12

Physical Phenomena Used in Sensors

Tiantian Xie
Auburn University

Bogdan M. Wilamowski
Auburn University

12.1 Introduction

A sensor is used to transform a nonelectrical stimulation to an electrical response that is suitable to be processed by electrical circuits [91W]. Sensors are related with everyday life, such as automobiles, airplanes, radios, and countless other applications [90BR,01RPSW]. Several transformation steps are required before the electric output signal can be generated. These steps involve changes of types of energy where the final step must produce an electrical signal of a desirable format. There are several physical effects that cause generation of electric signals in response to nonelectrical influences. In this chapter, the physical effects behind various sensor applications that can be used for conversion of stimuli into electric signals are introduced, including piezoresistive effect, thermoelectric effect, piezoelectric effect, pyroelectric effect, temperature effect in p-n junction, and Hall effect.

12.2 Piezoresistive Effect

Piezoresistive effect describes the changes of electrical resistance when the material is mechanically deformed. It occurs in crystals that have no polar axes and is well represented in semiconductors. Physically, piezoresistance comes from the anisotropic distribution of energy levels in the *k*-space of the angular wave vector. This phenomenon is successfully employed in sensors that are sensitive to stress.

The relationship between relative changes in resistance $\Delta R/R$ (where R is specific resistivity) and the mechanical stress applied is given by

$$\frac{\Delta R}{R} = \frac{\pi\sigma}{E}$$

(12.1)

where π is the so-called piezoresistive coefficient, which is dependent on the crystal orientation and the conditions of measurement, for example volume constancy.

The relationship between the stress σ and deformation of the material can be presented as

$$\sigma = E\frac{\Delta L}{L} \tag{12.2}$$

where E is Young's modulus of the material.

It is known that the resistance of a conductor can be calculated by

$$R = \frac{L}{W}R_s \tag{12.3}$$

where
  R_s is the unit surface resistance of the material
  L and W are the length and the width of the area, respectively

Considering the volume of the material as a constant, Equation 12.3 can be rewritten as

$$R = \frac{L^2}{A}R_s \tag{12.4}$$

where A is the area of the material.

Differentiating Equation 12.4, the following is obtained:

$$\frac{\Delta R}{\Delta L} = 2R_s\frac{L}{A} \tag{12.5}$$

By combining Equations 12.2, 12.4, and 12.5, the normalized resistance change of the wire can be rewritten as a linear function of the stress σ:

$$\frac{\Delta R}{R} = 2\frac{\Delta L}{L} = 2\frac{\sigma}{E} \tag{12.6}$$

In reality, the piezoresistive coefficients π contain 21 components. In the cubic system, only three of the components, π_{11}, π_{12}, and π_{44} are independent of each other. The same is true of monocrystalline silicon. The values of these coefficients depend on the type of conductor and the dosing level.

Let us derive the equations for $\langle 100 \rangle$ and $\langle 111 \rangle$ surface in a silicon wafer. For $\langle 100 \rangle$ surface of silicon, the changes of resistance can be measured by

$$\begin{aligned}
\frac{\Delta R}{R} = {} & 0.5\big[(\pi_{11}+\pi_{12}+\pi_{44})\sigma_{11}+(\pi_{11}+\pi_{12}-\pi_{44})\sigma_{22}\big]\cos^2\phi \\
& +0.5\big[(\pi_{11}+\pi_{12}-\pi_{44})\sigma_{11}+(\pi_{11}+\pi_{12}+\pi_{44})\sigma_{22}\big]\sin^2\phi \\
& +\pi_{12}\sigma_{33}+(\pi_{11}-\pi_{12})\sigma_{12}\sin(2\phi)+\alpha T
\end{aligned} \tag{12.7}$$

where
  π_{11}, π_{12}, and π_{44} are geometry-dependent constants (see Table 12.1)
  α is the temperature coefficient
  σ_{11}, σ_{22}, σ_{33}, and σ_{12} are geometry-dependent stresses

TABLE 12.1   Parameters for Silicon Wafer

	n-Type [1/TPa]	p-Type [1/TPa]
π_{11}	−1022	66
π_{12}	534	−11
π_{44}	−136	1381
$\pi_{11} + \pi_{12}$	−488	55
$\pi_{11} - \pi_{12}$	−1556	77
B_1	−312	718
B_2	297	−228
B_3	61	−446
C_1	−7	245
C_2	−305	473
C_3	670	615

It can be simplified to

$$\frac{\Delta R}{R} = 0.5\left(\pi_{11} + \pi_{12}\right)\left(\sigma_{11} + \sigma_{22}\right) + \pi_{12}\sigma_{33} + \alpha T$$
$$+ 0.5\pi_{44}\left(\sigma_{11} - \sigma_{22}\right)\cos\left(2\phi\right) + \left(\pi_{11} - \pi_{12}\right)\sigma_{12}\sin\left(2\phi\right) \tag{12.8}$$

For n-type resistors, using the data in Table 12.1,

$$\frac{\Delta R}{R} = -244\left(\sigma_{11} + \sigma_{22}\right) + 534\sigma_{33} + \alpha T - 68\left(\sigma_{11} - \sigma_{22}\right)\cos\left(2\phi\right)$$
$$-1556\sigma_{12}\sin\left(2\phi\right) \tag{12.9}$$

For p-type resistors, using the data in Table 12.1,

$$\frac{\Delta R}{R} = 27\left(\sigma_{11} + \sigma_{22}\right) - 11\sigma_{33} + \alpha T + 690\left(\sigma_{11} - \sigma_{22}\right)\cos\left(2\phi\right)$$
$$+77\sigma_{12}\sin\left(2\phi\right) \tag{12.10}$$

A sum of two perpendicular resistors is not a function of angular location and

$$\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} = 0.5\left(\pi_{11} + \pi_{12}\right)\left(\sigma_{11} + \sigma_{22}\right) + \pi_{12}\sigma_{33} + \alpha T \tag{12.11}$$

A difference is a function of angular location:

$$\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} = \pi_{44}\left(\sigma_{11} - \sigma_{22}\right)\cos\left(2\phi\right) + 2\left(\pi_{11} + \pi_{12}\right)\sigma_{12}\sin\left(2\phi\right) \tag{12.12}$$

To measure $\sigma_{11} - \sigma_{22}$, $\Phi = 0°$ and $90°$ should be used (preferably p-type); while to measure σ_{12}, $\Phi = -45°$ and $45°$ should be used (preferably n-type).

For $\langle 111 \rangle$ surface of silicon,

$$\frac{\Delta R}{R} = \left[B_1\sigma_{11} + B_2\sigma_{22} + B_3\sigma_{33} + 2\sqrt{2}\left(B_2 - B_3\right)\sigma_{23} \right]\cos^2\phi$$

$$+ \left[B_2\sigma_{11} + B_1\sigma_{22} + B_3\sigma_{33} - 2\sqrt{2}\left(B_2 - B_3\right)\sigma_{23} \right]\sin^2\phi$$

$$+ \left[2\sqrt{2}\left(B_2 - B_3\right)\sigma_{13} + \left(B_1 - B_2\right)\sigma_{12} \right]\sin\left(2\phi\right) + \alpha T \tag{12.13}$$

where

$$B_1 = \frac{\pi_{11} + \pi_{12} + \pi_{44}}{2}, \quad B_2 = \frac{\pi_{11} + 5\pi_{12} - \pi_{44}}{6}, \quad B_3 = \frac{\pi_{11} + 2\pi_{12} - \pi_{44}}{3} \tag{12.14}$$

It can be simplified to

$$\frac{\Delta R}{R} = C_1\left(\sigma_{11} + \sigma_{22}\right) + B_3\sigma_{33} + \alpha T + \left[C_2\left(\sigma_{11} - \sigma_{22}\right) + C_3\sigma_{23} \right]\cos\left(2\phi\right)$$

$$+ \left[2C_2\sigma_{12} + C_3\sigma_{13} \right]\sin\left(2\phi\right) \tag{12.15}$$

where

$$C_1 = \frac{2\pi_{11} + 4\pi_{12} + \pi_{44}}{6}, \quad C_2 = \frac{\pi_{11} - \pi_{12} + 2\pi_{44}}{6}, \quad C_3 = 2\sqrt{2}\,\frac{-\pi_{11} + \pi_{12} + \pi_{44}}{6} \tag{12.16}$$

For n-type resistors, using the data in Table 12.1,

$$\frac{\Delta R}{R} = -7\left(\sigma_{11} + \sigma_{22}\right) + 61\sigma_{33} + \alpha T + \left[-305\left(\sigma_{11} - \sigma_{22}\right) + 670\sigma_{23} \right]\cos\left(2\phi\right)$$

$$+ \left[-610\sigma_{12} + 670\sigma_{13} \right]\sin\left(2\phi\right) \tag{12.17}$$

For p-type resistors, using the data in Table 12.1,

$$\frac{\Delta R}{R} = 245\left(\sigma_{11} + \sigma_{22}\right) - 446\sigma_{33} + \alpha T + \left[473\left(\sigma_{11} - \sigma_{22}\right) + 750\sigma_{23} \right]\cos\left(2\phi\right)$$

$$+ \left[94\sigma_{12} + 750\sigma_{13} \right]\sin\left(2\phi\right) \tag{12.18}$$

The sum of two perpendicular resistors is

$$\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} = C_1\left(\sigma_{11} + \sigma_{22}\right) + B_3\sigma_{33} + \alpha T \tag{12.19}$$

For n-type resistor on $\langle 111 \rangle$ surface

$$\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} = -7\left(\sigma_{11} + \sigma_{22}\right) + 61\sigma_{33} + 2000\Delta T \tag{12.20}$$

where σ is in MPa and ΔT is in °C.

For p-type resistor on $\langle 111 \rangle$ surface is less preferable   AQ1

$$\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} = 245(\sigma_{11} + \sigma_{22}) - 446\sigma_{33} + 2000\Delta T \tag{12.21}$$

As shown above, the piezoresistive effect in silicon can be several orders of magnitudes larger than in metals, making it a good member for piezoresistive sensors. However, the silicon is very sensitive to temperature. Additional methods should be adopted to counteract the temperature effect to make the sensor more accurate.

12.3 Thermoelectric Effect

Thermoelectric effect is also called Seebeck effect in honor of East Prussian scientist Thomas Johann Seebeck (1770–1831) [92AE]. He observed that an electrical current was present in a series circuit of two different metals that were contacted and at different temperatures.

When different conductors A and B are connected together, free electrons behave like an ideal gas. For different materials, the energies and densities of free electrons are different. Kinetic energy of electrons is a function of the temperature. At the same temperature, when two different materials contact, free electrons diffuse through the junction (contacting point). The electric potential of the material accepting electrons becomes more negative at the interface, while the material emitting electrons becomes more positive. Different electronic concentrations across the junction set up an electric field that balances the diffusion process until equilibrium is established. When the two materials are at different temperatures, the voltage at the junction can be presented as (Figure 12.1)

$$\frac{dV_{AB}}{dT} = S_A - S_B \tag{12.22}$$

where S_A and S_B are the absolute Seebeck coefficients of the conductors A and B, respectively.

Conversely, when a voltage is applied to the conductors A and B in Figure 12.1, it creates a temperature difference between side A and side B.

The differential Seebeck coefficient $S_{AB} = S_A - S_B$ is called the sensitivity of a thermocouple junction. The Seebeck coefficient is independent on the characteristic of the junction, but only related with the materials. Therefore, to achieve the best sensitivity, the differential Seebeck coefficient of those junction materials should be as large as possible.

12.4 Piezoelectric Effect

Piezoelectric effect is the property of certain crystals that can generate a voltage subjected to a pressure and conversely generate a pressure due to an applied voltage [92DN]. The effect exists in crystals that do not have a symmetrical center.

Each molecule in piezoelectric crystal is polarized: one side is negatively charged while the other side is positively charged, which is also named as a dipole. This is due to the different atoms that make up the molecule, and the way in which the molecules are shaped. The polar axis runs through the center of both charges in the molecule and the molecule is electrically neutral under non-stress conditions. When external force is applied, the lattice is deformed and the electric field is built up; conversely, when the piezoelectric
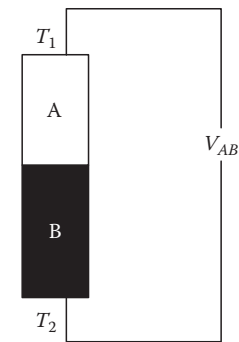

FIGURE 12.1 Voltage of two conductors with different temperatures connected in series.

crystal is under a strong electric field, most of the dipoles in the crystal are forced to line up in nearly the same direction, which results in mechanical stress.

Piezoelectricity is the combined effect of the electrical behavior of the crystal:

$$D = \varepsilon E \tag{12.23}$$

where
  D is the electric charge density displacement (electric displacement)
  ε is the permittivity
  E is the electric field strength

Using Hooke's law,

$$L = s\sigma \tag{12.24}$$

where
  L is strain
  s is compliance
  σ is the stress

By combining Equations 12.7 and 12.8, the piezoelectric effect can be described as

$$\{L\} = [s^E]\{\sigma\} + [d^T]\{E\}$$
$$\{D\} = [d]\{\sigma\} + [\varepsilon^\sigma]\{E\} \tag{12.25}$$

where
  {} and [] standard for vector and matrix separately
  d is the piezoelectric coefficient
  [d] is the matrix for the direct piezoelectric effect
  transposed [d^T] is the matrix for the converse piezoelectric effect
  the superscript E indicates under a zero or constant electric field
  the superscript σ indicates under a zero or constant stress field.

Piezoelectric crystals perform direct conversion between mechanical and electrical energy. Efficiency of the conversion is defined by the coupling coefficients k:

$$k = \sqrt{d\ h} \tag{12.26}$$

where h is the gradient of electric field E multiplied by Young's modulus.

The k-coefficient is an important characteristic for applications where energy efficiency is of prime importance, like in acoustics and ultrasonics.

12.5 Pyroelectric Effect

Pyroelectric effect is the phenomenon of generating a temporary electrical potential when materials are heated or cooled [78ZL]. Different from thermoelectric devices that produce steady voltages, pyroelectric devices generate dynamical charges corresponding to the change of temperatures. So a pyroelectric device is usually used as a heat flow detector rather than a heat detector.

Pyroelectric effect is very tightly connected to the piezoelectric effect. There are several mechanisms that will result in pyroelectricity. Temperature changes cause shortening or elongation of individual dipoles. It may also affect the randomness of the dipole orientations due to thermal agitation. This is the primary pyroelectricity. The second pyroelectricity is induced by the strain in material caused by thermal expansion, which may be described as a result of the piezoelectric effect. Of the classic 32 crystal structures, 10 of these exhibit pyroelectric properties.

The pyroelectric charge coefficient, P_Q, is defined as

$$P_Q = \frac{\partial P_S}{\partial T} \tag{12.27}$$

and the pyroelectric voltage coefficient, P_V, as

$$P_V = \frac{\partial E}{\partial T} \tag{12.28}$$

where
P_S is the spontaneous polarization
E is the electric field strength
T is the absolute temperature
AQ2 C is the parasitic capacitance of the device

By combining Equations 12.27 and 12.28,

$$\frac{P_Q}{P_V} = \frac{\partial P_S}{\partial E} = \varepsilon = \varepsilon_r \varepsilon_0 \tag{12.29}$$

where
ε_0 is the electric permittivity of vacuum
ε_r is the electric permittivity of the materials

If a pyroelectric material is exposed to a heat source, its temperature rises by ΔT and the corresponding charge and voltage changes can be calculated by

$$\Delta Q = P_Q A \Delta T$$
$$\Delta V = P_V L \Delta T \tag{12.30}$$

where A and L is the area and thickness of the material separately.
Since the capacitance is defined as

$$C = \frac{\Delta Q}{\Delta V} = \varepsilon_r \varepsilon_0 \frac{A}{L} \tag{12.31}$$

Integrating Equations 12.13, 12.14, and 12.15, the relationship between ΔV and ΔT can be described as

$$\Delta V = \Delta T \frac{P_Q L}{\varepsilon_0 \varepsilon_r} \tag{12.32}$$

One may notice that the output voltage is proportional to the sensor's temperature change, pyroelectric charge coefficient, and its thickness.

The equivalent electrical circuit of the pyroelectric sensor is shown in Figure 12.2. It consists of three components: the current source generating a heat induced current i; the parasitic capacitance C, and the leakage resistance R.
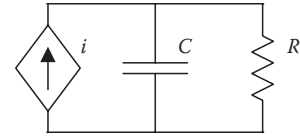
FIGURE 12.2 Equivalent circuit of pyroelectric sensor.

12.6 Photoelectric Effect in Semiconductors

A photoelectric effect is any effect in which light energy is converted to electricity. When light strikes certain light-sensitive materials, it may cause them to give electrons, or change their ability to conduct electricity, or may cause them to develop an electrical potential or voltage across two surfaces.

When a surface is exposed to electromagnetic radiation above a certain threshold frequency, the radiation is absorbed and electrons are emitted. This is called photoelectric effect, which was discovered by A. Einstein. The required photon energy must equal or exceed the energy of a single photon, which is given by

$$E = h\nu \tag{12.33}$$

where
  ν is the frequency of light
  h is Planck's constant equal to 6.63×10^{-34} J·s

Considering the relationship between frequency and the wavelength, the energy in Equation 12.33 can be written as the function of wavelength

$$E = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-7}}{\lambda} [\text{eV} \cdot \text{s}] \tag{12.34}$$

where
  c is the light speed in materials
  λ is the wavelength of light

For Si, the value of E at room temperature is 1.12 eV. However, in a real semiconductor crystal other excitation mechanisms are possible. These include absorption through transitions between the allowed bands and absorption through high levels of distortion in the forbidden band. However, the greatest excitation effect is still provoked by band-to-band absorption.

The electromagnetic radiation can be classified from low frequency to high frequency as infrared, visible, and ultraviolet, respectively. The electromagnetic frequency spectrum is shown in Figure 12.3.
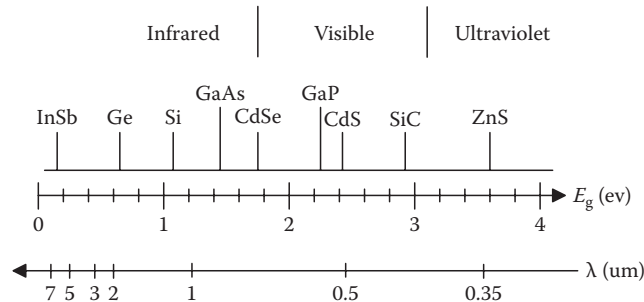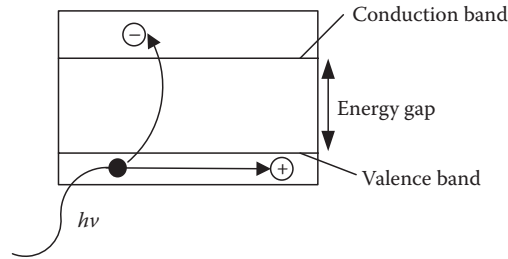
FIGURE 12.3 Electromagnetic frequency spectrum.

FIGURE 12.4 Photoconductive effect in semiconductor.

When light is absorbed by a semiconductor, a current can be induced and thus cause the change of resistance of the material. As shown in Figure 12.4, the semiconductor in thermal equilibrium contains free electrons and holes. The optical field to be detected is incident on and absorbed in the crystal, thereby exciting electrons into the conduction band or, in p-type semiconductors, holes into the valence band. The electronic deficiency thus created is acted upon by the electric field, and its drift along the field direction gives rise to the signal current.

12.7 Photoelectric Effect in p-n Junctions

When light strikes a semiconductor p-n junction, its energy is absorbed by electrons. Electrons and holes generated by light in the p-n junction are swept by junction electrical field. As a result, the current flows though the junction. Figure 12.5 shows the process of generating current in photo diode.

Figure 12.5a is the p-n junction with a depletion layer without light. When the p-n junction is exposed to light, the impinging photons create electron–hole pairs everywhere in n-type area, p-type area, and depletion layer. In the depletion layer, the electric field accelerates electrons toward the n-layer and the holes toward the p-layer (Figure 12.5b).

For the electron–hole pairs generated in the n-layer, the electrons, along with electrons that have arrived from the p-layer, are left in the n-layer conduction band. The holes at this time are being diffused through the n-layer up to the depletion layer while being accelerated, and collected in the p-layer valence band. In this manner, electron–hole pairs that are generated in proportion to the amount of incident light are collected in the n- and p-layers. This results in a positive charge in the p-layer and a negative charge in the n-layer. If an external circuit is connected between the p- and n-layers, electrons will flow away from the n-layer, and holes will flow away from the p-layer toward the opposite respective electrodes (Figure 12.5c). The current is thus generated.

12.8 Temperature Effect in p-n Junctions

The temperature effect of p-n junction can be utilized as a temperature sensor. The principal sensor is straightforward. The *I–V* characteristic of the diode is as follows:

$$I_D = I_S \left(\exp\left(\frac{qV_D}{kT} \right) - 1 \right)$$
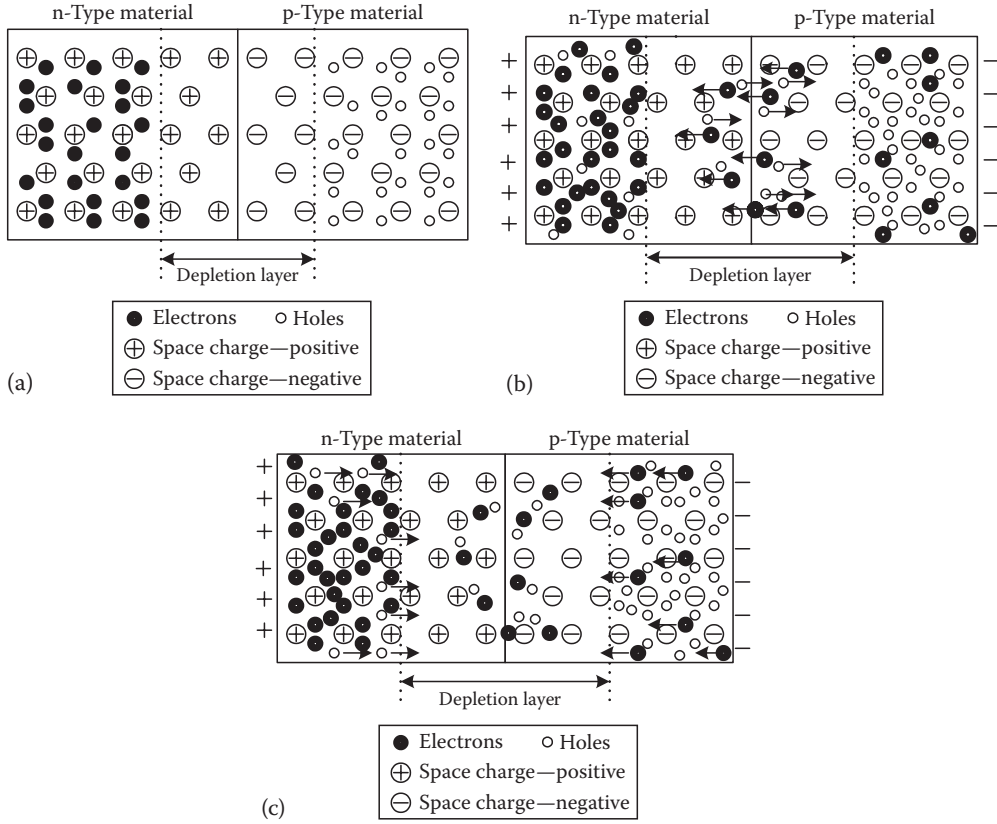
(12.35)

where
  V_D is the applied voltage
  q is electronic charge equal to 1.60×10^{-19} C
  k is Boltzman's constant equal to 1.38×10^{-23} J/K
  T is absolute temperature
  I_S is the reverse saturation current of diode

FIGURE 12.5   Photo diode: (a) p-n junction; (b) carrier generation in depletion layer associated with carrier sweep in electric field of depletion layer (fast process); (c) carrier generation in bulk material associated with minority carrier diffusion toward junction (slow process).

$$I_S = Aqn_i^2 \left(\frac{1}{\displaystyle\int_0^{x_n} (N/D_p)dx} + \frac{1}{\displaystyle\int_0^{x_p} (N/D_n)dx} \right) \tag{12.36}$$

where n_i is calculated by

$$n_i^2 = N_v N_c T^3 \exp\left(-\frac{E_g}{kT} \right) \tag{12.37}$$

By combining Equations 12.12 through 12.14, the voltage of the diode can be calculated by

$$V_D = \frac{kT}{q} \ln\left(\frac{I_D}{I_S} + 1 \right) \cong \frac{kT}{q} \ln\left(\frac{I_D}{I_S} \right) \tag{12.38}$$

Since I_S is proportional to n_i^2 (Equation 12.13), taking the derivative with respect to temperature yields
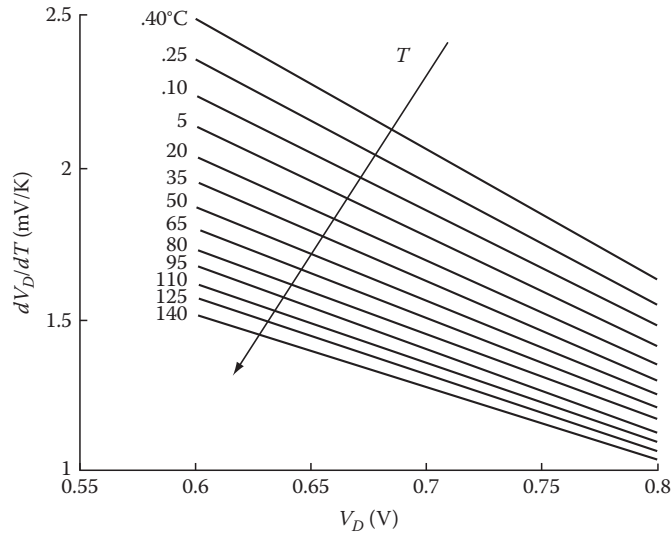
FIGURE 12.6 Relationship between dV_D/dT and V_D for a silicon diode.

$$\frac{\partial V_D}{\partial T} = \frac{k}{q}\ln\left(\frac{I_D}{I_S}\right) - \frac{kT}{q}\frac{1}{I_S}\frac{\partial I_S}{\partial T} = \frac{V_D - (3kT/q) - (E_g/q)}{T} \tag{12.39}$$

where E_g is the semiconductor bandgap energy. The relationship between dV_D/dT and V_D for a silicon diode is shown in Figure 12.6. dV_D/dT decreases with temperature increasing.

With a constant current applied, the voltage across a diode or p-n junction will decrease by approximately 1–2 mV/°C. The diode voltage vs. temperature can be characterized by placing the amplifier in a temperature chamber with a constant current applied to the diode junction.

12.9 Hall Effect

The Hall effect was discovered in 1879 in Johns Hopkins University by E.H. Hall. The effect is based on the interaction between moving electric carriers and an external magnetic field. When an electron moves through a magnetic field, it acts a sideways force

$$F = qvB \tag{12.40}$$

where

    q is an electronic charge
    v is the speed of an electron
    B is the magnetic field

When the electric current source I is perpendicular to the magnetic field B, as shown in Figure 12.7, the so called Hall voltage, V_H, is produced in the direction perpendicular to both B and I. V_H is given by
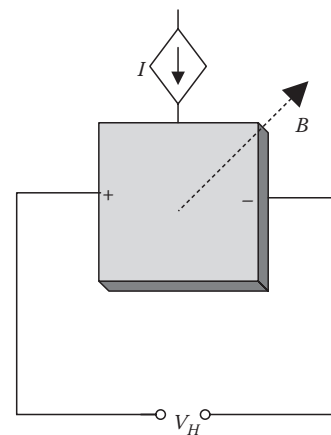

FIGURE 12.7 Principle of Hall effect.

$$V_H = \frac{IB}{qnd} \tag{12.41}$$

where
    d is the thickness of the hall plate
    n is the carrier density

One very important feature of the Hall effect is that it differentiates between positive charges moving in one direction and negative charges moving in the opposite.

Hall effect devices produce a very low signal level and thus require amplification. The Hall sensors can be used to detect magnetic fields, position, and displacement of objects.

12.10 Conclusion

The physical phenomena described above are frequently used in various sensors. For example, piezoresistive effect and piezoelectric effect can be used to measure both pressure and acceleration. Thermoelectric effect, pyroelectric effect, and temperature effect in p-n junction are usually applied in temperature sensors, bolometers, and so on. Photoelectric effect is used in light detectors such as photodiodes and thermal detectors. Hall effect is always used in measuring magnetic fields and sensing position and motion.

References

[91W]. White, R. W., A sensor classification scheme, In *Microsensors*, IEEE Press, New York, pp. 3–5, 1991.

[90BR]. Barshan, B. and Kuc, R., Differentiating sonar reflections from corners and planes by employing an intelligent sensor, *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 1990(12), 560–569.

[01RPSW]. Leonhard, M. R., Alfred, P., Gerd, S., and Robert, W., SAW-based radio sensor systems, *IEEE Sensors Journal*, 2001(1), 69–78.

[92AE]. Boyer, A. and Cisse, E., Properties of thin films thermoelectric materials: Application to sensors using the Seebeck effect, *Materials Science & Engineering*, 1992(13), 103–111.

[92DN]. Damjanovic, D. and Newnham, R. E., Electrostrictive and piezoelectric materials for actuator applications, *Journal of Intelligent Material Systems and Structures*, 1992(3), 190–208.

[78ZL]. Zook, J. D. and Liu, S. T., Pyroelectric effects in thin film, *Journal of Applied Physics*, 1978(49), 4604.