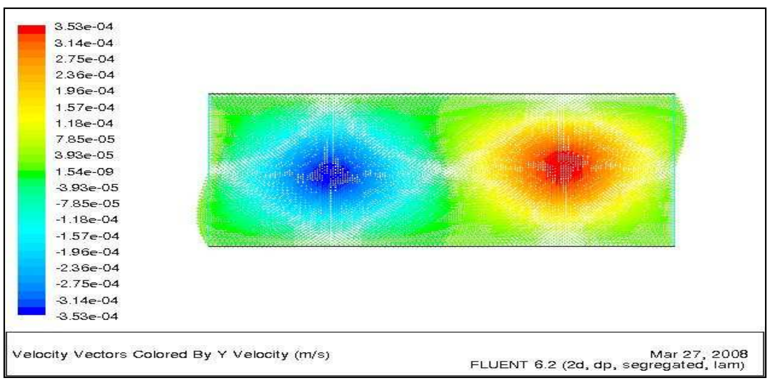

Figure 11. Contour plot of velocity in y direction at Ra = 6000

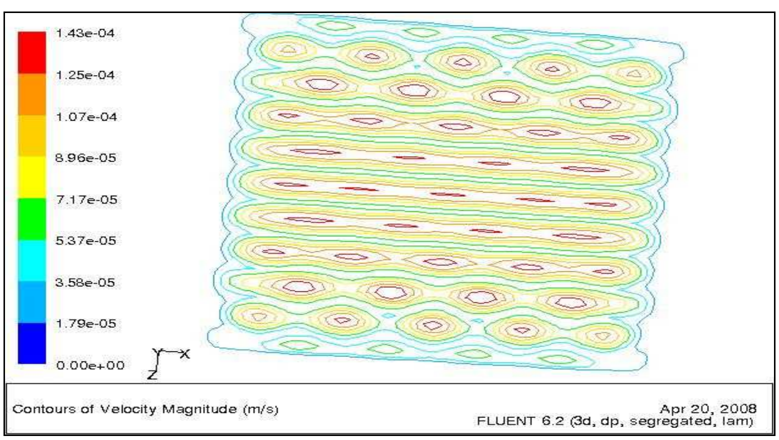

Figure 12. Contour plot of the velocity in the 3 d case showing the cross section of the domain, Ra=6000

V. Conclusion

Raleigh Benard convection was simulated numerically through the commercial package Fluent. The basic aim was to find the effect of aspect ratio change and initial velocity condition on the onset of convection. It was found that increase in the aspect ratio brings down the critical Rayleigh number. But the wavelength of the roll was always consistent with the depth of the fluid layer. The three dimensional simulation showed that along with the x and y component of wavelength it has got a periodicity in the z direction also. The rolls formed in all the cases were square rolls which are the ideal solution of the normal mode solution. The plot of Nusselt number vs. Rayleigh number showed good correspondence between the theoretical and fluent result. For turbulent convection it was found that the plot shows divergence between the theoretical and computational result. A proper choice of turbulence model with appropriate coefficients may solve this problem.

References

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