

## Further Information

Tabulations of representative standard filter specification functions appear in the sources in the References by Schaumann *et al.* [1990] and Bowron and Stephenson [1979], but more extensive tabulations, including prototype passive filter component values, are given in A. I. Zverev, *Handbook of Filter Synthesis* (New York: John Wiley, 1967). More generally, the Schaumann text provides an admirable, up-to-date coverage of filter design with an extensive list of references.

The field of active filter design remains active, and new developments appear in *IEEE Transactions on Circuits and Systems* and *IEE Proceedings Part G (Circuits and Systems)*. The IEE publication *Electronic Letters* provides for short contributions. A number of international conferences (whose proceedings can be borrowed through technical libraries) feature active filter and related sessions, notably the *IEEE International Symposium on Circuits and Systems* (ISCAS) and the *European Conference on Circuit Theory and Design* (ECCTD).

## 2 Realization

J. W. Steadman and B. M. Wilamowski

After the appropriate low-pass form of a given filter has been synthesized, the designer must address the realization of the filter using **operational amplifiers**. If the required filter is not low-pass but high-pass, bandpass, or bandstop, transformation of the prototype function is also required [Budak, 1974; Van Valkenburg, 1982]. While a detailed treatment of the various transformations is beyond the scope of this work, most of the filter designs encountered in practice can be accomplished using the techniques given here.

When the desired filter function has been determined, the corresponding electronic circuit must be designed. Many different circuits can be used to realize any given transfer function. For purposes of this handbook, we present several of the most popular types of realizations. Much more detailed information on various circuit realizations and the advantages of each may be found in the literature, in particular Van Valkenburg [1982], Huelseman and Allen [1980], and Chen [1986]. Generally the design trade-offs in making the choice of circuit to be used for the realization involve considerations of the number of elements required, the sensitivity of the circuit to changes in component values, and the ease of tuning the circuit to given specifications. Accordingly, limited information is included about these characteristics of the example circuits in this section.

Each of the circuits described here is commonly used in the realization of **active filters**. When implemented as shown and used in the appropriate gain and bandwidth specifications of the amplifiers, they will provide excellent performance. Computer-aided filter design programs are available which simplify the process of obtaining proper element values and simulation of the resulting circuits [Krobe *et al.*, 1989; Wilamowski *et al.*, 1992].

### Transformation from Low-Pass to Other Filter Types

To obtain a high-pass, bandpass, or bandstop filter function from a low-pass prototype, one of two general methods can be used. In one of these, the circuit is realized and then individual circuit elements are replaced by other elements or subcircuits. This method is more useful in **passive filter** designs and is not discussed further here. In the other approach, the transfer function of the low-pass prototype is transformed into the required form for the desired filter. Then a circuit is chosen to realize the new filter function. We give a brief description of the transformation in this section, then give examples of circuit realizations in the following sections.

$$S - S_j \rightarrow \frac{(s - s_3)(s - s_4)}{H(s - s_1)(s - s_2)} \quad (28.19)$$

where

$$H = \frac{1}{S_j}; s_{1,2} = \pm j\omega_c; s_{3,4} = \omega_c(\beta \pm \sqrt{\beta^2 - 1}); \text{ and } \beta = \frac{B}{2\omega_c S_j} \quad (28.20)$$

Once the desired transfer function has been obtained through obtaining the appropriate low-pass prototype and transformation, if necessary, to the associated high-pass, bandpass or bandstop function, all that remains is to obtain a circuit and the element values to realize the transfer function.

## Circuit Realizations

Various electronic circuits can be found to implement any given transfer function. Cascade filters and ladder filters are two of the basic approaches for obtaining a practical circuit. Cascade realizations are much easier to find and to tune, but ladder filters are less sensitive to element variations. In cascade realizations, the transfer function is simply factored into first- and second-order parts. Circuits are built for the individual parts and then cascaded to produce the overall filter. For simple to moderately complex filter designs, this is the most common method, and the remainder of this section is devoted to several examples of the circuits used to obtain the first- and second-order filters. For very high-order transfer functions, ladder filters should be considered, and further information can be obtained by consulting the literature.

In order to simplify the circuit synthesis procedure, very often  $\omega_0$  is assumed to be equal to one and then after a circuit is found, the values of all capacitances in the circuit are divided by  $\omega_0$ . In general, the following magnitude and frequency transformations are allowed:

$$R_{\text{new}} = K_M R_{\text{old}} \text{ and } C_{\text{new}} = \frac{1}{K_F K_M} C_{\text{old}} \quad (28.21)$$

where  $K_M$  and  $K_F$  are magnitude and frequency scaling factors, respectively.

Cascade filter designs require the transfer function to be expressed as a product of first- and second-order terms. For each of these terms a practical circuit can be implemented. Examples of these circuits are presented in Figs. 28.12–28.22. In general the following first- and second-order terms can be distinguished:

(a) First-order low-pass:

$$T(s) = \frac{H\omega_0}{s + \omega_0}$$

Assumption :  $r_1 = 1$

$$c_1 = \frac{1}{\omega_0} \quad r_2 = |H| \omega_0$$

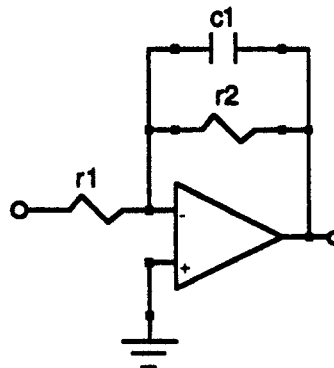


FIGURE 28.12 First-order low-pass filter.

### Low-Pass to High-Pass Transformation

Suppose the desired filter is, for example, a high-pass Butterworth. Begin with the low-pass Butterworth transfer function of the desired order and then *transform* each pole of the original function using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{Hs}{s - s_j} \quad (28.12)$$

which results in one complex pole and one zero at the origin for each pole in the original function. Similarly, each zero of the original function is transformed using the formula

$$S - S_j \rightarrow \frac{s - s_j}{Hs} \quad (28.13)$$

which results in one zero on the imaginary axis and one pole at the origin. In both equations, the scaling factors used are

$$H = \frac{1}{S_j} \quad \text{and} \quad s_j = \frac{\omega_0}{S_j} \quad (28.14)$$

where  $\omega_0$  is the desired cut-off frequency in radians per second.

### Low-Pass to Bandpass Transformation

Begin with the low-pass prototype function in factored, or *pole-zero*, form. Then each pole is transformed using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{Hs}{(s - s_1)(s - s_2)} \quad (28.15)$$

resulting in one zero at the origin and two conjugate poles. Each zero is transformed using the formula

$$S - S_j \rightarrow \frac{(s - s_1)(s - s_2)}{Hs} \quad (28.16)$$

resulting in one pole at origin and two conjugate zeros. In Eqs. (28.15) and (28.16)

$$H = -B; \quad s_{1,2} = \omega_c \left( \alpha \pm \sqrt{\alpha^2 - 1} \right); \quad \text{and} \quad \alpha = \frac{BS_j}{2\omega_c} \quad (28.17)$$

where  $\omega_c$  is the center frequency and  $B$  is the bandwidth of the bandpass function.

### Low-Pass to Bandstop Transformation

Begin with the low-pass prototype function in factored, or *pole-zero*, form. Then each pole is transformed using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{H(s - s_1)(s - s_2)}{(s - s_3)(s - s_4)} \quad (28.18)$$

transforming each pole into two zeros on the imaginary axis and into two conjugate poles. Similarly, each zero is transformed into two poles on the imaginary axis and into two conjugate zeros using the formula

This filter is inverting, i.e.,  $H$  must be negative, and the scaling factors shown in Eq. (28.21) should be used to obtain reasonable values for the components.

(b) First-order high-pass:

$$T(s) = \frac{Hs}{s + \omega_0}$$

Assumption :  $r_1 = 1$

$$c_1 = \frac{1}{\omega_0} \quad r_2 = |H|$$

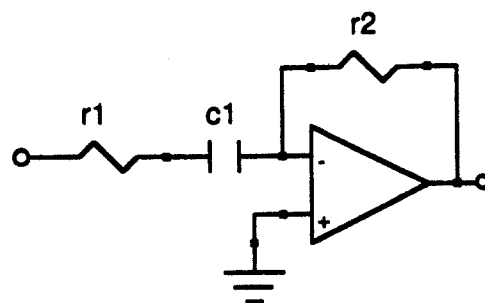


FIGURE 28.13 First-order high-pass filter.

This filter is inverting, i.e.,  $H$  must be negative, and the scaling factors shown in Eq. (28.21) should be used to obtain reasonable values for the components.

While several passive realizations of first-order filters are possible (low-pass, high-pass, and lead-lag), the active circuits shown here are inexpensive and avoid any loading of the other filter sections when the individual circuits are cascaded. Consequently, these circuits are preferred unless there is some reason to avoid the use of the additional operational amplifier. Note that a second-order filter can be realized using one operational amplifier as shown in the following paragraphs, so it is common practice to choose even-order transfer functions, thus avoiding the use of any first-order filters.

(c) There are several second-order low-pass circuits:

$$T(s) = \frac{H\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Assumption :  $r_1 = r_2 = 1$

$$c_1 = \frac{2Q}{\omega_0} \quad c_2 = \frac{1}{2Q\omega_0}$$

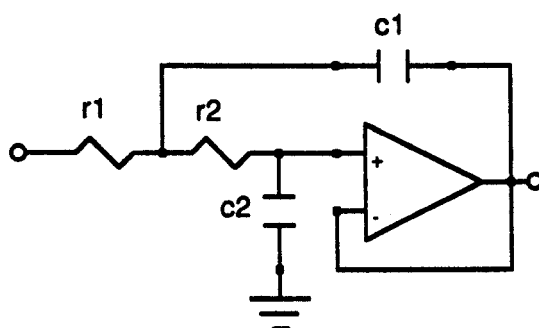


FIGURE 28.14 Second-order low-pass Sallen-Key filter.

This filter is noninverting and unity gain, i.e.,  $H$  must be one, and the scaling factors shown in Eq. (28.21) should be used to obtain reasonable element values. This is a very popular filter for realizing second-order functions because it uses a minimum number of components and since the operation amplifier is in the unity gain configuration it has very good bandwidth.

Another useful configuration for second-order low-pass filters uses the operational amplifier in its inverting "infinite gain" mode as shown in Fig. 28.15.

$$T(s) = \frac{H\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\text{Assumption : } r_1 = r_2 = r_3 = 1$$

$$c_1 = \frac{3Q}{\omega_0} \quad c_2 = \frac{1}{3Q\omega_0}$$

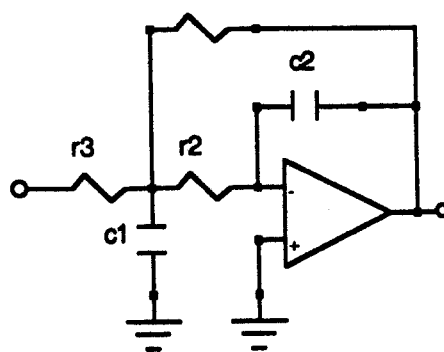


FIGURE 28.15 Second-order low-pass filter using the inverting circuit.

This circuit has the advantage of relatively low sensitivity of  $\omega_0$  and  $Q$  to variations in component values. In this configuration the operational amplifier's gain-bandwidth product may become a limitation for high- $Q$  and high-frequency applications [Budak, 1974]. There are several other circuit configurations for low-pass filters. The references given at the end of the section will guide the designer to alternatives and the advantages of each.

(d) Second-order high-pass filters may be designed using circuits very much like those shown for the low-pass realizations. For example, the Sallen-Key low-pass filter is shown in Fig. 28.16.

$$T(s) = \frac{Hs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\text{Assumption : } r_3 = 1$$

$$c_1 = c_2 = 1$$

$$r_1 = r_2 = \frac{1}{\omega_0} \quad r_4 = 2 - \frac{1}{Q}$$

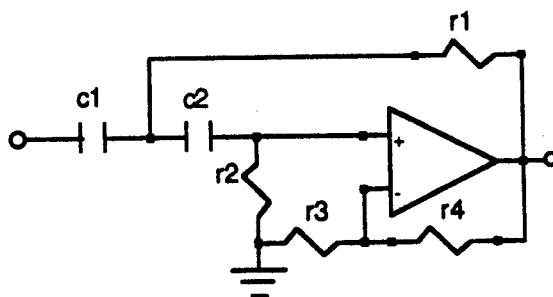


FIGURE 28.16 A second-order high-pass Sallen-Key filter.

As in the case of the low-pass Sallen-Key filter, this circuit is noninverting and requires very little gain from the operational amplifier. For low to moderate values of  $Q$ , the **sensitivity functions** are reasonable and the circuit performs well.

The inverting *infinite gain* high-pass circuit is shown in Fig. 28.17 and is similar to the corresponding low-pass circuit.

$$T(s) = \frac{Hs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\text{Assumption : } r_1 = 1$$

$$r_2 = 9Q^2 \quad c_1 = c_2 = c_3 = \frac{1}{3Q^2}$$

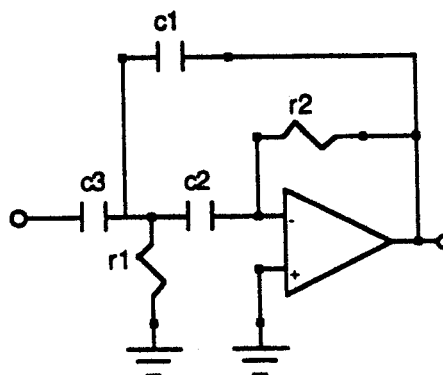


FIGURE 28.17 An inverting second-order high-pass circuit.

This circuit has relatively good sensitivity figures. The principal limitation occurs with high- $Q$  filters since this requires a wide spread of resistor values.

Both low-pass and high-pass frequency response circuits can be achieved using three operational amplifier circuits. Such circuits have some sensitivity function and tuning advantages but require far more components. These circuits are used in the sections describing bandpass and bandstop filters. The designer wanting to use the three-operational-amplifier realization for low-pass or high-pass filters can easily do this using simple modifications of the circuits shown in the following sections.

(e) Second-order bandpass circuits may be realized using only one operational amplifier. The Sallen-Key filter shown in Fig. 28.18 is one such circuit.

$$T(s) = \frac{H \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Assumption :  $c_1 = c_2 = 1$ ;  $r_5 = 1$

$$r_2 = r_3 = \frac{\sqrt{2}}{\omega_0} \quad r_1 = \frac{\frac{4Q}{\sqrt{2}} - 1}{H} \frac{\sqrt{2}}{\omega_0}$$

$$r_4 = \frac{\frac{4Q}{\sqrt{2}} - 1}{\frac{4Q}{\sqrt{2}} - 1 - H} \frac{\sqrt{2}}{\omega_0} \quad r_6 = 3 - \frac{\sqrt{2}}{\omega_0}$$

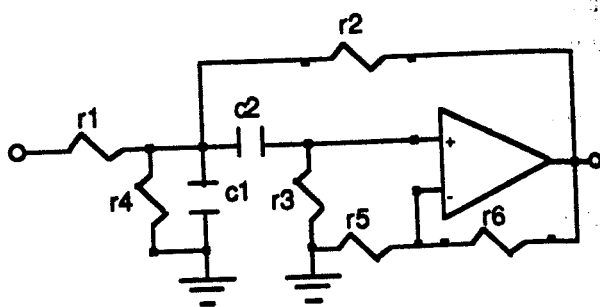


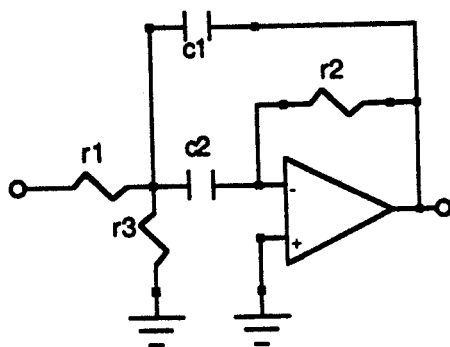
FIGURE 28.18 A Sallen-Key bandpass filter.

This is a noninverting amplifier which works well for low- to moderate- $Q$  filters and is easily tuned [Budak, 1974]. For high- $Q$  filters the sensitivity of  $Q$  to element values becomes high, and alternative circuits are recommended. One of these is the bandpass version of the inverting amplifier filter as shown in Fig. 28.19.

$$T(s) = \frac{H \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

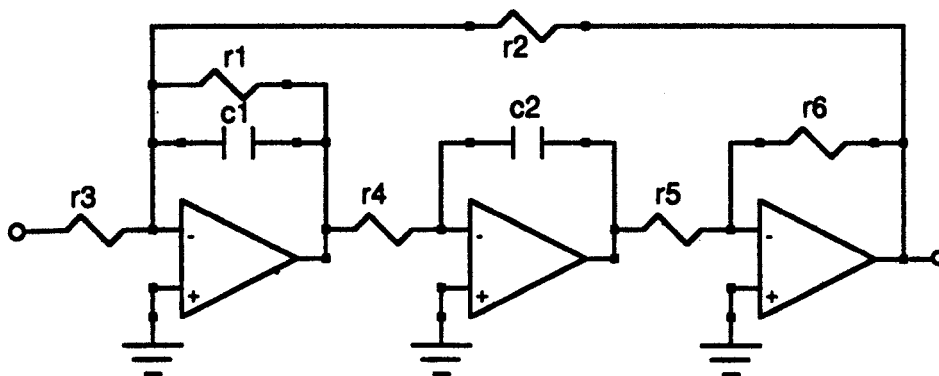
Assumption :  $c_1 = c_2 = \frac{1}{2Q\omega_0}$

$$r_1 = \frac{2Q^2}{H} \quad r_2 = 4Q^2 \quad r_3 = \frac{1}{1 - \frac{H}{2Q^2}}$$



This circuit has few components and relatively small sensitivity of  $\omega_0$  and  $Q$  to variations in element values. For high- $Q$  circuits, the range of resistor values is quite large as  $r_1$  and  $r_2$  are much larger than  $r_3$ .

When ease of tuning and small sensitivities are more important than the circuit complexity, the three-operational-amplifier circuit of Fig. 28.20 may be used to implement the bandpass transfer function.



$$T(s) = \frac{H \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \quad c_1 = c_2 = \frac{1}{\omega_0} \quad r_1 = Q \quad r_2 = r_4 = r_5 = r_6 = 1 \quad r_3 = \frac{Q}{|H|}$$

FIGURE 28.20 The three-operational-amplifier bandpass filter.

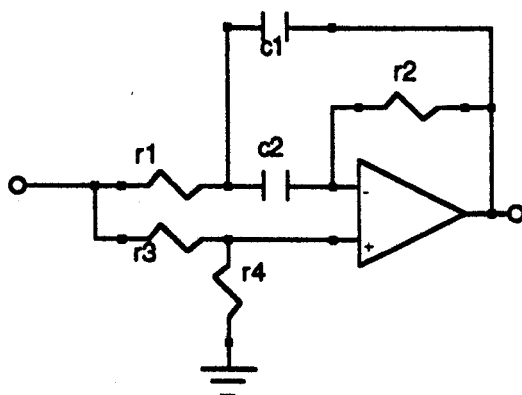
The filter as shown in Fig. 28.20 is inverting. For a noninverting realization, simply take the output from the middle amplifier rather than the right one. This same configuration can be used for a three-operational-amplifier low-pass filter by putting the input into the summing junction of the middle amplifier and taking the output from the left operational amplifier. Note that  $Q$  may be changed in this circuit by varying  $r_1$  and that this will not alter  $\omega_0$ . Similarly,  $\omega_0$  can be adjusted by varying  $c_1$  or  $c_2$  and this will not change  $Q$ . If only variable resistors are to be used, the filter can be tuned by setting  $\omega_0$  using any of the resistors other than  $r_1$  and then setting  $Q$  using  $r_1$ .

(f) Second-order bandstop filters are very useful in rejecting unwanted signals such as line noise or carrier frequencies in instrumentation applications. Such filters are implemented with methods very similar to the bandpass filters just discussed. In most cases, the frequency of the zeros is to be the same as the frequency of the poles. For this application, the circuit shown in Fig. 28.21 can be used.

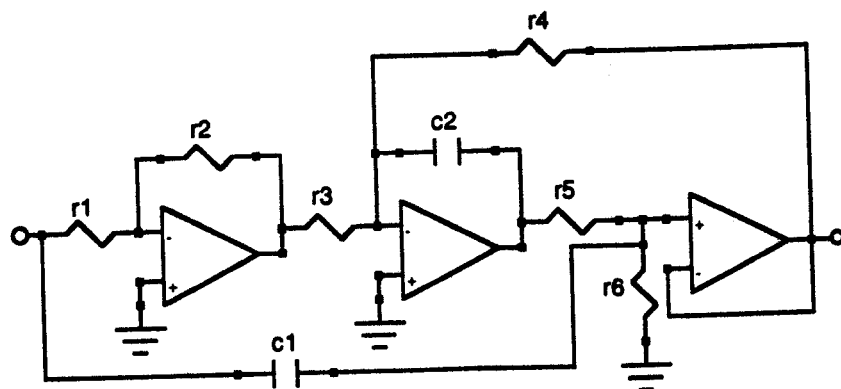
$$T(s) = \frac{H(s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

Assumption :  $c_1 = c_2 = 1$

$$r_1 = \frac{1}{2Q\omega_0} \quad r_3 = \frac{1}{Q\omega_0} \quad r_2 = r_4 = \frac{2Q}{\omega_0}$$



The primary advantage of this circuit is that it requires a minimum number of components. For applications where no tuning is required and the  $Q$  is low, this circuit works very well. When the bandstop filter must be tuned, the three-operational-amplifier circuit is preferable.



$$T(s) = \frac{H(s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad c_1 = c_2 = \frac{1}{\omega_0} \quad r_1 = 1 \quad r_2 = H \quad r_5 = r_6 = 2Q \quad r_3 = \frac{H\omega_0^2}{2Q\omega_z^2} \quad r_4 = \frac{1}{2Q}$$

FIGURE 28.22 A three-operational-amplifier bandstop filter.

The foregoing circuits provide a variety of useful first- and second-order filters. For higher-order filters, these sections are simply cascaded to realize the overall transfer function desired. Additional detail about these circuits as well as other circuits used for active filters may be found in the references.

## Defining Terms

- Active filter:** A filter circuit which uses active components, usually operational amplifiers.
- Filter:** A circuit which is designed to be frequency selective. That is, the circuit will emphasize or "pass" certain frequencies and attenuate or "stop" others.
- Operational amplifier:** A very high-gain differential amplifier used in active filter circuits and many other applications. These monolithic integrated circuits typically have such high gain, high input impedance, and low output impedance that they can be considered "ideal" when used in active filters.
- Passive filter:** A filter circuit which uses only passive components, i.e., resistors, inductors, and capacitors. These circuits are useful at higher frequencies and as prototypes for ladder filters that are active.
- Sensitivity function:** A measure of the fractional change in some circuit characteristic, such as center frequency, to variations in a circuit parameter, such as the value of a resistor. The sensitivity function is normally defined as the partial derivative of the desired circuit characteristic with respect to the element value and is usually evaluated at the nominal value of all elements.

## References

- A. Budak, *Passive and Active Network Analysis and Synthesis*, Boston: Houghton Mifflin, 1974.
- W.K. Chen, *Passive and Active Filters, Theory and Implementations*, New York: Wiley, 1986.



- M.R. Krobe, J. Ramirez-Angulo, and E. Sanchez-Sinencio, "FIESTA—A filter educational synthesis teaching aid," *IEEE Trans. on Education*, vol. 12, no. 3, pp. 280–286, August 1989.
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- B.M. Wilamowski, S.F. Legowski, and J.W. Steadman, "Personal computer support for teaching analog filter analysis and design," *IEEE Trans. on Education*, vol. 35, no. 4, November 1992.

## Further Information

The monthly journal *IEEE Transactions on Circuits and Systems* is one of the best sources of information on new active filter functions and associated circuits.

The British journal *Electronics Letters* also often publishes articles about active circuits.

The *IEEE Transactions on Education* has carried articles on innovative approaches to active filter synthesis as well as computer programs for assisting in the design of active filters.